Fermion Dipole Moments from R-parity Violating Parameters

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We have developed an efficient formulation for the study of the generic supersymmetric standard model, which admits all kind of R-parity violating terms. Using the formulation, we discuss all sources of fermion dipole moment contributions from R-parity violating, or rather lepton number violating, parametersand the constraints obtained. Stringent constraints comparable to those from neutrino masses are resulted in some cases.

I. INTRODUCTION

Fermion electric dipole moments (EDMs) are known to be extremely useful constraints on (the CP violating part of) models depicting interesting scenarios of beyond Standard Model (SM) physics. In particular, the experimental bounds on neutron EDM (d_n) and electron EDM (d_e) are very stringent. The current numbers are given by $d_n < 6.3 \cdot 10^{-26}\, e \cdot$ cm and $d_e < 4.3 \cdot 10^{-27}\, e \cdot$ cm. The SM contributions are known to be very small, given that the only source of CP violation has to come from the KM phase in (charged current) quark flavor mixings : $d_n \sim 10^{-32}\, e \cdot$ cm and $d_e \sim 8 \cdot 10^{-41}\, e \cdot$ cm.

Extensions of the SM normally are expected to have potentially large EDM contributions. For instance, for the minimal supersymmetric standard model (MSSM), there are a few source of such new contributions. For example, they can come in through LR sfermion mixings. The latter have two parts, an A-term contribution as well as a F-term contribution. The F-term is a result of the complex phase in the so-called μ-term. The resulted constraints on MSSM have been studied extensively. We are interested here in the modified version with R parity not imposed. We will illustrate that there are extra contributions at the same level and discuss the class of important constraints hence resulted.

II. FORMULATION AND NOTATION

A theory built with the minimal superfield spectrum incorporating the SM particles, the admissible renormalizable interactions dictated by the SM (gauge) symmetries together with the idea that supersymmetry (SUSY) is softly broken is what should be called the the generic supersymmetric standard model (GSSM). The popular MSSM differs from the generic version in having a discrete symmetry, called R parity, imposed by hand to enforce baryon and lepton number conservation. With the strong experimental hints at the existence of lepton number violating neutrino masses, such a theory of SUSY without R-parity deserves ever more attention. The GSSM contains all kinds of (so-called) R-parity violating (RPV) parameters. The latter includes the more popular trilinear (λ_{ijk}, λ'_{ijk}, and λ''_{ijk}) and bilinear (μ_i) couplings in the superpotential, as well as soft SUSY breaking parameters of the trilinear, bilinear, and soft mass (mixing) types. In order not to miss any plausible RPV phenomenological features, it is important that all of the RPV parameters be taken into consideration without *a priori* bias. We do, however, expect some sort of symmetry principle to guard against the very dangerous proton decay problem. The emphasis is hence put on the lepton number violating phenomenology.

The renormalizable superpotential for the GSSM can be written as

$$W = \varepsilon_{ab}\Big[\mu_\alpha \hat{H}_u^a \hat{L}_\alpha^b + h_{ik}^u \hat{Q}_i^a \hat{H}_u^b \hat{U}_k^C + \lambda'_{\alpha jk} \hat{L}_\alpha^a \hat{Q}_j^b \hat{D}_k^C + \frac{1}{2}\,\lambda_{\alpha\beta k} \hat{L}_\alpha^a \hat{L}_\beta^b \hat{E}_k^C\Big] + \frac{1}{2}\,\lambda''_{ijk} \hat{U}_i^C \hat{D}_j^C \hat{D}_k^C\,, \quad (1)$$

where (a,b) are $SU(2)$ indices, (i,j,k) are the usual family (flavor) indices, and (α,β) are extended flavor indices going from 0 to 3. At the limit where λ_{ijk}, λ'_{ijk}, λ''_{ijk} and μ_i all vanish, one recovers the expression for the R-parity preserving MSSM, with $\hat{L}_0$ identified as $\hat{H}_d$. Without R-parity imposed, the latter is not *a priori* distinguishable from the $\hat{L}_i$'s. Note that λ is antisymmetric in the first two indices, as required by the $SU(2)$ product rules, as shown explicitly here with $\varepsilon_{12} = -\varepsilon_{21} = 1$. Similarly, λ'' is antisymmetric in the last two indices, from $SU(3)_C$.

