Lepton Flavor Violating Processes in Bi-maximal Texture of Neutrino Mixings

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We investigate the lepton flavor violation in the framework of the MSSM with right-handed neutrinos taking the large mixing angle MSW solution in the quasi-degenerate and the inverse-hierarchical neutrino masses. We predict the branching ratio of $\mu \to e + \gamma$ processes assuming the degenerate right-handed Majorana neutrino masses. We find that the branching ratio in the quasi-degenerate neutrino mass spectrum is 100 times smaller than the ones in the inverse-hierarchical and the hierarchical neutrino spectra. We emphasize that the magnitude of U_{e3} is one of important ingredients to predict $\mathrm{BR}(\mu \to e + \gamma)$. The effect of the deviation from the complete-degenerate right-handed Majorana neutrino masses are also estimated.

1 Introduction

If neutrinos are massive and mixed in the SM, due to the smallness of the neutrino masses, the predicted branching ratios for the lepton flavor violation (LFV) are so tiny that they are completely unobservable. On the other hand, in the supersymmetric framework the situation is quite different. Many authors have already studied the LFV in the minimal supersymmetric standard model (MSSM) with right-handed neutrinos assuming the relevant neutrino mass matrix [1, 2, 3, 4]. In the MSSM with soft breaking terms, there exist lepton flavor violating terms such as off-diagonal elements of slepton mass matrices $(\mathbf{m}^2_{\tilde{L}})_{ij}$, $(\mathbf{m}^2_{\tilde{e}_R})_{ij}$ and trilinear couplings $\mathbf{A}^e_{ij}$. Strong bounds on these matrix elements come from requiring branching ratios for LFV processes to be below observed ratios. For the present, the most stringent bound comes from the $\mu \to e + \gamma$ decay ($\mathrm{BR}(\mu \to e + \gamma) < 1.2 \times 10^{-11}$) [5]. However, if the LFV occurs at tree level in the soft breaking terms, the branching ratio of this process exceeds the experimental bound considerably. Therefore one assumes that the LFV does not occur at tree level in the soft parameters. This is realized by taking the assumption that soft parameters such as $(\mathbf{m}^2_{\tilde{L}})_{ij}$, $(\mathbf{m}^2_{\tilde{e}_R})_{ij}$, $\mathbf{A}^e_{ij}$, are universal *i.e.*, proportional to the unit matrix. However, even though there is no flavor violation at tree level, it is generated by the effect of the renormalization group equations (RGE's) via neutrino Yukawa couplings. Suppose that neutrino masses are produced by the see-saw mechanism [6], there are the right-handed neutrinos above a scale M_R. Then neutrinos

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have the Yukawa coupling matrix $\mathbf{Y}_\nu$ with off-diagonal entries in the basis of the diagonal charged-lepton Yukawa couplings. The off-diagonal elements of $\mathbf{Y}_\nu$ drive off-diagonal ones in the $(\mathbf{m}^2_{\tilde{L}})_{ij}$ and $\mathbf{A}^e_{ij}$ matrices through the RGE's running [7].

One can construct $\mathbf{Y}_\nu$ by the recent data of neutrino oscillations. Assuming that oscillations need only accounting for the solar and the atmospheric neutrino data, we take the LMA-MSW solution of the solar neutrino. Then, the lepton mixing matrix, which may be called the MNS matrix or the MNSP matrix [8, 9], is given in ref.[10]. Since the data of neutrino oscillations only indicate the differences of the mass square Δm^2_{ij}, neutrinos have three possible mass spectra: the hierarchical spectrum $m_{\nu 3} \gg m_{\nu 2} \gg m_{\nu 1}$, the quasi-degenerate one $m_{\nu 1} \simeq m_{\nu 2} \simeq m_{\nu 3}$ and the inverse-hierarchical one $m_{\nu 1} \simeq m_{\nu 2} \gg m_{\nu 3}$.

