Global Electroweak Fits in the SM, MSSM, and CMSSM

Wim de Boer and Christian Sander
Institut für Experimentelle Kernphysik, University of Karlsruhe
Postfach 6980, D-76128 Karlsruhe, Germany

Abstract

A global statistical χ^2 analysis of all electroweak data including new data on the anomalous magnetic moment of the muon and the $b \to X_s\gamma$ decay rate in both the SM and the MSSM has been performed. The total χ^2 of the MSSM is better than in the SM, mainly because of the W-mass and a_μ, although the total probability is similar in both models due to the larger number of parameters in the MSSM. In addition the fit is performed in the supergravity inspired Constrained MSSM (CMSSM).

1 Introduction

A few years ago a complete electroweak fit program including all possible supersymmetric corrections in the Minimal Supersymmetric Model (MSSM) was developed, mainly to investigate the so-called R_b deviation of the Standard Model (SM) [1]. At present R_b shows no significant deviation from the SM, but the present total χ^2 of all electroweak data is not excellent [2]. In addition, if the new measurements of M_W, the anomalous magnetic moment of the muon a_μ [3], and $b \to X_s\gamma$ [4],[5] are included, the SM fit becomes worse. We include these new measurements in our analysis and compare the SM fit with the MSSM fit.

2 Experimental Data

A summary of the most recent electroweak data from colliders can be found in the report of the Electroweak Working Group (EWWG) [2]. As mentioned above, we included in addition the anomalous magnetic moment of the muon a_μ, which was determined by the E821 collaboration from a measurement of $g-2$ using the polarization in the decays of muons in a muon storage ring. They found a_μ to be slightly above the SM prediction [6]. The new value of $\Delta a_\mu = a_\mu - a_\mu^{\rm SM}$ corresponds to a 1.6 to 3.0σ deviation from the SM, depending on which SM prediction is used [7]. If e^+e^- data is used to calculate the vacuum polarization correction to the fine structure constant, one gets $\Delta a_\mu = (338 \pm 112) \cdot 10^{-11}$ which corresponds to a 3.0σ deviation. If on the other hand hadronic τ-decays are used to calculate this correction one obtains a 1.6σ deviation. Clearly since a 1.6σ deviation

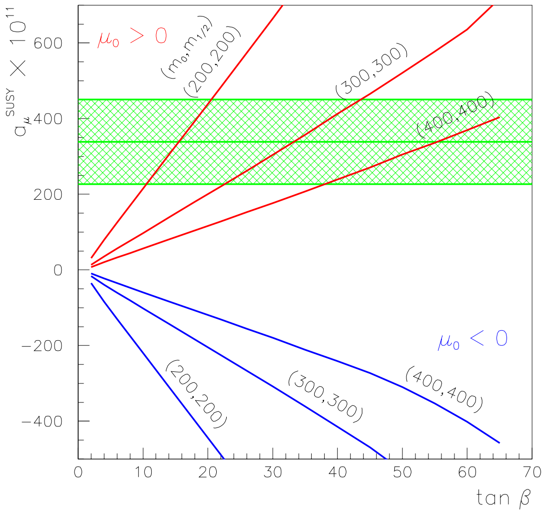


Figure 1: Dependence of the anomalous magnetic moment of the muon a_μ on $\tan\beta$ for different supersymmetric sparticle masses, parameterized by the common GUT scale masses for the spin 0 and spin 1/2 sparticles, called m_0 and $m_{1/2}$ respectively. The horizontal band represent the experimental measurement $\Delta a_\mu = (338 \pm 112) \cdot 10^{-11}$

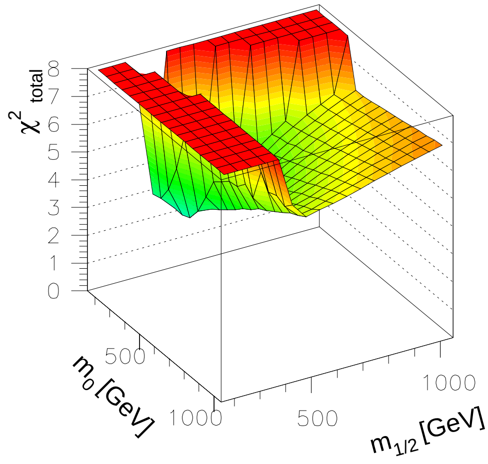
has a probability above 5%, one cannot use Δa_μ to obtain 95% C.L. limits in this case. For the 3.0σ deviation, relying purely on e^+e^- data, the 95% C.L. will be given.

