$B \to X_s\gamma$ in minimal supersymmetric standard model with general flavor mixing*

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Abstract

We present the results of our analysis on $b \to s\gamma$ in the minimal supersymmetric standard model with a general flavor mixing in the squark sector. We identify the next–to–leading order focusing effect, which considerably reduces the gluino contribution to $b \to s\gamma$ relative to the leading order calculation. We illustrate that $BR(B \to X_s\gamma)$ is still very sensitive to the flavor mixing parameters, δ^d_{LL} and δ^d_{LR} so that the lower bound for $m_{1/2}$ obtained in the minimal flavor violation can be easily removed even at $\mu < 0$, while it is rather stable against δ^d_{RR} and δ^d_{RL}.

I Introduction

Radiative decays of B–mesons provide a unique opportunity for exploring physics beyond the SM. In the SM, a leading contribution is from a virtual W–top (charm/up) loop accompanied by a real photon with GIM suppression. New physics effects, which are also loop induced, could be same order if mass scale of the new physics is not far from that of W and top-quark. This is exactly the case of softly broken low–energy supersymmetry (SUSY).

At early stage of the investigation, $b \to s\gamma$ was expected to provide a clear signature of supersymmetry [2], however, experimental and theoretical progress have eventually revealed that there remains little room for a new physics contribution. The world average of the branching ratio has reached an accuracy of 10 % level,

$$BR(B \to X_s\gamma) = (3.41 \pm 0.36) \times 10^{-4}, \tag{1}$$

where four independent measurements [3] are combined, including a recent result from BaBar collaboration. ‡ Theoretical calculation of the inclusive decay in the SM has improved over several stages and next–to–leading order (NLO) QCD calculation has completed recently [5],

$$BR(B \to X_s\gamma) = (3.70 \pm 0.30) \times 10^{-4}. \tag{2}$$

There are a variety of numbers quoted by different authors possibly because of their choice of input, however, they are overlapping with the measured value within their 1 σ errors, whose magnitude is similar to those of the experiments. The SM again successfully explain the process with this level of accuracy and therefore new physics effects should be confined to lie within the remaining uncertainty. In particular, possible mass spectrum and flavor mixing of SUSY particles are expected to be strongly constrained from these results. To discuss them with the quoted accuracy, it becomes important to introduce (SUSY–) QCD corrections beyond LO.

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*This talk is based on reference [1].

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‡We add up all the errors of a single measurement in quadrature choosing a conservative value from asymmetric errors.

Going beyond the SM, NLO calculation has also been completed in two Higgs doublet model (2HDM) [6, 7]. In supersymmetry, NLO formulas are available only for limited parameter space [8], while NLO-level analysis has been performed assuming minimal flavor violation (MFV) [9]. In the analysis, two dominant effects, resummation of QCD logarithms and threshold correction to the bottom quark mass [10] have been identified. On the other hand, most of the analysis including general flavor mixing (GFM) in squark sector so far has been done using LO formulas or, at most, with LO matching of SUSY contribution [11, 12, 13].

In this paper, we present the results of our analysis of $b \to s\gamma$ in the minimal supersymmetric standard model (MSSM) with GFM in the squark sector. In addition to the NLO contributions from the SM and the 2HDM, we introduce NLO QCD corrections and leading NLO SUSY–QCD effects to dominant SUSY contributions, which enable us to compare the results with the experiments and existing NLO-level analysis in MFV.

II Minimal Flavor Violation

In the SM, after integrating out W and top quark, the short-distance physics is described by the following effective Lagrangian [4],

$$\mathcal{L} = \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} \left[C_i(\overline{\mu}) P_i + C_i'(\overline{\mu}) P_i' \right], \tag{3}$$

where P_i' represent chirality partners of P_i, which are suppressed by m_s/m_b in the SM while this is not necessarily true in its extensions. At $\overline{\mu} = \overline{\mu}_{\rm W}$, the Wilson coefficient of the four-fermion operator C_2 has a tree level contribution and dominant loop contribution comes to the coefficient of (chromo–) magnetic operator, C_7 (C_8). These coefficients mix with each other by renormalization group evolution between $\overline{\mu}_{\rm W}$ and $\overline{\mu}_{\rm b}$. Then $BR(B \to X_s\gamma)$ is evaluated at $\overline{\mu}_{\rm b}$ based on the heavy–quark expansion. For this part of calculation, we follow the NLO calculation of [4] with updated numbers in [5].

