Improved Model–Independent Analysis of Semileptonic and Radiative Rare B Decays

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Based on: A. Ali, E. L., C. Greub, G. Hiller, hep-ph/0112300

Motivations & Questions

Plenty of data from the B–Factories:

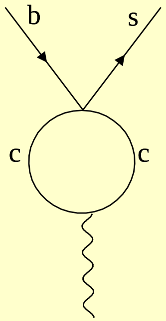
$$
\begin{aligned}
\mathcal{B}(B\to X_s\gamma)_{\rm WA} &= (3.22 \pm 0.40) \times 10^{-4} \\
\mathcal{B}(B\to K\mu^+\mu^-) &= (0.99^{+0.40+0.13}_{-0.32-0.14}) \times 10^{-6} \\
\mathcal{B}(B\to K e^+e^-) &= (0.48^{+0.32+0.09}_{-0.24-0.11}) \times 10^{-6} \\
\mathcal{B}(B\to K\ell^+\ell^-) &= \begin{cases} (0.75^{+0.25}_{-0.21} \pm 0.09) \times 10^{-6} \\ (0.84^{+0.30+0.10}_{-0.24-0.18}) \times 10^{-6} \end{cases} \\
\mathcal{B}(B\to K^*\mu^+\mu^-) &\leq 3.0 \times 10^{-6} \text{ at 90\% C.L.} \\
\mathcal{B}(B\to K^* e^+e^-) &\leq 5.1 \times 10^{-6} \text{ at 90\% C.L.} \\
\mathcal{B}(B\to X_s\mu^+\mu^-) &= (8.9^{+2.3+1.6}_{-2.4-1.7}) \times 10^{-6} \\
\mathcal{B}(B\to X_s e^+e^-) &= (5.1^{+2.6+1.3}_{-2.4-1.2}) \times 10^{-6} \\
\mathcal{B}(B\to X_s\ell^+\ell^-) &= (7.1^{+1.6+1.4}_{-1.6-1.2}) \times 10^{-6}
\end{aligned}
$$

Questions to be addressed:

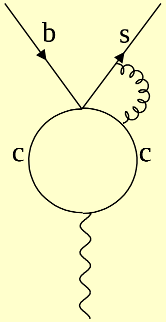
- Which observables can establish New Physics?
- Experiment vs Theory!
- What is the impact of the combined $b \to s\gamma$ and $b \to s\ell\ell$ measurements?
- What about SUSY?
  - Is it possible to establish SUSY with rare decays only?
  - Can they exclude specific models?
  - What happens in Minimal Flavour Violating SUSY models?
  - And in more general models?

$B \to X_s \gamma$

- NLO precision: $\langle \mathcal{A} \rangle = A_0 + A_1 \alpha_s + O(\alpha_s^2)$
- Power corrections: $\frac{\Lambda^2}{m_b^2}, \frac{\Lambda^2}{m_c^2}$
- Issue of the charm mass:



$= 0$ but



$\neq 0$

SM Prediction(s)

For $\hat{m}_c = m_c/m_b = \underline{0.29} \pm 0.02$ (pole mass):
$\mathcal{B}(B \to X_s \gamma)^{SM} = (\underline{3.29} \pm 0.33) \times 10^{-4}$

For $\hat{m}_c = \underline{0.22} \pm 0.04$ ($\overline{MS}$ mass at $\mu = \mu_b$):
$\mathcal{B}(B \to X_s \gamma)^{SM} = (\underline{3.73} \pm 0.30) \times 10^{-4}$

For $\hat{m}_c = 0.25$ (average):
$\mathcal{B}(B \to X_s \gamma)^{SM} = (3.50 \pm 0.50) \times 10^{-4}$

$B \to K^{(*)} \ell^+ \ell^-$

- NNLO precision is at hand.
- Large Form Factors uncertainties.

$B \to K^* \gamma$ problem: data require a value of the FF smaller than its typical QCD Sum Rule estimate!
$\Rightarrow$ We choose to use the minimum allowed Form Factors.