The most popular explanation for contributions to Δa_μ outside the SM is given in the framework of SUSY theories [8]. Extensive references can be found in [9]. In Fig. 1 the dependence of Δa_μ on $\tan\beta$ for different supersymmetric particle masses is shown. Note the preferred positive sign of μ and the relatively light sparticle spectrum needed to be consistent with the experimental value of Δa_μ.

SUSY contributions are also expected to affect the $b \to X_s\gamma$ rate, for which the most recent world average is: $Br(b \to X_s\gamma) = (3.43 \pm 0.35) \cdot 10^{-4}$. This value is dominated by the recently published results from BaBar ($(3.88 \pm 0.36_{\rm stat} \pm 0.37_{\rm syst} \pm 0.36_{\rm mod}) \cdot 10^{-4}$) [4] and CLEO ($(3.21 \pm 0.43_{\rm stat} \pm 0.27_{\rm syst} \pm 0.14_{\rm mod}) \cdot 10^{-4}$) [5].

The world average is slightly below, but consistent with a recent SM prediction by Gambino and Misiak of $(3.73 \pm 0.30) \cdot 10^{-4}$ [10]. This value is somewhat higher than previous predictions, since it uses the running mass for the charm quark in the loops, while keeping the pole mass for the bottom quark in the external lines. This gives an additional uncertainty, but the authors found a reduced scale dependence. In our present analysis we conservatively keep a theoretical error of $\pm 0.40 \cdot 10^{-4}$, but use $m_c(\mu)/m_b = 0.22$. This is not critical, since with the present large errors $b \to X_s\gamma$ hardly constrains the fit.

In Fig. 2 the allowed CMSSM parameter region in the m_0-$m_{1/2}$-plane is shown for $\tan\beta = 35$. One can see that a rather big area is allowed (light shaded region on the right hand side). In the fit the following constraints are included: electroweak symmetry breaking, 3. generation of fermion masses, the lightest supersymmetric particle (LSP) should be neutral, present Higgs limit ($m_h > 114.6$ GeV), $b \to X_s\gamma$ and a_μ [9]. The trilinear



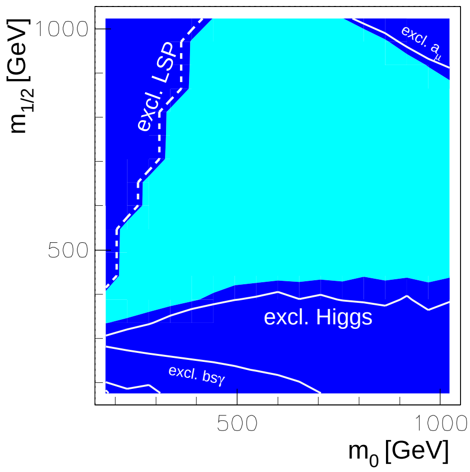


Figure 2: The total χ^2 in the m_0-$m_{1/2}$-plane of a global fit performed in the CMSSM with $\tan\beta = 35$ and positive sign of μ. Parameter regions with $\chi^2 > \chi^2_{\rm min} + 4$ are excluded, which corresponds to a two sided 95% confidence level. If a more conservative value of a_μ is used, the excluded region on the right top corner vanishes as described in the text.

coupling A_0 at the GUT scale is a free parameter in our fit, which then prefers positive values of A_0. In this case the Higgs limit becomes important. If A_0 is fixed to 0 the Higgs limit becomes less important but in exchange $b \to X_s\gamma$ becomes the dominant lower limit.

The value of M_W becomes higher in the MSSM than in the SM, as shown in Fig. 3, in agreement with the direct measurement of M_W at LEP II and $p\bar{p}$ colliders. It should be mentioned that the W mass from LEP is still preliminary.

