On the Renormalization of the Minimal Supersymmetric Standard Model

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Abstract:
The renormalization of the Minimal Supersymmetric Standard Model is discussed. In particular we focus on the soft supersymmetry breaking sector of the MSSM and comment on non-renormalization theorems.

Introduction

The Minimal Supersymmetric Standard Model (MSSM) [1] is certainly the best motivated and conceptually most elaborated extension of the Standard Model (SM). Both models can be studied with very high precision at future e^+e^- colliders [2]. To account for the high experimental accuracy the full $\mathcal{O}(\alpha)$ corrections for most particle reactions and even the full $\mathcal{O}(\alpha^2)$ in specific cases have to be included into the theoretical predictions.

However in supersymmetric theories like the MSSM, no regularization method is known which is mathematically consistent (that obeys the quantum action principle) and preserves all symmetries. Particularly dimensional regularization, which is mathematically consistent in the form of Ref. [3], violates supersymmetry and chiral symmetry. For consistency such symmetry breakings have to be restored by adding suitable (non-invariant) counterterms whose values are determined by the algebraic method. On the other hand dimensional reduction [5] which is often used in phenomenological applications suffers from the fact that it is mathematically not well defined [6]. Explicit examples of inconsistencies are known at the three-loop level [7] and, best to our knowledge, even at one loop it is not yet completely proven that dimensional reduction conserves all symmetries.

The restoration of symmetries is in practise an extremely complicated and time consuming work [4]. From the abstract point of view, the Slavnov-Taylor (ST) identity can be broken in the procedure of renormalization

$$\mathcal{S}(\Gamma) = \Delta. \tag{1}$$

Owing to the quantum action principle, Δ consists of local field polynomials with dimension $D \leq 4$ and Faddeev-Popov charge $Q_{\Phi\Pi} = 1$ (see e.g. Ref. [8] for details) satisfying the equation

$$\mathcal{S}_\Gamma \Delta = 0. \tag{2}$$

If the cohomology problem

$$\mathcal{S}_\Gamma \hat{\Delta} = \Delta \tag{3}$$

can be solved, the theory is free of anomalies and the ST identity can be restored by adding the symmetry-restoring counterterms $-\hat{\Delta}$ to the vertex function Γ:

$$\mathcal{S}(\Gamma - \hat{\Delta}) = \mathcal{S}(\Gamma) - \mathcal{S}_\Gamma \hat{\Delta} = \Delta - \mathcal{S}_\Gamma \hat{\Delta} = 0. \tag{4}$$

In case of the MSSM the algebraic analysis, which is the necessary prerequisite for the application in explicit calculations, has been worked out in Ref. [8]. All symmetry requirements that have to be respected to all orders in perturbation theory have been explicitly given. Furthermore, the symmetric counterterms of the MSSM have been systematically constructed, a complete set renormalization conditions has been defined, and infrared finiteness has been proven by power-counting arguments.

In the following we want to concentrate on the soft supersymmetry breaking part of the MSSM and briefly discuss non-renormalization theorems at the end.

Soft supersymmetry breaking

In realistic models like the MSSM supersymmetry has to be broken owing to the large mass difference between SM particles and their superpartners and to trigger the spontaneous breakdown of the $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge group. However in a quantum theory even a (softly) broken supersymmetry has to be maintained in a mathematically consistent way in higher orders. In order to fomulate symmetry identities that take into account soft supersymmetry breaking we couple the soft symmetry breaking part of the MSSM to external fields that possess finite vacuum expectation values. Two kinds of external fields are used in the literature.

In Ref. [9] a BRS doublet u, v has been used, where the field v has a finite vacuum expecation value v_A, and the field u is a Faddeev-Popov ghost. The special form of the BRS transformation law of BRS doublets

$$\mathbf{s}u = v, \qquad \mathbf{s}v = 0 \tag{5}$$

accounts for some extraordinary properties (see e.g. Ref. [10]):

- BRS doublets do not contribute to anomalies of the ST identity.
- Their contribution to the action can be written as BRS variations that fulfil the ST identity due to the nilpotency of the ST operator. The action and ST identity can be decomposed as follows (see e.g. Ref. [10]):

$$\Gamma = \Gamma|_{u,v=0} + \int \mathrm{d}^4x \, v \frac{\delta}{\delta u} \tilde{\Delta}(u, v), \tag{6}$$

$$\mathcal{S}(\Gamma) = \mathcal{S}^0(\Gamma) + \int \mathrm{d}^4x \, v \frac{\delta}{\delta u} \Gamma. \tag{7}$$