R-parity is exactly an *ad hoc* symmetry put in to make $\hat{L}_0$, stand out from the other $\hat{L}_i$'s as the candidate for $\hat{H}_d$. It is defined in terms of baryon number, lepton number, and spin as, explicitly, $\mathcal{R} = (-1)^{3B+L+2S}$. The consequence is that the accidental symmetries of baryon number and lepton number in the SM are preserved, at the expense of making particles and superparticles having a categorically different quantum number, R parity. The latter is actually not the most effective discrete symmetry to control superparticle mediated proton decay[1], but is most restrictive in terms of what is admitted in the Lagrangian, or the superpotential alone. On the other hand, R parity also forbides neutrino masses in the supersymmetric SM. The strong experimental hints for the existence of (Majorana) neutrino masses is an indication of lepton number violation, hence suggestive of R-parity violation.

The soft SUSY breaking part of the Lagrangian is more interesting, if only for the fact that many of its interesting details have been overlooked in the literature. However, we will postpone the discussion till after we address the parametrization issue.

Doing phenomenological studies without specifying a choice of flavor bases is ambiguous. It is like doing SM quark physics with 18 complex Yukawa couplings, instead of the 10 real physical parameters. As far as the SM itself is concerned, the extra 26 real parameters are simply redundant, and attempts to relate the full 36 parameters to experimental data will be futile. In the GSSM, the choice of an optimal parametrization mainly concerns the 4 $\hat{L}_\alpha$ flavors. We use here the single-VEV parametrization[2, 3] (SVP), in which flavor bases are chosen such that : 1/ among the $\hat{L}_\alpha$'s, only $\hat{L}_0$, bears a VEV, *i.e.* $\langle \hat{L}_i \rangle \equiv 0$; 2/ $h_{jk}^e (\equiv \lambda_{0jk}) = \frac{\sqrt{2}}{v_0}\,\mathrm{diag}\{m_1, m_2, m_3\}$; 3/ $h_{jk}^d (\equiv \lambda'_{0jk} = -\lambda_{j0k}) = \frac{\sqrt{2}}{v_0}\mathrm{diag}\{m_d, m_s, m_b\}$; 4/ $h_{ik}^u = \frac{\sqrt{2}}{v_u} V_{\text{CKM}}^T\,\mathrm{diag}\{m_u, m_c, m_t\}$, where $v_0 \equiv \sqrt{2}\,\langle \hat{L}_0 \rangle$ and $v_u \equiv \sqrt{2}\,\langle \hat{H}_u \rangle$. The big advantage of the SVP is that it gives the complete tree-level mass matrices of all the states (scalars and fermions) the simplest structure[3, 4].

III. LEPTONS IN GSSM

The SVP gives quark mass matrices exactly in the SM form. For the masses of the color-singlet fermions, all the RPV effects are paramatrized by the μ_i's only. For example, the five charged fermions (gaugino + Higgsino + 3 charged leptons), we have

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \frac{g_2 v_0}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{g_2 v_u}{\sqrt{2}} & \mu_0 & \mu_1 & \mu_2 & \mu_3 \\ 0 & 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & m_3 \end{pmatrix} . \tag{2}$$

Moreover each μ_i parameter here characterizes directly the RPV effect on the corresponding charged lepton ($\ell_i = e$, μ, and τ). This, and the corresponding neutrino-neutralino masses and mixings, has been exploited to implement a detailed study of the tree-level RPV phenomenology from the gauge interactions, with interesting results[2].

Neutrino masses and oscillations is no doubt one of the most important aspects of the model. Here, it is particularly important that the various RPV contributions to neutrino masses, up to 1-loop level, be studied in a framework that takes no assumption on the other parameters. Our formulation provides such a framework. Interested readers are referred to Refs.[4–8].