None of these measurements, a_μ, $b \to X_s\gamma$ and M_W, shows by itself a significant deviation from the SM, but since they all point to supersymmetric contributions, it is interesting to compare a global fit of all data in SM, MSSM and CMSSM.

3 Fit Results

The fits to the electroweak precision data are performed in three different models:

- Standard Model (SM) with 5 parameters: $\alpha_s(M_Z)$, M_Z, m_t, m_h and $\Delta\alpha^{(5)}_{\rm had}$
- Minimal Supersymmetric Model (MSSM): In the most general case all sfermions masses can be chosen independently, because they are not constrained by GUT relations. For simplification we assume a common slepton and common squark mass scale with the exception of the left and right handed stop mass. In the third generation sfermion sector mass splitting due to Yukawa couplings is taken into account. The chargino and neutralino matrices have as free parameters μ and M_2, while M_1 was taken to be $\frac{5}{3}\frac{\sin^2\theta_W}{\cos^2\theta_W}M_2$, as expected from RGE. For details see Ref. [1].

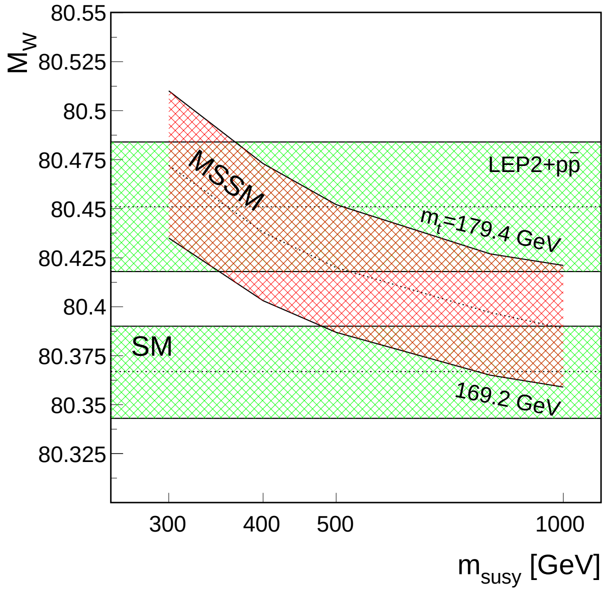


Figure 3: W mass versus sparticle masses, assuming all sparticles have the same mass. The horizontal bands represent the SM prediction from LEP I data and the direct measurement from LEP II and $p\bar{p}$ colliders. The curved band is the MSSM prediction for the case that all sparticles have a given mass m_{SUSY}. Its width is determined by the uncertainty from the top mass. The SM value of M_W is a function of M_Z, $\sin^2\theta_W$, m_t and m_h without a constraint to the direct measurement. With a constraint one gets a somewhat higher value.

- Constrained Minimal Supersymmetric Model (CMSSM): Supersymmetry is broken by gravity mediation (mSUGRA). This lowers the number of free parameters. The sfermion masses are unified at the GUT scale M_{GUT} to m_0, just as the gaugino masses are unified to $m_{1/2}$, while $|\mu|$ is determined by electroweak symmetry breaking. M_{GUT} is determined by gauge unification ($\alpha_1 = \alpha_2 = \alpha_3 = \alpha_{\mathrm{GUT}}$). Precise gauge unification is not a necessary constraint since threshold corrections from heavy higgs bosons can contribute to the running of the couplings so we did not use it as a constraint. Also the trilinear couplings of the third generation are unified to A_0 at the GUT scale. The low energy values are determined by RGEs [11].