2 LFV in the MSSM with Right-handed Neutrinos

The superpotential of the lepton sector is described as follows:

$$W_{\rm lepton} \;=\; \mathbf{Y}_e L H_d e^c_R + \mathbf{Y}_\nu L H_u \nu^c_R + \frac{1}{2}\nu^{cT}_R \mathbf{M_R} \nu^c_R \;, \tag{2.1}$$

where H_u, H_d are chiral superfields for Higgs doublets, L is the left-handed lepton doublet, e_R and ν_R are the right-handed charged lepton and the neutrino superfields, respectively. The $\mathbf{Y}_e$ is the Yukawa coupling matrix for the charged lepton, $\mathbf{M_R}$ is Majorana mass matrix of the right-handed neutrinos. We take $\mathbf{Y}_e$ and $\mathbf{M_R}$ to be diagonal.

It is well-known that the neutrino mass matrix is given as $\mathbf{m}_\nu = (\mathbf{Y}_\nu v_u)^{\rm T} \mathbf{M}^{-1}_{\mathbf{R}} (\mathbf{Y}_\nu v_u)$, via the see-saw mechanism, where v_u is the vacuum expectation value (VEV) of Higgs H_u. The neutrino mass matrix $\mathbf{m}_\nu$ is diagonalized by a single unitary matrix $\mathbf{m}^{\rm diag}_\nu \equiv \mathbf{U}^{\rm T}_{\rm MNS} \mathbf{m}_\nu \mathbf{U}_{\rm MNS}$, where $\mathbf{U}_{\rm MNS}$ is the lepton mixing matrix. Following the expression in ref.[4], we write the neutrino Yukawa coupling as

$$\mathbf{Y}_\nu = \frac{1}{v_u}\sqrt{\mathbf{M}^{\rm diag}_{\mathbf{R}}}\; \mathbf{R}\; \sqrt{\mathbf{m}^{\rm diag}_\nu}\; \mathbf{U}^{\rm T}_{\rm MNS} \;, \tag{2.2}$$

where $\mathbf{R}$ is a 3×3 orthogonal matrix, which depends on models.

At first, let us take the degenerate right-handed Majorana masses $M_{R1} = M_{R2} = M_{R3} \equiv M_R$. This assumption is reasonable for the case of the quasi-degenerate neutrino masses. Otherwise a big conspiracy would be needed between $\mathbf{Y}_\nu$ and $\mathbf{M_R}$. This assumption is also taken for cases of the inverse-hierarchical and the hierarchical neutrino masses. We also discuss later the effect of the deviation from the degenerate right-handed Majonara neutrino masses. Then, we get

$$\mathbf{Y}^\dagger_\nu \mathbf{Y}_\nu \;=\; \frac{M_R}{v_u^2}\mathbf{U}_{\rm MNS}\begin{pmatrix} m_{\nu 1} & 0 & 0 \\ 0 & m_{\nu 2} & 0 \\ 0 & 0 & m_{\nu 3}\end{pmatrix}\mathbf{U}^{\rm T}_{\rm MNS} \;. \tag{2.3}$$

It is remarked that $\mathbf{Y}^\dagger_\nu \mathbf{Y}_\nu$ is independent of $\mathbf{R}$ in the case of $M_{R1} = M_{R2} = M_{R3} \equiv M_R$.

As mentioned in the previous section, there are three possible neutrino mass spectra. The hierarchical type $(m_{\nu 1} \ll m_{\nu 2} \ll m_{\nu 3})$ gives the neutrino mass spectrum as

$$m_{\nu 1} \sim 0 \;, \qquad m_{\nu 2} = \sqrt{\Delta m^2_\odot} \;, \qquad m_{\nu 3} = \sqrt{\Delta m^2_{\rm atm}} \;, \tag{2.4}$$

the quasi-degenerate type ($m_{\nu 1} \sim m_{\nu 2} \sim m_{\nu 3}$) gives

$$m_{\nu 1} \equiv m_\nu \ , \qquad m_{\nu 2} = m_\nu + \frac{1}{2m_\nu}\Delta m^2_\odot \ , \qquad m_{\nu 3} = m_\nu + \frac{1}{2m_\nu}\Delta m^2_{\rm atm} \ , \tag{2.5}$$

and the inverse-hierarchical type ($m_{\nu 1} \sim m_{\nu 2} \gg m_{\nu 3}$) gives

$$m_{\nu 2} \equiv \sqrt{\Delta m^2_{\rm atm}} \ , \quad m_{\nu 1} = m_{\nu 2} - \frac{1}{2m_{\nu 2}}\Delta m^2_\odot \ , \quad m_{\nu 3} \simeq 0 \ . \tag{2.6}$$