IV. SOFT SUSY BREAKING TERMS AND THE SCALAR MASSES

Obtaining the squark and slepton masses is straightforward, once all the admissible soft SUSY breaking terms are explicitly written down[4]. The soft SUSY breaking part of the Lagrangian can be written as

$$\begin{aligned} V_{\rm soft} \;=\; & \epsilon_{ab} B_\alpha \, H_u^a \tilde{L}_\alpha^b + \epsilon_{ab} \left[A_{ij}^U \, \tilde{Q}_i^a H_u^b \tilde{U}_j^C + A_{ij}^D H_d^a \tilde{Q}_i^b \tilde{D}_j^C + A_{ij}^E H_d^a \tilde{L}_i^b \tilde{E}_j^C \right] + \text{h.c.} \\ + \; & \epsilon_{ab} \left[A_{ijk}^{\lambda'} \tilde{L}_i^a \tilde{Q}_j^b \tilde{D}_k^C + \frac{1}{2} \, A_{ijk}^{\lambda} \tilde{L}_i^a \tilde{L}_j^b \tilde{E}_k^C \right] + \frac{1}{2} \, A_{ijk}^{\lambda''} \tilde{U}_i^C \tilde{D}_j^C \tilde{D}_k^C + \text{h.c.} \\ + \; & \tilde{Q}^\dagger \tilde{m}_Q^2 \, \tilde{Q} + \tilde{U}^\dagger \tilde{m}_U^2 \, \tilde{U} + \tilde{D}^\dagger \tilde{m}_D^2 \, \tilde{D} + \tilde{L}^\dagger \tilde{m}_L^2 \tilde{L} + \tilde{E}^\dagger \tilde{m}_E^2 \, \tilde{E} + \tilde{m}_{H_u}^2 \, |H_u|^2 \\ & + \frac{M_1}{2} \tilde{B}\tilde{B} + \frac{M_2}{2} \tilde{W}\tilde{W} + \frac{M_3}{2} \tilde{g}\tilde{g} + \text{h.c.} \, , \end{aligned} \tag{3}$$

where we have separated the R-parity conserving A-terms from the RPV ones (recall $\hat{H}_d \equiv \hat{L}_0$). Note that $\tilde{L}^\dagger \tilde{m}_L^2 \tilde{L}$, unlike the other soft mass terms, is given by a 4×4 matrix. Explicitly, $\tilde{m}_{L_{00}}^2$ corresponds to $\tilde{m}_{H_d}^2$ of the MSSM case while $\tilde{m}_{L_{0k}}^2$'s give RPV mass mixings.

The only RPV contribution to the squark masses is given by a $-(\,\mu_i^* \lambda'_{ijk}\,)\ \frac{v_u}{\sqrt{2}}$ term in the LR mixing part. Note that the term contains flavor-changing ($j \neq k$) parts which, unlike the A-terms ones, cannot be suppressed through a flavor-blind SUSY breaking spectrum. Hence, it has very interesting implications to quark electric dipole moments (EDMs) and related processses such as $b \to s\,\gamma$[9–11].

The mass matrices are a bit more complicated in the scalar sectors[4, 12]. We illustrated explicitly here only the charged scalare mass matrix. The $1+4+3$ charged scalar masses are given in terms of the blocks

$$\begin{aligned}
\widetilde{\cal M}_{Hu}^2 &= \tilde{m}_{Hu}^2 + \mu_\alpha^* \mu_\alpha + M_Z^2\,\cos 2\beta \left[\frac{1}{2} - \sin^2\!\theta_W\right] + M_Z^2\,\sin^2\!\beta\,\left[1 - \sin^2\!\theta_W\right]\,, \\
\widetilde{\cal M}_{LL}^2 &= \tilde{m}_L^2 + m_L^\dagger m_L + M_Z^2\,\cos 2\beta \left[-\frac{1}{2} + \sin^2\!\theta_W\right] \\
&\qquad + \begin{pmatrix} M_Z^2\,\cos^2\!\beta\,\left[1 - \sin^2\!\theta_W\right] & 0_{1\times 3} \\ 0_{3\times 1} & 0_{3\times 3} \end{pmatrix} + (\mu_\alpha^* \mu_\beta)\,, \\
\widetilde{\cal M}_{RR}^2 &= \tilde{m}_E^2 + m_E m_E^\dagger + M_Z^2\,\cos 2\beta \left[-\sin^2\!\theta_W\right]\;;
\end{aligned} \tag{4}$$