In the 2HDM, the charged Higgs boson H^- also contributes to $C_{7,8}$ replacing W in the SM. This always adds constructively to the contribution from the SM, which pushes up the mass of H^- until it almost decouples form the process unless other contributions cancel it. We use matching condition in [14] at $\overline{\mu}_{\rm W}$ to estimate this contribution at NLO.

In the MSSM, SUSY doubles the number of physical states and the contributions to $b \to s\gamma$ of CKM origin. SUSY partners of W and H^- mix with each other and form charginos, which contribute to the process with up-type squarks as the corresponding contributions in the SM/2HDM. This contribution comes constructively or destructively depending on whether the higgsino mass parameter $\mu < 0$ or $\mu > 0$. If we assume the CKM matrix is a unique source of flavor mixing in both quark and squark and limit ourselves to the above contributions (MFV framework), $b \to s\gamma$ provides a strict bound on mass spectrum of SUSY partners.

We illustrate this in Constrained MSSM (CMSSM). In FIG. 1, we take $\tan\beta = 10$ (left) and $\tan\beta = 40$ (right) and fix $A_0 = 0$. We scan m_0 over the region below 1000 GeV and impose the constraints from direct SUSY searches. The green band shows a calculation, where the one–loop chargino contribution to $C_{7,8}$ is estimated at $\overline{\mu}_{\rm W}$. The blue band shows a improved calculation, where the SUSY contribution is estimated at the common mass scale of squarks, $\overline{\mu}_{\rm SUSY}$ using NLO QCD matching condition in [14] and evolved down to $\overline{\mu}_{\rm W}$ using NLO anomalous dimensions in [15] with 6 quark flavor. Here we also take into account the effect of threshold correction to the bottom quark mass in the charged Higgs and higgsino

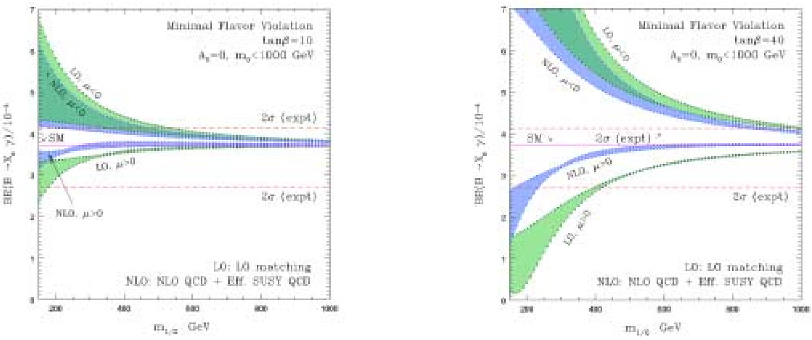

FIG. 1: We plot $BR(B \to X_s\gamma)$ *vs.* $m_{1/2}$ in the Constrained MSSM (MFV) for $\tan\beta = 10$ (left) and $\tan\beta = 40$ (right). We show the bands obtained by varying m_0 in the approximation of using only LO matching conditions (green) and of including NLO corrections (purple). The SM prediction and 2σ CL experimental limits are also marked.

couplings as described later, which corresponds to a leading $\tan\beta$ enhanced part of the NLO SUSY–QCD correction. This calculation reproduces well the results of the second reference of [9] in MFV. The dashed two lines indicate the 2σ experimental bounds and the solid line shows the prediction in the SM. As clearly seen, the μ negative case and low $m_{1/2}$ region are disfavored, especially, at larger value of $\tan\beta$. Note that the NLO–level corrections are playing a crucial role in setting a lower bound of $m_{1/2}$ from the plot at $\tan\beta = 40$ and $\mu > 0$.