SM Predictions

$$\mathcal{B}(B \to K \ell^+ \ell^-) = (0.35 \pm 0.12) \times 10^{-6} \overset{\text{exp}}{=} (0.79 \pm 0.21) \times 10^{-6}$$

$$\mathcal{B}(B \to K^* e^+ e^-) = (1.58 \pm 0.49) \times 10^{-6} \overset{\text{exp}}{\leq} 5.1 \times 10^{-6}$$

$$\mathcal{B}(B \to K^* \mu^+ \mu^-) = (1.19 \pm 0.39) \times 10^{-6} \overset{\text{exp}}{\leq} 3.0 \times 10^{-6}$$

$B \to X_s \ell^+ \ell^-$ (Full Spectrum)

- NNLO precision in the low-s region:
  $\langle \mathcal{A} \rangle = A_0 + A_1 \alpha_s + O(\alpha_s^2)$
- Power corrections: $\frac{\Lambda^2}{m_b^2}$, $\frac{\Lambda^2}{m_c^2}$
- No charm mass issue! ($c\bar{c}$ resonances)
- in the high-s region we drop the problematic NNLO corrections to the matrix elements and set $\mu_b = 2.5$ GeV

SM Predictions

$$\mathcal{B}^{\rm full}(B \to X_s e^+ e^-) = (6.89 \pm 1.01) \times 10^{-6}$$

$$\stackrel{\rm exp}{=} (8.9^{+2.3+1.6}_{-2.4-1.7}) \times 10^{-6}$$

$$\mathcal{B}^{\rm full}(B \to X_s \mu^+ \mu^-) = (4.15 \pm 0.70) \times 10^{-6}$$

$$\stackrel{\rm exp}{=} (5.1^{+2.6+1.3}_{-2.4-1.2}) \times 10^{-6}$$

$$\mathcal{B}^{\rm full}(B \to X_s \ell^+ \ell^-) = (5.52 \pm 0.94) \times 10^{-6}$$

$$\stackrel{\rm exp}{=} (7.1^{+1.6+1.4}_{-1.6-1.2}) \times 10^{-6}$$

$B \to X_s \ell^+ \ell^-$ (Low-s region)

- We define the low-s regions as follow:

$$0.2 \text{ GeV} \le \sqrt{s_{ee}} \le M_{J/\Psi} - 0.60 \text{ GeV}$$

$$2\, m_\mu \le \sqrt{s_{\mu\mu}} \le M_{J/\Psi} - 0.35 \text{ GeV}$$

SM Predictions

$$\mathcal{B}^{\rm low}(B \to X_s e^+ e^-) = (2.47 \pm 0.40) \times 10^{-6}$$

$$\mathcal{B}^{\rm low}(B \to X_s \mu^+ \mu^-) = (2.75 \pm 0.45) \times 10^{-6}$$

SM Operator Basis for $b \to s\gamma$ and $b \to s\ell^+\ell^-$ decays

$$\begin{aligned}
\mathcal{H}_{\rm eff} &= -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu)\, O_i(\mu) \\
O_7 &= \frac{e}{g_s^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \\
O_8 &= \frac{1}{g_s} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu} \\
O_9 &= \frac{e^2}{g_s^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell) \\
O_{10} &= \frac{e^2}{g_s^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma_5 \ell)
\end{aligned}$$