and

$$\widetilde{\cal M}_{LH}^2 \;=\; (B_\alpha^*) + \begin{pmatrix} \frac{1}{2}\,M_Z^2\,\sin 2\beta\,\left[1 - \sin^2\!\theta_W\right] \\ 0_{3\times 1} \end{pmatrix}\,, \tag{5}$$

$$\widetilde{\cal M}_{RH}^2 \;=\; -\,(\,\mu_i^* \lambda_{i0k}\,)\,\frac{v_0}{\sqrt{2}}\,, \tag{6}$$

$$(\widetilde{\cal M}_{RL}^2)^T \;=\; \begin{pmatrix} 0 \\ A^E \end{pmatrix} \frac{v_0}{\sqrt{2}} - (\,\mu_\alpha^* \lambda_{\alpha\beta k}\,)\,\frac{v_u}{\sqrt{2}}\,. \tag{7}$$

Note that $\tilde{m}_L^2$ here is a $4{\times}4$ matrix of soft masses for the L_α, and B_α's are the corresponding bilinear soft terms of the μ_α's. A^E is just the 3×3 R-parity conserving leptonic A-term. There is no contribution from the admissible RPV A-terms under the SVP. Also, we have used $m_L \equiv \mathrm{diag}\{\,0, m_E\,\} \equiv \mathrm{diag}\{\,0, m_1, m_2, m_3\,\}$.

V. NEUTRON ELECTRIC DIPOLE MOMENT

Let us take a look first at the quark dipole operator through 1-loop diagrams with LR squark mixing. A simple direct example is given by the gluino diagram. Comparing with the MSSM case, the extra (RPV) to the d squark LR mixing in GSSM obvious modified the story. If one naively imposes the constraint for this RPV contribution itself not to exceed the experimental bound on neutron EDM, one gets roughly $\mathrm{Im}(\mu_i^* \lambda'_{i11}) \lesssim 10^{-6}\,\mathrm{GeV}$, a constraint that is interesting even in comparison to the bounds on the corresponding parameters obtainable from asking no neutrino masses to exceed the super-Kamiokande atmospheric oscillation scale[9].

In fact, there are important contributions beyond the gluino diagram and without LR squark mixings involved. For the MSSM, it is well-known that there is such a contribution from the chargino diagram, which is likely to be more important than the gluino one when a unification type gaugino mass relationship is imposed. The question then is if the GSSM has a similar RPV analog. A RPV version of the chargino diagram is given in Fig.1. The diagram, however, looks ambiguous. Looking at the diagram in terms of the electroweak states involved under our formulation, it seems like a l_k^{-}–$\tilde{W}^{+}$ mass insertion is required, which is however vanishing. However, putting in extra mass insertion, with a μ_i flipping the l_k^{-} into a $\tilde{h}_u^{+}$ first seems to give a non-zero result. The structure obviously indicates a GIM-like cancellation at worked, and we have to check its violation due to the lack of mass degeneracy.

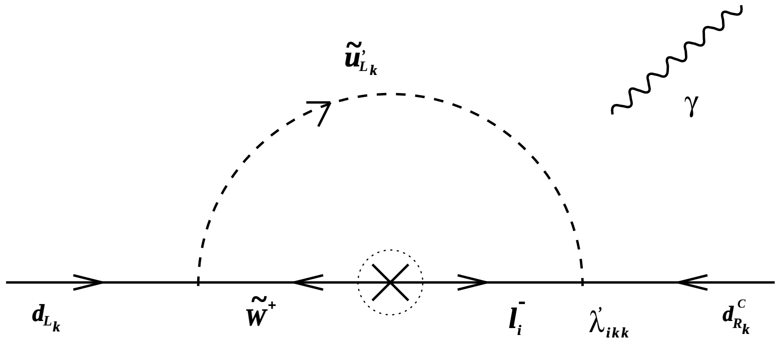

FIG. 1: The new charginolike diagram.