We take the typical values $\Delta m^2_{\rm atm} = 3 \times 10^{-3}{\rm eV}^2$ and $\Delta m^2_\odot = 7 \times 10^{-5}{\rm eV}^2$ in our calculation of the LFV.

We take the typical mixing angles of the LMA-MSW solution such as $s_{23} = 1/\sqrt{2}$ and $s_{12} = 0.6$ [10], in which the lepton mixing matrix is given in terms of the standard parametrization of the mixing matrix [13]. The reacter experiment of CHOOZ [11] presented a upper bound of s_{13}. We use the constraint from the two flavor analysis, which is $s_{13} \leq 0.2$ in our calculation. If we take account of the recent result of the three flavor analysis [12], the upper bound of s_{13} may be smaller than 0.2. Then, if we use the results in [12], our results of $\mu \to e + \gamma$ are reduced at most by a factor of two. In our calculation, the CP violating phase is neglected for simplicity.

Since SUSY is spontaneously broken at the low energy, we consider the MSSM with the soft SUSY breaking terms:

$$-\mathcal{L}_{\rm soft} = (\mathbf{m}^2_{\tilde{L}})_{ij}\tilde{L}^\dagger_i\tilde{L}_j + (\mathbf{m}^2_{\tilde{e}})_{ij}\tilde{e}^*_{Ri}\tilde{e}_{Rj} + (\mathbf{m}^2_{\tilde{\nu}})_{ij}\tilde{\nu}^*_{Ri}\tilde{\nu}_{Rj} + \mathbf{A}^e_{ij}H_d\tilde{e}^*_{Ri}\tilde{L}_j + \mathbf{A}^\nu_{ij}H_u\tilde{\nu}^*_{Ri}\tilde{L}_j \ , \tag{2.7}$$

where $\mathbf{m}^2_{\tilde{L}}, \mathbf{m}^2_{\tilde{e}}$ and $\mathbf{m}^2_{\tilde{\nu}}$ are mass-squares of the left-handed charged slepton, the right-handed charged slepton and the sneutrino, respectively. $\mathbf{A}_e$ and $\mathbf{A}_\nu$ are A-parameters.

Note that the lepton flavor violating processes come from diagrams including non-zero off-diagonal elements of the soft parameter. In this paper we assume the m-SUGRA, therefore we put the assumption of universality for soft SUSY breaking terms at the unification scale:

$$(\mathbf{m}^2_{\tilde{L}})_{ij} = (\mathbf{m}^2_{\tilde{e}})_{ij} = (\mathbf{m}^2_{\tilde{\nu}})_{ij} = \cdots = \delta_{ij}m^2_0 \ , \ \ \mathbf{A}^\nu = \mathbf{Y}_\nu a_0 m_0, \ \mathbf{A}^e = \mathbf{Y}_e a_0 m_0 \ , \tag{2.8}$$

where m_0 and a_0 stand for the universal scalar mass and the universal A-parameter, respectively.

The RGE's for the left-handed slepton soft mass are given by

$$\begin{aligned} \mu\frac{d}{d\mu}(\mathbf{m}^2_{\tilde{L}})_{ij} &= \mu\frac{d}{d\mu}(\mathbf{m}^2_{\tilde{L}})_{ij}\Big|_{\rm MSSM} \\ &+ \frac{1}{16\pi^2}\left[(\mathbf{m}^2_{\tilde{L}}\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu + \mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu\mathbf{m}^2_{\tilde{L}})_{ij} + 2(\mathbf{Y}^\dagger_\nu\mathbf{m}_{\tilde{\nu}}\mathbf{Y}_\nu + \tilde{m}^2_{H_u}\mathbf{Y}^\dagger_\nu\mathbf{Y}_\nu + \mathbf{A}^\dagger_\nu\mathbf{A}_\nu)_{ij}\right] , \end{aligned} \tag{2.9}$$

while the first term in the right hand side is the normal MSSM term which has no LFV, and the second one is a source of the LFV through the off-diagonal elements of neutrino Yukawa couplings.