These bounds in MFV are rather model dependent, however, often used as one of reliable constraint to estimate other processes in supersymmetry, especially, where flavor mixing is less important, such as in detection of LSP cold dark matter [16]. These bounds often exclude parameter regions, which otherwise predict interesting signature in on–going or future experiments. If we take this seriously, a question arises how robust the bounds really are, if we go beyond MFV.

III General Flavor Mixing

Without any ad–hoc ansatz, gauge symmetry of the SM can not fix the flavor mixing in the soft SUSY breaking terms or, in turn, the squark mass matrices. This general flavor mixing in down-type squarks induces new neutralino and gluino contributions to $b \to s\gamma$. The latter is important because of enhancement by strong coupling constant. To parameterize this new flavor mixing, it is convenient to introduce following dimensionless parameters,

$$\delta^d_{LL} = \frac{(m^2_Q)_{23}}{\sqrt{(m^2_Q)_{22}(m^2_Q)_{33}}}, \quad \delta^d_{LR} = \frac{m_b(A_d)^*_{32}}{\sqrt{(m^2_D)_{33}(m^2_Q)_{22}}}, \tag{4}$$

and similarly for δ^d_{RR} and δ^d_{RL}. Counter–parts of these flavor violating (FV) terms between 1–2 generations are severely constrained by CP violating parameter in K–$\overline{K}$ mixing, while these 2–3 FV–terms have been rather stringently constrained by $BR(B \to X_s\gamma)$ [11, 12].

In this article we extend the analyses including leading NLO corrections. As explained in the previous section with MFV, we introduce NLO resummation of QCD logs to $C^{(\prime)}_{7,8}$ assuming the common mass scale, $\overline{\mu}_{\rm SUSY}$ for squarks and gluinos, which tend to be heavier than other states in most of interesting SUSY breaking models. As with the chargino contribution, we follow [14] for NLO QCD matching of the neutralino and gluino contributions. We checked that SUSY loop–induced contributions to $C^{(\prime)}_{1-6}$ is numerically less important in the estimation of $BR(B \to X_s\gamma)$. [§] NLO matching also requires $O(\alpha_s)$ corrections from gluino. To include this NLO SUSY–QCD corrections, we introduce the effective coupling constants for W and H^- after decoupling of squarks and gluino [7]. We have re-calculated them including full flavor mixing. On the other hand, this procedure is not justified for the SUSY contributions themselves which 'decouple' in its framework. Instead of calculating full two-loop diagrams, we take into account leading $\tan\beta$ enhanced parts of them. We insert finite counter–terms into the LO SUSY contributions, which are required by the renormalizaton of the quark mass matrix to keep their flavor–diagonal form. These 'two–loop' level corrections are enhanced by an explicit $\tan\beta$ factor relative to the corresponding two–loop diagrams. This insertion is performed on higgsino coupling [9] and the F–term contributions of squark mass matrix. In the calculation, we include all sources of flavor mixing by numerically diagonalizing mass matrices, instead of using the mass–insertion approximation. Technical details of the calculation can be found in the reference [1]. In the following section, we will illustrate the impact of these corrections beyond LO and robustness of the MFV prediction against GFM with them.