We have performed an extensive analytical and numerical study, including the complete charginolike contributions, as well as the neutralinolike contributions, to the neutron EDM[10]. The charginolike part is given by the following formula :

$$\left(\frac{d_f}{e}\right)_{\chi^-} = \frac{\alpha_{\rm em}}{4\pi\,\sin^2\!\theta_W}\sum_{\tilde{f}'\mp}\sum_{n=1}^{5}{\rm Im}(\mathcal{C}_{fn\mp})\,\frac{M_{\chi_n^-}}{M^2_{\tilde{f}'\mp}}\left[\mathcal{Q}_{\tilde{f}'}\;B\!\left(\frac{M^2_{\chi_n^-}}{M^2_{\tilde{f}'\mp}}\right)+(\mathcal{Q}_f-\mathcal{Q}_{\tilde{f}'})\;A\!\left(\frac{M^2_{\chi_n^-}}{M^2_{\tilde{f}'\mp}}\right)\right]\,, \tag{8}$$

for f being u (d) quark and f' being d (u), where

$$\begin{aligned}
\mathcal{C}_{un-} &= \frac{y_u}{g_2}\,\boldsymbol{V}^*_{2n}\,\mathcal{D}_{d11}\left(-\boldsymbol{U}_{1n}\,\mathcal{D}^*_{d11}+\frac{y_d}{g_2}\,\boldsymbol{U}_{2n}\,\mathcal{D}^*_{d21}+\frac{\lambda'_{k11}}{g_2}\,\boldsymbol{U}_{(k+2)n}\,\mathcal{D}^*_{d21}\right)\,,\\
\mathcal{C}_{un+} &= \frac{y_u}{g_2}\,\boldsymbol{V}^*_{2n}\,\mathcal{D}_{d12}\left(-\boldsymbol{U}_{1n}\,\mathcal{D}^*_{d12}+\frac{y_d}{g_2}\,\boldsymbol{U}_{2n}\,\mathcal{D}^*_{d22}+\frac{\lambda'_{k11}}{g_2}\,\boldsymbol{U}_{(k+2)n}\,\mathcal{D}^*_{d22}\right)\,,\\
\mathcal{C}_{dn-} &= \left(\frac{y_d}{g_2}\,\boldsymbol{U}_{2n}+\frac{\lambda'_{k11}}{g_2}\,\boldsymbol{U}_{(k+2)n}\right)\mathcal{D}_{u11}\left(-\boldsymbol{V}^*_{1n}\,\mathcal{D}^*_{u11}+\frac{y_u}{g_2}\,\boldsymbol{V}^*_{2n}\,\mathcal{D}^*_{u21}\right)\,,\\
\mathcal{C}_{dn+} &= \left(\frac{y_d}{g_2}\,\boldsymbol{U}_{2n}+\frac{\lambda'_{k11}}{g_2}\,\boldsymbol{U}_{(k+2)n}\right)\mathcal{D}_{u12}\left(-\boldsymbol{V}^*_{1n}\,\mathcal{D}^*_{u12}+\frac{y_u}{g_2}\,\boldsymbol{V}^*_{2n}\,\mathcal{D}^*_{u22}\right)\,,
\end{aligned}$$

(only repeated index i is to be summed) ; (9)