3 Numerical Analyses of Branching Ratios

Let us calculate the branching ratio of $e_i \to e_j + \gamma$ $(j < i)$. The decay rate can be calculated using $A^{L,R}$ as

$$\Gamma(e_i \to e_j + \gamma) = \frac{e^2}{16\pi} m_{e_i}^5 (|A^L|^2 + |A^R|^2) \ . \tag{3.1}$$

Since we know the relation $m_{e_i}^2 \gg m_{e_j}^2$, then we can expect $|A^R| \gg |A^L|$. The decay amplitude is approximated as

$$|A^R|^2 \simeq \frac{\alpha_2^2}{16\pi^2} \frac{|(\triangle \mathbf{m}_{\tilde{L}}^2)_{ij}|^2}{m_S^8} \tan^2\beta \ , \tag{3.2}$$

where α_2 is the gauge coupling constant of $SU(2)_L$ and m_S is a SUSY particle mass. The RGE's develop the off-diagonal elements of the slepton mass matrix and A-term:

$$(\triangle \mathbf{m}_{\tilde{L}}^2)_{ij} \simeq -\frac{(6+2a_0^2)m_0^2}{16\pi^2} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \ln \frac{M_X}{M_R} \ , \tag{3.3}$$

where M_X is the GUT scale.

We present a qualitative discussion on $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$ before predicting the branching ratio $\mathrm{BR}(\mu \to e+\gamma)$. This is given in terms of neutrino masses and mixings at the electroweak scale as follows:

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} = \frac{M_R}{v_u^2} \left[U_{\mu 2} U_{e2}^* (m_{\nu 2} - m_{\nu 1}) + U_{\mu 3} U_{e3}^* (m_{\nu 3} - m_{\nu 1}) \right] \ , \tag{3.4}$$

where $v_u \equiv v \sin\beta$ with $v = 174\mathrm{GeV}$ is taken as an usual notation and the unitarity condition of the lepton mixing matrix elements is used. Taking the three cases of the neutrino mass spectra, one obtains the following forms, respectively,

$$\begin{aligned}
(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} &\simeq \frac{M_R}{\sqrt{2} v_u^2} \frac{\Delta m_{\mathrm{atm}}^2}{2 m_\nu} \left[\frac{1}{\sqrt{2}} U_{e2}^* \frac{\Delta m_\odot^2}{\Delta m_{\mathrm{atm}}^2} + U_{e3}^* \right] \ , \quad (\text{Degenerate}) \\
&\simeq \frac{M_R}{\sqrt{2} v_u^2} \sqrt{\Delta m_{\mathrm{atm}}^2} \left[\frac{1}{2\sqrt{2}} U_{e2}^* \frac{\Delta m_\odot^2}{\Delta m_{\mathrm{atm}}^2} - U_{e3}^* \right] \ , \quad (\text{Inverse}) \\
&\simeq \frac{M_R}{\sqrt{2} v_u^2} \sqrt{\Delta m_{\mathrm{atm}}^2} \left[\frac{1}{\sqrt{2}} U_{e2}^* \sqrt{\frac{\Delta m_\odot^2}{\Delta m_{\mathrm{atm}}^2}} + U_{e3}^* \right] \ , \quad (\text{Hierarchy})
\end{aligned} \tag{3.5}$$

where we take the maximal mixing for the atmospheric neutrinos. Since $U_{e2} \simeq 1/\sqrt{2}$ for the bi-maximal mixing matrix, the first terms in the square brackets of the right hand sides of eqs.(3.5) can be estimated by putting the experimental data. For the case of the degenerate neutrino masses, $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$ depends on the unknown neutrino mass scale m_ν. As one takes the smaller m_ν, one predicts the larger branching ratio. In our calculation, we take $m_\nu = 0.3\mathrm{eV}$, which is close to the upper bound from the neutrinoless double beta decay experiment [14], and also leads to the smallest branching ratio.