3.1 Comparison of SM and MSSM Fits

All electroweak variables were calculated in the SM using ZFITTER 6.11 [12] and in the MSSM using MSSMFITTER [1]. The deviation between data and theoretical prediction in the two models are shown in Fig. 4 for $\tan\beta = 35$.
Clearly, the largest deviations occur in the forward-backward asymmetry A^b_{FB} for b-quarks and the left-right asymmetry A_{LR}, as measured with the polarized electron beam at SLAC. Both can be translated into a measurement of the electroweak mixing angle, which than turns out to be 3σ apart [2]. In the MSSM the situation does not improve. Since there is no preference for any of the data, we followed the procedure from the Particle Data Group to rescale the errors of A^b_{FB} and A_{LR} in such a way that their χ^2 contributions are about one [13]. This hardly influences any of the other variables, as shown on the right

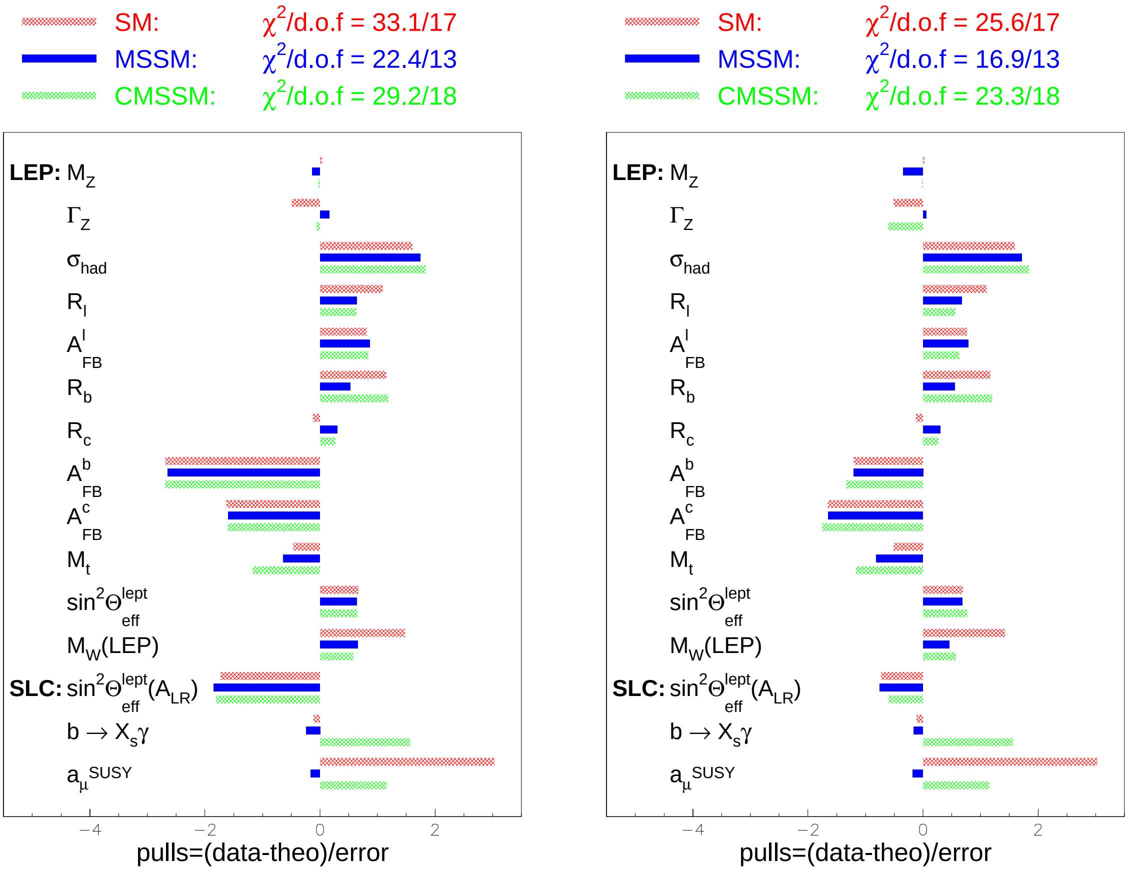


Figure 4: The pulls of the electroweak data for the SM, MSSM and CMSSM. On the left hand side the full errors are taken into account. On the right hand side the errors for A^b_{FB} and A_{LR} are rescaled as described in the text.

hand side of Fig. 4, but increases the probability from 1% (5%) to 8% (20%) in the SM (MSSM).
The χ^2/d.o.f. in the MSSM is better than in the SM (22.4/13 for MSSM versus 33.1/17 for SM), mainly because of a_μ and M_W (see Fig. 4), but the probabilitiy is similar due to the larger number of parameters in the MSSM (5% for MSSM versus 1% for SM). The MSSM fits are not very sensitive to $\tan\beta$, if it is large. Large values of $\tan\beta$ are preferred from a_μ (see Fig. 1).