$\boldsymbol{V}^\dagger\mathcal{M}_C\,\boldsymbol{U} = {\rm diag}\{M_{\chi_n^-}\}\equiv{\rm diag}\{M_{c1},M_{c2},m_e,m_\mu,m_\tau\}$ while $\mathcal{D}_u$ and $\mathcal{D}_d$ diagonalize the $\tilde{u}$ and $\tilde{d}$ squark mass-squared matrices respectively; and

$$A(x)=\frac{1}{2\,(1-x)^2}\left(3-x+\frac{2\ln x}{1-x}\right)\,,\qquad B(x)=\frac{1}{2\,(x-1)^2}\left[1+x+\frac{2\,x\ln x}{(1-x)}\right]. \tag{10}$$

To extract the contribution from the diagram of Fig. 1, we have to look at the pieces in $\mathcal{C}_{dn\mp}$ with a $\boldsymbol{V}^*_{1n}$ and a $\boldsymbol{U}_{(k+2)n}$. It is easy to see that the $n = 1$ and 2 mass eigenstates, namely the chargino states, do give the dominating contribution. With the small μ_i mixings strongly favored by the sub-eV neutrino masses, we have

$$\boldsymbol{U}_{(k+2)1} = \frac{\mu_k^*}{M_{c1}} R_{R_{21}} \qquad \text{and} \qquad \boldsymbol{U}_{(k+2)2} = \frac{\mu_k^*}{M_{c2}} R_{R_{22}} \tag{11}$$

where the R_R denotes the right-handed rotation that would diagonalize the first 2×2 block of $\mathcal{M}_C$. The latter rotation matrix is expected to have elements of order 1. Hence, we have the dominating result proportional to

$$\sum_{n=1,2} R^*_{R_{12}}\, R_{R_{2n}}\, \mu_k^*\, \lambda'_{k11}\, F_{BA}\Big(M^2_{c_n}\Big)$$

where F_{BA} denotes the mass eigenvalue dependent part. The result agrees with what we say above. It vanishes for $M_{c1} = M_{c2}$, showing a GIM-like mechanism. However, with unequal chargino masses, our numerical results indicate that the cancellation is generically badly violated. More interestingly, it can be seen from the above analysis that a complex phase in $\mu_k^*\, \lambda'_{k11}$ is actually no necessary for this potentially dominating chargino contribution to be there, so long as complex CP violating phases exist in the R_R matrix, *i.e.* in the R-parity conserving parameters such as μ_0.

An illustration of the result is given in Fig. 3 in which variations of the EDM contribution against the tanβ value is plotted. On the whole, the magnitude of the parameter combination $\mu_i^* \lambda'_{i11}$ is shown to be responsible for the RPV 1-loop contribution to neutron EDM and is hence well constrained. This applies not only to the complex phase, or imaginary part of, the combination. Readers are referred to Ref.[10] for more details.

VI. DIPOLE MOMENTS OF THE ELECTRON AND OTHER FERMIONS

There is in fact a second class of 1-loop diagrams contributing to the quark EDMs. These are diagrams with quarks and scalars in the loop, and hence superpartners of the charginolike and neutralinolike diagrams discussed above. The R-parity conserving analog of the class of diagrams has no significance, due to the unavoidable small Yukawa couplings involved. With the latter replaced by flavor-changing λ'-couplings. We can have a t quark loop contributing to neutron EDM, for example.

For the case of the charged leptons, the two classes of superpartner diagrams merges into one. But then, all scalars has to be included. The assumption hidden, in our quark EDM formula above, that only the (two) superpartner sfermions have a significant role to play does not stand any more.

The above quark EDM formula obviously applies with some trivial modifications to the cases of the other quarks. For the charge leptons, while the exact formulae would be different, there are major basic features that are more or less the same. For instance, for the charged lepton, the λ-couplings play the role of the λ'-couplings. The $\mu_i^* \lambda_{i11}$

combination contributes to electron EDM while the $\mu_i^* \lambda_{i22}$ combination contributes to that of the muon. As we have no explicit numerical results to show at the moment, we refrain from showing any details here. However, we have finished a $\mu \to e\,\gamma$ study[12], from which the charged lepton EDM formula could be extracted without too much effort. Interested readers may check the reference for details.