We also note that the degenerate case gives the smallest branching ratio BR($\mu \to e+\gamma$) among the three cases as seen in eqs.(3.5) owing to the scale of m_ν. It is easy to see the fact that the second terms in eqs.(3.5) are dominant as far as $U_{e3} \gtrsim 0.01$(degenerate), 0.01 (inverse) and 0.07(hierarchy), respectively. The magnitude and the phase of U_{e3} are important in the comparison between cases of the inverse-hierarchical and the normal hierarchical masses. In the limit of $U_{e3} = 0$, the predicted branching ratio in the case of the normal hierarchical masses is larger than the other one. However, for $U_{e3} \simeq 0.2$ the predicted branching ratios are almost the same in both cases.

At first, we present numerical results in the case of the degenerate neutrino masses assuming $\mathbf{M_R} = M_R\mathbf{1}$. The magnitude of M_R is constrained considerably if we impose the $b-\tau$ unification of Yukawa couplings [15]. In the case of $\tan\beta \leq 30$, the lower bound of M_R is approximately 10^{12}GeV. We take also $M_R \leq 10^{14}$GeV, in order that neutrino Yukawa couplings remain below $\mathcal{O}(1)$. Therefore, we use $M_R = 10^{12},\ 10^{14}$GeV in our following calculation.

We take a universal scalar mass (m_0) for all scalars and $a_0 = 0$ as a universal A-term at the GUT scale ($M_X = 2 \times 10^{16}$ GeV). The branching ratio of $\mu \to e + \gamma$ is given versus the left-handed selectron mass $m_{\tilde{e}_L}$ for each $\tan\beta = 3,\ 10,\ 30$ and a fixed wino mass M_2 at the electroweak scale. In fig.1, the branching ratios are shown for $M_2 = 150,\ 300$ GeV in the case of $U_{e3} = 0.2$ with $M_R = 10^{14}$GeV and $m_\nu = 0.3$eV, in which the solid curves correspond to $M_2 = 150$GeV and the dashed ones to $M_2 = 300$GeV. The threshold of the selectron mass is determined by the recent LEP2 data [16] for $M_2 = 150$GeV, however, for $M_2 = 300$GeV, determined by the constraint that the left-handed slepton should be heavier than the neutralinos. As the $\tan\beta$ increases, the branching ratio increases because the decay amplitude from the SUSY diagrams is approximately proportional to $\tan\beta$ [1]. It is found that the branching ratio is almost larger than the experimental upper bound in the case of $M_2 = 150$GeV. On the other hand, the predicted values are smaller than the experimental bound except for $\tan\beta = 30$ in the case of $M_2 = 300$GeV.

Our predictions depend on M_R strongly, because the magnitude of the neutrino Yukawa coupling is determined by M_R as seen in eq.(2.2). If M_R reduces to 10^{12}GeV, the branching ratio becomes 10^4 times smaller since it is proportional to M_R^2. The numerical result is shown in fig.2.

Next we show results in the case of the inverse-hierarchical neutrino masses. As expected in eq.(3.5), the branching ratio is much larger than the one in the degenerate case. In fig.3, the branching ratio is shown for $M_2 = 150,\ 300$ GeV in the case of $U_{e3} = 0.2$ with $M_R = 10^{14}$GeV. In fig.4, the branching ratio is shown for $U_{e3} = 0.05$ with $M_R = 10^{14}$GeV. The M_R dependence is the same as the case of the quasi-degenerate neutrino masses. The predictions almost exceed the experimental bound as far as $U_{e3} \geq 0.05$, $\tan\beta \geq 10$ and $M_R \simeq 10^{14}$GeV. This result is based on the assumption $\mathbf{M_R} = M_R\mathbf{1}$, however, it is not guaranteed in the case of the inverse-hierarchical neutrino masses.