3.2 CMSSM

The parameters in the CMSSM are determined by minimization of a χ^2 function, which includes constraints to various experimental limits and measurements [11].
The gauge unification prefers higher values of $\alpha_s(M_Z)$ and $\sin^2\theta_W$ than derived from electroweak precision data in the SM. But different observables lead to somewhat different values of α_s and $\sin^2\theta_W$. If only M_Z, $\Gamma_{\rm tot}$ and $\sigma_{\rm had}$ are used a small value of $\alpha_s = 0.1153(40)$ is found. However R_l yields a higher value $\alpha_s = 0.1225(37)$. One should note that the observable $\sigma_{\rm had}$ depends on the luminosity and its error in contrast to R_l, which

is a ratio. The measurement of A^b_{FB} yields $\sin^2\theta_W^{\rm eff} = 0.23226(31)$, which is above the value determined by A_{LR} of $\sin^2\theta_W^{\rm eff} = 0.23098(26)$.
A global fit in the CMSSM to electroweak precision data including electroweak symmetry breaking, third generation fermion masses and the LSP constraint is shown in Fig. 4 in comparison with the SM and MSSM fits. The χ^2 is in the CMSSM larger, but this causes no decrease in probability since the smaller number of parameters increases the degree of freedom (MSSM: 5.0% $\rightarrow$ CMSSM: 4.6%).

4 Conclusion

It has been shown that a SM electroweak fit including anomalous magnetic moment of the muon and $Br(b \rightarrow X_s\gamma)$ yields a probability below 5%, even with conservative error estimates. The total χ^2 is improved in the MSSM, mainly because of a_μ and M_W, but the probability does not drop as much due to the larger number of free parameters in the MSSM. However, in both cases the 3σ discrepancy in $\sin^2\theta_W$ from A^b_{FB} and A_{LR} is the main source for the low probability. Since at present no arguments to doubt any of the measurements can be found, we tested the Particle Data Group's procedure to rescale the errors of these two measurements by the corresponding pulls. This yields considerably improved errors, both in the SM and MSSM, without significant changes in the fitted parameters.

References

[1] W. de Boer *et al.*, Z. Phys. C **75** (1997) 627 [arXiv:hep-ph/9607286].
[2] LEP Electroweak Working group, LEPEWWG/2001-01 `http://lepewwg.web.cern.ch/LEPEWWG/stanmod/`.
[3] H. N. Brown *et al.* [Muon g-2 Collaboration], Phys. Rev. Lett. **86** (2001) 2227 [arXiv:hep-ex/0102017].
[4] B. Aubert *et al.* [BaBar Collaboration], arXiv:hep-ex/0207076.
[5] S. Chen *et al.* [CLEO Collaboration], Phys. Rev. Lett. **87** (2001) 251807 [arXiv:hep-ex/0108032].
[6] G. W. Bennett [Muon g-2 Collaboration], arXiv:hep-ex/0208001.
[7] M. Davier *et al.*, arXiv:hep-ph/0208177.
[8] A. Czarnecki and W. J. Marciano, Phys. Rev. D **64** (2001) 013014 [arXiv:hep-ph/0102122].
[9] W. de Boer *et al.*, Phys. Lett. B **515** (2001) 283. arXiv:hep-ph/0109131.
[10] P. Gambino and M. Misiak, Nucl. Phys. B **611** (2001) 338 [arXiv:hep-ph/0104034].
[11] W. de Boer, Prog. Part. Nucl. Phys. **33** (1994) 201 [arXiv:hep-ph/9402266].
[12] D. Y. Bardin *et al.*, Comput. Phys. Commun. **133** (2001) 229 [arXiv:hep-ph/9908433].
[13] D. E. Groom *et al.* [Particle Data Group Collaboration], Eur. Phys. J. C **15** (2000) 1.
[14] The LEP Collaborations, LEP Higgs Working Group, CERN-EP/2001-55, `http://lephiggs.web.cern.ch/LEPHIGGS/papers/index.html`.