VII. NEUTRINO DIPOLE MOMENTS

Another topic we want to discuss briefly here is the dipole moments of the neutrinos. Neutrinos as Majorana fermions have vanishing dipole moments. However, flavor off-diagonal dipole moments, or known as transition dipole moments are interesting. There are good terrestial as well as astrophysical and cosmological bounds available[13].

The same set of diagrams giving rise to 1-loop neutrino masses within the model give rise also to dipole moments when an extra photon line is attached. There are two types of

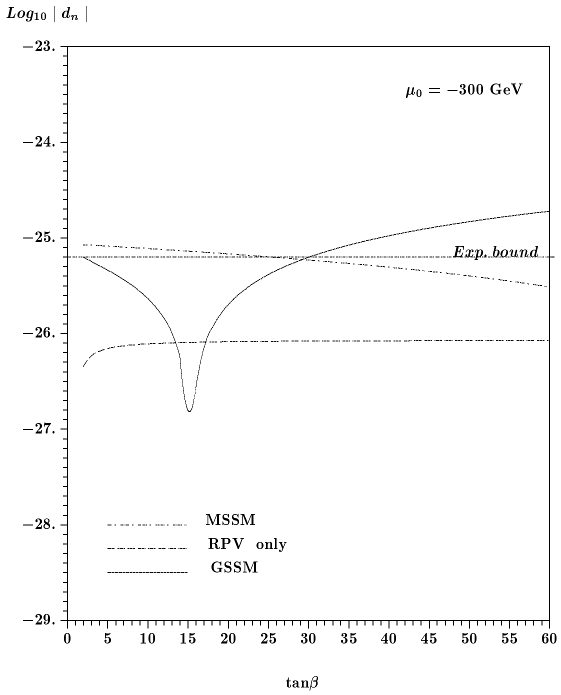

FIG. 2: Logarithmic plot of (the magnitude of) the neutron EDM result verses tanβ. We show here the MSSM result, our general result with RPV phase only, and the generic result with complex phases of both kinds. In particular, the A and μ_0 phases are chosen as 7^o and 0.1^o respectively, for the MSSM line. They are zero for the RPV-only line, with which we have a phase of $\frac{\pi}{4}$ for λ'_{311}. All the given nonzero values are used for the three phases for the generic result (from our complete formulae) marked by GSSM.

such neutrino mass diagrams, the charged and neutral loop ones. A neutral loop diagram has, of course, no place to attach a photon line. Hence, only the charged loop diagrams contribute. Checking parameter fits to both neutrino masses and their implications on dipole moments would be very interesting.

We give in Ref.[7], all contributions to 1-loop neutrino masses within GSSM under a systematic framework. For example, each diagram composes of two (external) neutrino interaction vertices. The charged vertices are given by

$$\begin{aligned}
\mathcal{C}^{R}_{inm} &= \frac{y_{e_i}}{g_2}\,\boldsymbol{V}_{(i+2)n}\,D^{l^*}_{2m} - \frac{\lambda^*_{ikh}}{g_2}\boldsymbol{V}_{(h+2)n}\,D^{l^*}_{(k+2)m}\;, \\
\mathcal{C}^{L}_{inm} &= -\boldsymbol{U}_{1n}\,\mathcal{D}^{l^*}_{(i+2)m} + \frac{y_{e_i}}{g_2}\,\boldsymbol{U}_{2n}\,\mathcal{D}^{l^*}_{(i+5)m} - \frac{\lambda_{ihk}}{g_2}\,\boldsymbol{U}_{(h+2)n}\,\mathcal{D}^{l^*}_{(k+5)m}\;.
\end{aligned} \tag{12}$$

A $\mathcal{C}^{R^*}_{jnm}\,\mathcal{C}^{L}_{inm}$ combination plays the role of $\mathcal{C}_{fn\mp}$ in the formula of Eq.(8), for ν_i and ν_j. Here, we are interested not only in the imaginary part; the real part contribute magnetic moments. Nevertheless, we have to switch back to the mass eigenstate basis for the neutrinos to better understand and use the dipole moment results[14].

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