For comparison, we show the branching ratio in the case of the hierarchical neutrino masses in fig.5. It is similar to the case of the inverse-hierarchical neutrino masses. The branching ratio in the case of the degenerate neutrino masses is 10^2 times smaller than the one in the inverse-hierarchical and the hierarchical neutrino spectra.

The analyses in the previous section depend on the assumption of $M_{R1} = M_{R2} =$

$M_{R3} \equiv M_R$. In the case of the quasi-degenerate neutrino masses in eq.(2.5) this complete degeneracy of $\mathbf{M}_R$ may be deviated in the following magnitude without fine-tuning:

$$\frac{M_{R3}^2}{M_{R1}^2} \simeq 1 \pm \frac{\Delta m_{\rm atm}^2}{m_\nu^2} \ , \qquad \frac{M_{R2}^2}{M_{R1}^2} \simeq 1 \pm \frac{\Delta m_\odot^2}{m_\nu^2} \ . \tag{3.6}$$

By using eq.(2.2), we obtain

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} = (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}\Big|_{\mathbf{M_R \propto 1}} + \Delta(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} \ , \tag{3.7}$$

where the first term is the $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$ element in eq.(3.4), which corresponds to the $\mathbf{M_{R\propto 1}}$, while the second term stands for the deviation from it as follows:

$$\Delta(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} = \frac{M_R}{v_u^2} \sum_{i,j}^{3} U_{2i} U_{1j} \sqrt{m_{\nu i}} \sqrt{m_{\nu j}} (\varepsilon_2 R_{2i} R_{2j} + \varepsilon_3 R_{3i} R_{3j}) \ . \tag{3.8}$$

In order to estimate the second term, we use $\varepsilon_2 = 0.0001$ and $\varepsilon_3 = 0.01$ taking account of $\varepsilon_2 \simeq \Delta m_\odot^2 / 2m_\nu^2$ and $\varepsilon_3 \simeq \Delta m_{\rm atm}^2 / 2m_\nu^2$, where $m_\nu = 0.3$eV is put. Since $m_{\nu i} \simeq m_{\nu j}$ and $R_{ij} \leq 1$, we get

$$\Delta(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} \leq 3.5 \times 10^{-3} \ . \tag{3.9}$$

Taking this maximal value, we can estimate the branching ratio as follows:

$$\frac{BR(\text{non-degenerate } M_R)}{BR(\text{degenerate } M_R)} \leq \left(\frac{2.6 + 3.5}{2.6}\right)^2 \simeq 5.5 \ . \tag{3.10}$$

Therefore, the enhancement due to the second term is at most factor 5. This conclusion does not depend on the specific form of $\mathbf{R}$

Consider the case of the inverse-hierarchical type of neutrino masses. We take $\varepsilon_2 \sim 0.01$ with the similar argument of the quasi-degenerate type neutrino masses, because $m_{\nu 1}$ and $m_{\nu 2}$ are almost degenerate and $\varepsilon_2 \simeq \Delta m_\odot^2 / 2\Delta m_{\rm atm}^2$ in this case. Then, we get

$$\Delta(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} \leq 0.063 \times 10^{-2} \ , \tag{3.11}$$

where we assume $\varepsilon_2 \geq \varepsilon_3$ and use $m_{\nu 3} \simeq 0$, $m_{\nu 1} \simeq m_{\nu 2} \simeq 0.054$eV and $R_{ij} \leq 1$. Taking the maximal value, we get

$$\frac{BR(\text{non-degenerate } M_R)}{BR(\text{degenerate } M_R)} \leq \left(\frac{2.7 + 0.063}{2.7}\right)^2 \simeq 1.04 \ . \tag{3.12}$$

Thus, the effect of the $\Delta(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$ is very small in the case of the inverse hierarchical neutrino masses.