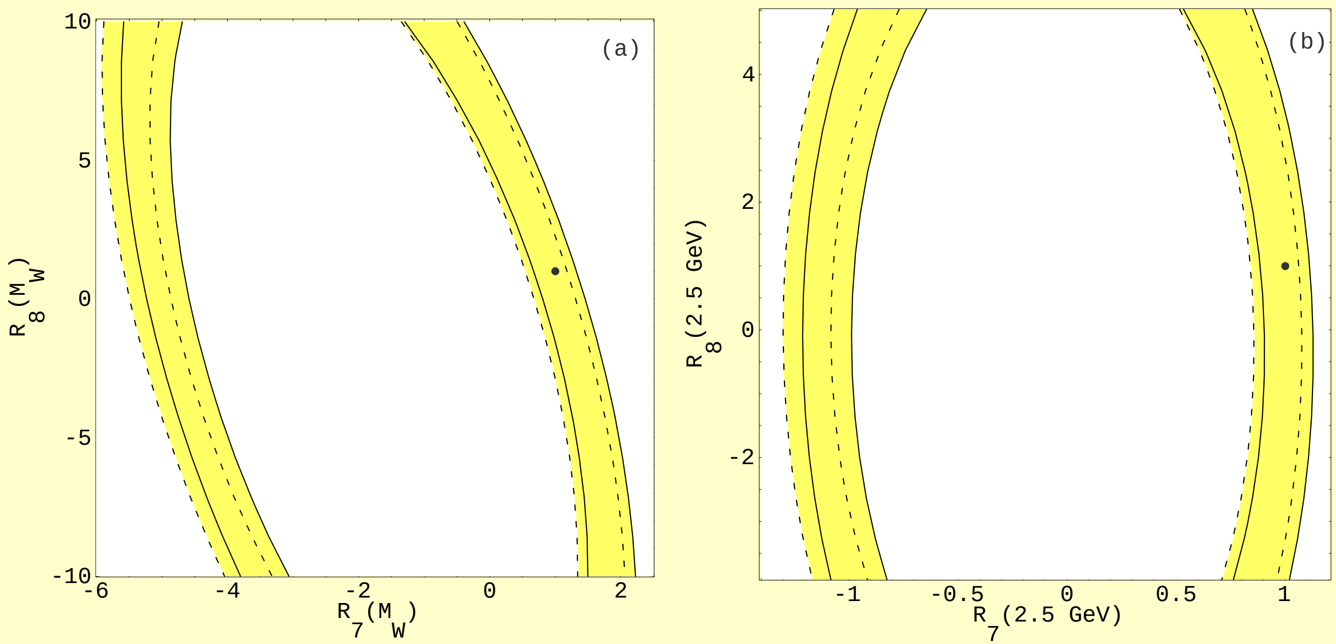
$$C_i(\mu) = C_i^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)}(\mu) + \ldots \Longrightarrow C_i^{(0)} \text{ is } \begin{cases} \neq 0 & i = 1 \div 6, 9 \\ = 0 & i = 7, 8, 10 \end{cases}$$

From γ to $\ell^+\ell^-$: I round

- At 90% C.L.: $2.56 \times 10^{-4} \leq \mathcal{B}(B \to X_s \gamma) \leq 3.88 \times 10^{-4}$
- $\hat{m}_c = 0.25$
- $-10 \leq C_8(M_W)/C_8^{SM}(M_W) \leq 10$ from $b \to sg$ and $B \to X_{\not c}$
- The theoretical error ($\delta_{\mathcal{B}}^{\rm th} = 14\%$) is included

Note that $b \to s\ell^+\ell^-$ decays depend on $C_7(\mu_b)$ only.