IV Focusing effect

We have identified NLO focusing effect especially in large $\tan\beta$ and $\mu > 0$ region, which considerably relaxes the constraints on the squark flavor–mixing. We illustrate this in CMSSM inserting the FV–terms at $\overline{\mu}_{\rm SUSY}$. FIG. 2 shows similar plots of $BR(B \to X_s\gamma)$ as FIG. 1 with GFM for $\tan\beta = 40$. The upper middle window is a case for $\delta^d_{LR} = \pm 0.02$ at $\mu > 0$. The green bands correspond to a calculation with LO matching condition and the blue bands show that with the leading NLO corrections. The branching ratio is strongly focusing toward its SM value, once the NLO-level corrections are included. Similar strong focusing at $\mu > 0$ is generic for $\delta^d_{RL} = 0.05$ (upper right), $\delta^d_{LL} = \pm 0.1$ (lower middle) and $\delta^d_{RR} = 0.5$ (lower right). We do not plot the case for $\delta^d_{RL} = -0.05$ and $\delta^d_{RR} = -0.5$ because the branching ratio is almost symmetric about these FV–terms. At $\mu < 0$, focusing is not obvious as seen in the cases of δ^d_{LR} (upper left) and δ^d_{LL} (lower left) and we suppress δ^d_{RL} and δ^d_{RR} cases here.

Focusing effect comes from two sources. Firstly, renormalization group evolution of $C_{7,8}$ from $\overline{\mu}_{\rm SUSY}$ to $\overline{\mu}_{\rm W}$ reduces the overall amplitude of these coefficients. NLO QCD matching of them at $\overline{\mu}_{\rm SUSY}$ brings further suppression. Secondly, gluino contribution to the mass matrix of down-type quark shows remarkable alignment with the dominant Wilson coefficients at $\mu > 0$, which is taken into account by including the leading NLO SUSY–QCD corrections at $\overline{\mu}_{\rm SUSY}$ as described in the previous section. This considerably reduces the LO gluino contribution to $C^{(\prime)}_{7,8}$. This alignment is enhanced by $\tan\beta$ and dominates the focusing effect in the figure at $\mu > 0$. At $\mu < 0$, the SUSY–QCD corrections cause anti–alignment instead, which competes with the overall suppression by renormalization group evolution and results in small focusing (or de–focusing) in the figure. Some NLO suppression of SUSY contribution already exits in

[§] Detailed analysis at LO [12] shows that a complete analysis with GFM introduces new operators other than those in Eq.(3). Inclusion of them is out of scope of this paper, while the analysis has shown that they are less important at LO.

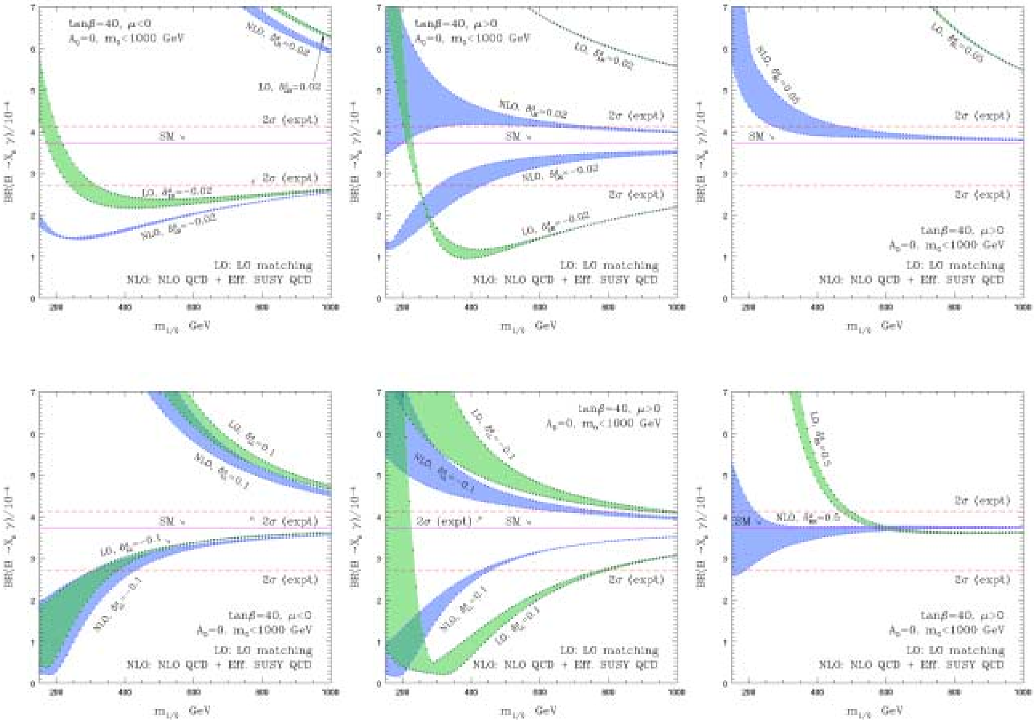

FIG. 2: Similar plots as in FIG. 1 with General Flavor Mixing in down-type squark.