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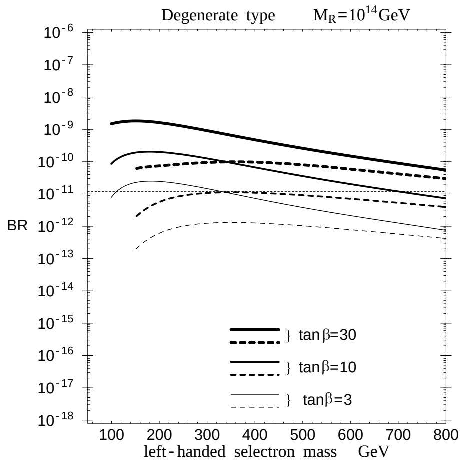

Figure 1: BR($\mu \to e + \gamma$) in the case of the degenerate neutrino masses.

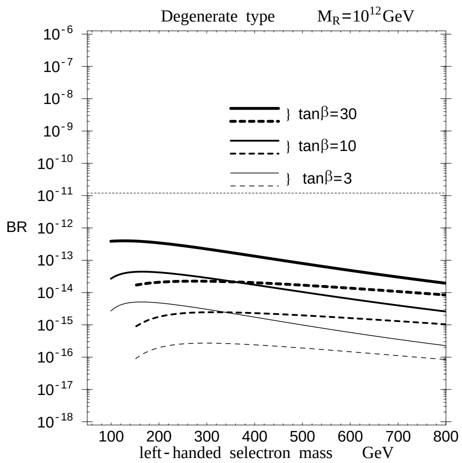

Figure 2: BR($\mu \to e + \gamma$) in the case of the case of degenerate neutrino masses.

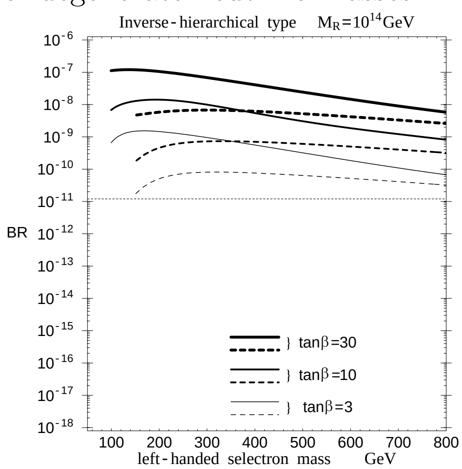

Figure 3: BR($\mu \to e + \gamma$) in the case of the inverse-hierarchical neutrino masses.

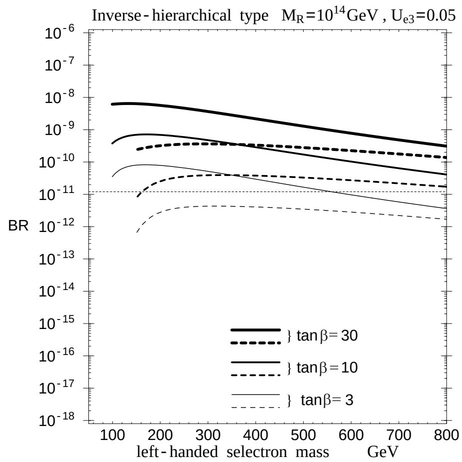

Figure 4: BR($\mu \to e + \gamma$) in the case of the inverse-hierarchical neutrino masses.

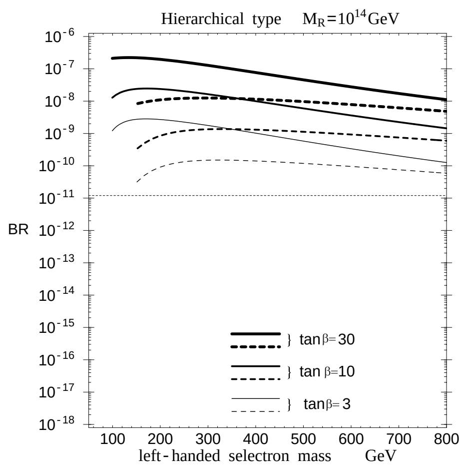

Figure 5: BR($\mu \to e + \gamma$) in the case of the hierarchical neutrino masses.