If we assume that the ST identity for $u, v = 0$ is already solved

$$\mathcal{S}(\Gamma|_{u,v=0}) = 0, \tag{8}$$

and if we add the symmetry-restoring counterterm $\mathcal{S}^0_\Gamma\tilde{\Delta}(u,v)$ to the action:

$$\Gamma \to \Gamma = \Gamma|_{u,v=0} + \mathcal{S}^0_\Gamma\tilde{\Delta}(u,v) + \int \mathrm{d}^4x\, v\frac{\delta}{\delta u}\tilde{\Delta} = \Gamma|_{u,v=0} + \mathcal{S}_\Gamma\tilde{\Delta}(u,v), \tag{9}$$

the full ST identity is fulfilled:

$$\mathcal{S}(\Gamma) = 0. \tag{10}$$

Owing to the nilpotency of the ST operator, $\tilde{\Delta}(u,v)$ does not contribute to the ST identity at all. Thus, the cohomology problem of the soft supersymmetry breaking part of the MSSM is trivial. In the doublet approach, however, it is not possible to distinguish physical soft breakings from unphysical breakings. Indeed, in Ref. [10] an additional R symmetry has to be used to exclude the soft breaking of gauge symmetry as for example a gauge boson mass. Then one remains with a restricted class of soft breaking terms, which again include unphysical as well as physical parameters. Those couplings that contribute to soft supersymmetry breaking terms are clearly important for the calculation of observables since they are (at least partly) determined by the renormalization condition of physical mass parameters. All other couplings are unphysical in the sense that their renormalization conditions do not effect physical observables. These unphysical couplings can be savely neglected.[1]

In a second approach [8,11,12] the soft supersymmetry breaking terms are generated from a massless external chiral superfield called spurion

$$\hat{A} = e^{-\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu}\left[A + \sqrt{2}\theta^\alpha a_\alpha + \theta^2(F_A + v_A)\right], \tag{11}$$

where the θ^2 component has an finite real vacuum expectation value v_A. In this way all soft supersymmetry breaking terms of the MSSM are generated. For instance, the gaugino mass originates from (see Ref. [8] for notations)

$$-\frac{1}{128g'^2}\int \mathrm{d}^6z\,\hat{A}\hat{F}'^\alpha\hat{F}'_\alpha = \frac{v_A}{2}\int \mathrm{d}^4x\,\lambda'^\alpha\lambda'_\alpha + \text{spurion terms}. \tag{12}$$

Since the spurion is dimensionless it can appear in arbitrary powers $\hat{A}^n$ in the effective action yielding an infinite number of new coupling constants and counterterms and, hence, it is not clear if such a theory is predictive at all. However it has been show in Ref. [12] that indeed only a small number of the spurion couplings are physically relevant and contribute in the limit of vanishing spurion fields.

In the following we want to show that the renormalization of the spurion part of the MSSM is trivial and can be traced back to the case of two BRS doublets. The spurion superfield obeys the following BRS transformations:

$$\mathbf{s}A = \sqrt{2}\epsilon^\alpha a_\alpha - \mathrm{i}\xi^\mu\partial_\mu A, \tag{13}$$

$$\mathbf{s}a_\alpha = \sqrt{2}\epsilon_\alpha(F_A + v_A) + \mathrm{i}\sqrt{2}\sigma^\mu_{\alpha\dot{\alpha}}\bar{\epsilon}^{\dot{\alpha}}\partial_\mu A - \mathrm{i}\xi^\mu\partial_\mu a_\alpha, \tag{14}$$

$$\mathbf{s}F_A = \mathrm{i}\sqrt{2}\partial_\mu a^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\epsilon}^{\dot{\alpha}} - \mathrm{i}\xi^\mu\partial_\mu F_A. \tag{15}$$

[1] It should be noted that both the symmetry breaking Δ and the symmetry-restoring counterterm $\hat{\Delta}$ involve in general all kinds of couplings related to u, v, but in a regularization-scheme dependent way. Therefore, this dependence is purely artificial and does not effect physical observables.

If we decompose the Weyl spinor a_α into two auxiliary fields a_1, a_2 according to

$$a_\alpha = \sqrt{2}\left(\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha}\partial_\mu a_1 + \epsilon_\alpha a_2\right), \tag{16}$$

we obtain the following BRS transformations:

$$\begin{aligned} \mathbf{s}a_1 &= A - \mathrm{i}\xi^\mu\partial_\mu a_1, & \mathbf{s}A &= 2\mathrm{i}\epsilon^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha}\partial_\mu a_1 - \mathrm{i}\xi^\mu\partial_\mu A, \\ \mathbf{s}a_2 &= F_A + v_A - \mathrm{i}\xi^\mu\partial_\mu a_2, & \mathbf{s}F_A &= 2\mathrm{i}\epsilon^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\epsilon^{\dot\alpha}\partial_\mu a_2 - \mathrm{i}\xi^\mu\partial_\mu F_A. \end{aligned} \tag{17}$$