$R_i = C_i/C_i^{\rm SM}$

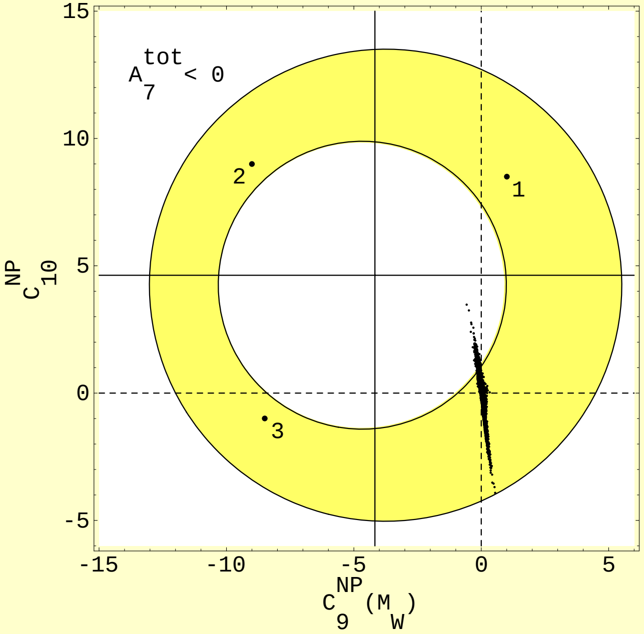


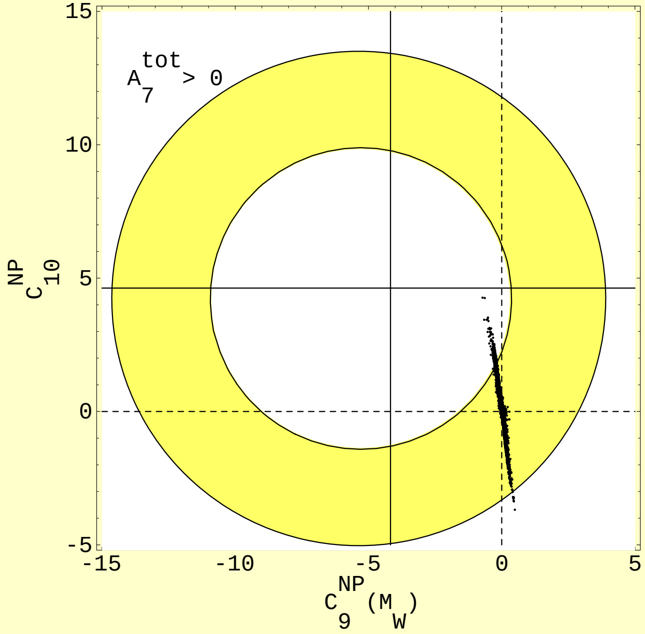
From γ to $\ell^+\ell^-$: II round

- The 90% C.L. bounds that come from $B \to X_s\gamma$ are:

$$-0.37 \le C_7(\mu_b) \le -0.23 \quad \& \quad 0.24 \le C_7(\mu_b) \le 0.38$$

- The operator basis is the same as in the SM
- The most stringent bounds come from $B \to X_s e^+e^-$ and $B \to K\ell^+\ell^-$
- We add the theory errors ($\delta_{see} = 15\%$, $\delta_{k\ell\ell} = 34\%$)



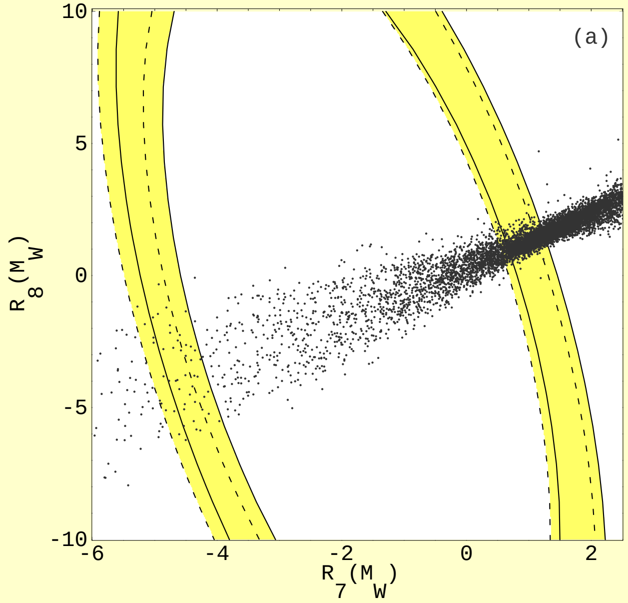


Minimal Flavour Violating MSSM

- The MFV parameter space is:

$M_{\tilde{t}} = 90$ GeV $\div$ 1 TeV
$\theta_{\tilde{t}} = -\pi/2 \div \pi/2$
$\mu = -1$ TeV $\div$ 1 TeV
$M_2 = 0 \div 1$ TeV
$M_{H^\pm} = 78.6$ GeV $\div$ 1 TeV
$M_{\tilde{q}} \simeq 1$ TeV, $M_{\tilde{\nu}} \geq 50$ GeV

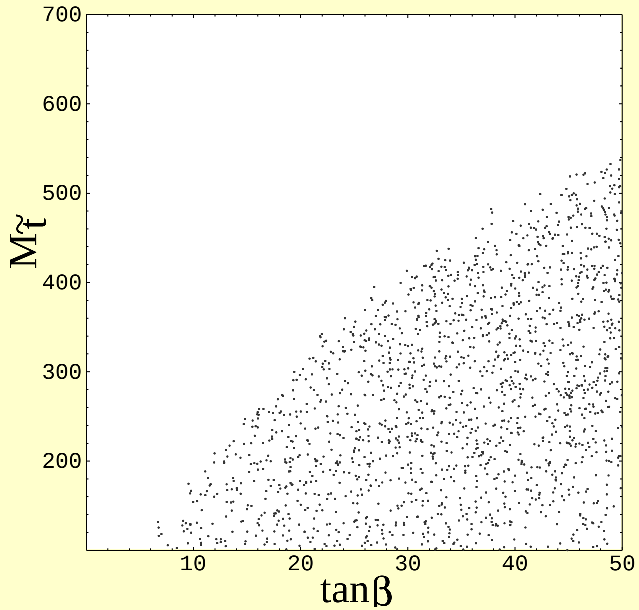
- Correlation between C_7 and C_8 in MFV:



- Allowed ranges for C_9 and C_{10} compatible with the $B \to X_s\gamma$ constraint:

  - $C_7 > 0 \Rightarrow \begin{cases} C_9 = -0.2 \div 0.3 \\ C_{10} = -0.8 \div 0.5 \end{cases}$
  - $C_7 < 0 \Rightarrow \begin{cases} C_9 = -0.2 \div 0.4 \\ C_{10} = -1.0 \div 0.7 \end{cases}$

- Points that satisfy the $B \to X_s\gamma$ constraint with a positive (opposite to the SM) C_7:



Extended–MFV MSSM

- Heavy squark–gluino mass spectrum
- The MFV hypothesis is not imposed
- In the Mass Insertion Approximation, only 2 insertions can play a role:

$$\delta_{\tilde{u}_L \tilde{t}_2} \equiv \frac{M^2_{\tilde{u}_L \tilde{t}_2}}{M_{\tilde{q}} M_{\tilde{t}_2}} \frac{|V_{td}|}{V^*_{td}}$$

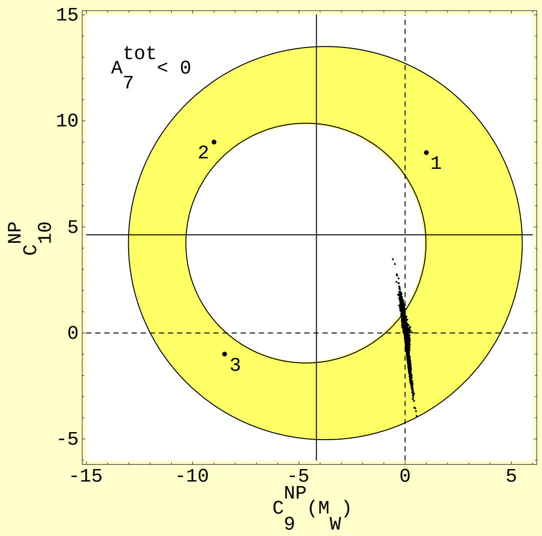
$$\delta_{\tilde{c}_L \tilde{t}_2} \equiv \frac{M^2_{\tilde{c}_L \tilde{t}_2}}{M_{\tilde{q}} M_{\tilde{t}_2}} \frac{|V_{ts}|}{V^*_{ts}}$$

- Parameters: μ, M_2, $\tan\beta_S$, $M_{H^\pm}$, $M_{\tilde{t}_2}$, $\theta_{\tilde{t}}$, $\delta_{\tilde{u}_L \tilde{t}_2}$, $\delta_{\tilde{c}_L \tilde{t}_2}$

$\delta_{\tilde{u}_L \tilde{t}_2}$: $b \to d\gamma$, $b \to d\ell^+\ell^-$, $B_d \to \ell^+\ell^-$, ΔM_{B_d}, $\Delta M_{B_s}/\Delta M_{B_d}$, $a_{\psi K_s}$, ϵ_K

The detailed analysis is presented in:
A. Ali and E. L., Eur. Phys. J. C21, 683 (2001)

$\delta_{\tilde{c}_L \tilde{t}_2}$: $b \to s\gamma$, $b \to s\ell^+\ell^-$, $B_s \to \ell^+\ell^-$, $\Delta M B_s/\Delta M B_d$



Conclusions

- Data on radiative and semileptonic rare B decays provide tight model independent bounds on the relevant Wilson Coefficients
- There are still open theoretical problems:
  - $B \to X_s \gamma$: Issue of the charm mass
  - $B \to X_s \ell^+ \ell^-$: NNLO precision only available in the low-s region
  - $B \to K^{(*)} \ell^+ \ell^-$: Validity of the QCD sum rules Form Factors
- Minimal Flavour Violating models:
  - Contributions to C_9 and C_{10} are small
  - The detection of a positive C_7 would imply the presence of a light stop, i.e. $M_{\tilde{t}} \leq 500$ GeV