We identify the two BRS doublets u, v and U, V by

$$\begin{aligned} U &= a_1, & V &= A - \mathrm{i}\xi^\mu\partial_\mu a_1, \\ u &= a_2, & v &= F_A + v_A - \mathrm{i}\xi^\mu\partial_\mu a_2, \end{aligned} \tag{18}$$

which respect the usual BRS transformation law for BRS doublets. The fields u, v correspond exactly to the BRS doublet of Ref. [9]. The additional fields U, V are dimensionless and can appear infinite many times in the action.

In order to retain the superfield character of the original spurion field we have to require in addition that u, U has to appear in the action only in the combination (16). This implies in particular that Γ depends only on the combination $u\epsilon_\alpha$. In the approach of Ref. [9] such a requirement is missing and even with R symmetry additional interaction terms are present which to not fit in the superfield formulation.

As a result, all spurion terms are BRS variations. The physical relevant terms can be found in Appendix E of Ref. [8]. They have a one-to-one correspondence to the terms in the superfield notation (see equation (2.18) of Ref. [8]). For instance the gaugino mass of (12) corresponds to

$$\mathcal{S}_\Gamma \int \mathrm{d}^4x\,[u\lambda'^\alpha\lambda'_\alpha]\,. \tag{19}$$

Hence the cohomology problem of the spurion part of the MSSM is as trivial as in the case of the BRS doublet of Ref. [9]. Particularly only terms up to second order in the spurion field contribute to soft supersymmetry breaking terms and are important for the calculation of physical observables. All other spurion terms can be savely ignored.

The main difference between the formulation with BRS doublets u, v and U, V and those with the spurion superfield $\hat{A}$ is the actual form of the ST operator. In the formulation with BRS doublets the ST operator reads

$$\mathcal{S}(\Gamma) = \mathcal{S}^0(\Gamma) + \int \mathrm{d}^4x\left(V\frac{\delta}{\delta U} + v\frac{\delta}{\delta u} + \text{c.c.}\right), \tag{20}$$

while in the superfield notation it yields

$$\mathcal{S}(\Gamma) = \mathcal{S}^0(\Gamma) + \int \mathrm{d}^4x\left(\mathbf{s}A\frac{\delta}{\delta A} + \mathbf{s}a^\alpha\frac{\delta}{\delta a^\alpha} + \mathbf{s}F_A\frac{\delta}{\delta F_A} + \text{c.c.}\right). \tag{21}$$

For practical purposes it can be more convenient to stay within the superfield notation. In this case it is sufficient to solve the ST identity in the limit $A, a_\alpha \to 0$:

$$\mathcal{S}(\Gamma)|_{A,a_\alpha=0} = 0. \tag{22}$$

In addition one has then to use the R Ward identity

$$\left(\mathcal{W}^R\Gamma\right)\Big|_{A,a_\alpha=0} = \left(\mathcal{W}^R\Gamma\right)\Big|_{A,a_\alpha,F_A=0} - \int \mathrm{d}^4x \left(2\mathrm{i}F_A\frac{\delta\Gamma}{\delta F_A} + \mathrm{c.c.}\right)\Bigg|_{A,a_\alpha=0} = 0, \tag{23}$$

yielding a relation between vertex functions including F_A fields and those without spurion fields.

Non-renormalization theorems

From superspace calculations [13] and renormalization group arguments [14] it has been known that soft supersymmetry breakings have special renormalization properties: Up to some D-term contributions it is possible to express divergences of soft breaking parameters in terms of renormalization constants of the couplings and supersymmetric masses. These improved renormalization properties escape the doublet approach as well as the spurion approach.

Recently the non-renormalization theorems of supersymmetric field theories have been proven in the Wess-Zumino gauge by extending coupling constants to external fields and including softly broken axial symmetry as an additional defining symmetry into the model [15]. The extended model already contains the soft breaking parameters and yields the relations between the renormalization constants of supersymmetric parameters and of soft breaking parameters [16]. In addition, the algebraic derivation exhibits a deep relation between two anomalies — the axial anomaly and a supersymmetry anomaly — and the explicit form of the non-renormalization theorems.

However, in this approach the symmetries and in particular the ST identity have a more complicated form as in the spurion and doublet approach. Hence, one could use the simpler formulation with spurion fields for practical calculations and take the information on the renormalization constants obtained from the abstract approach in the extended model as an additional input and check of explicit calculations.

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