MFV as shown in FIG. 1, however, flavor mixing is essential for the focusing effect with GFM as described above. More detailed discussion can be found in reference [1].

V Relaxation of Constraints on SUSY and FV Terms

The introduction of GFM changes the whole picture of $b \to s\gamma$ constraint on SUSY parameters. Therefore, the focusing effect has strong implication on analysis of SUSY models beyond MFV. We again illustrate this in CMSSM. FIG. 3 shows contour–plots of $BR(B \to X_s\gamma)$ in the $m_{1/2}$ and FV–term plane. Here, we choose the CMSSM parameters as $\tan\beta = 40$, $A_0 = 0$ and $m_0 = 500$ GeV. The upper left (middle) window is for δ^d_{LR} at $\mu < 0$ ($\mu > 0$). The light (dark) blue bands show allowed regions within the 1σ (2σ) experimental error. Outside of these regions is labeled as 'excluded'. Comparing the plot at $\mu < 0$, remarkably wider allowed–regions at $\mu > 0$ roughly measure an impact of the focusing effect. It is worth to comment that the 'excluded' region between the two bands is very shallow and small reduction of experimental value (for example, if we do not include the recent BaBar data) easily accommodates most of this region with 2σ error except for small $m_{1/2}$ part. Similar NLO-level relaxations exist in the plots for δ^d_{RL} (upper right) and δ^d_{RR} (lower right). These plots are almost symmetric about the FV–terms because new gluino contributions appear in $C'_{7,8}$ and contribute to the branching ratio quadratically without interfering the MFV contributions in $C_{7,8}$. In these cases, new contributions always come constructively and almost all of the region in the plot is not allowed at $\mu < 0$, which is omitted in the figure. In the case of

δ^d_{LL} (lower left and middle windows), the strong suppression of gluino contribution is rather unclear because of a new chargino contribution generated by coexisting SU(2)–related GFM among up-type squarks. In these plots, $BR(B \to X_s\gamma)$ seems to be more sensitive to δ^d_{LR} and δ^d_{RL} than δ^d_{LL} and δ^d_{RR}. However, it should be noted that natural scale of δ^d_{LR} (δ^d_{RL}) is suppressed by $m_b/M_{\rm SUSY} \lesssim O(10^{-2})$ in Eq. (4), where we follow the definitions widely used in the literature. Even though the focusing effect considerably reduces the gluino contribution generated by GFM, the branching ratio still shows strong dependence on δ^d_{LL} and δ^d_{LR} because of the interference between the gluino contributions and the MFV contributions to $C_{7,8}$. In addition, the new chargino contribution is present in the δ^d_{LL} case. We can read the lower bound of $m_{1/2}$ in MFV as $m_{1/2} \gtrsim 200$ (600) GeV at $\mu > 0$ ($\mu < 0$). However, FV–term of $|\delta^d_{LL}| \sim 0.05$ ($|\delta^d_{LR}| \sim 0.01$) could easily remove (or double) this bound, which is not necessary considered as a fine tuning. This is more striking at $\mu < 0$, where the focusing effect is not so obvious. On the other hand, this bound does not change much with δ^d_{RR} or δ^d_{RL} if $\delta^d_{RR} \lesssim 0.4$ and $\delta^d_{RL} \lesssim 0.015$, where the focusing effect plays crucial role in relaxing the constraints. This means that NLO-level calculation is essential to test flavor models, for example, based on SO(10) SUSY GUT, which tend to have large $\tan\beta$.

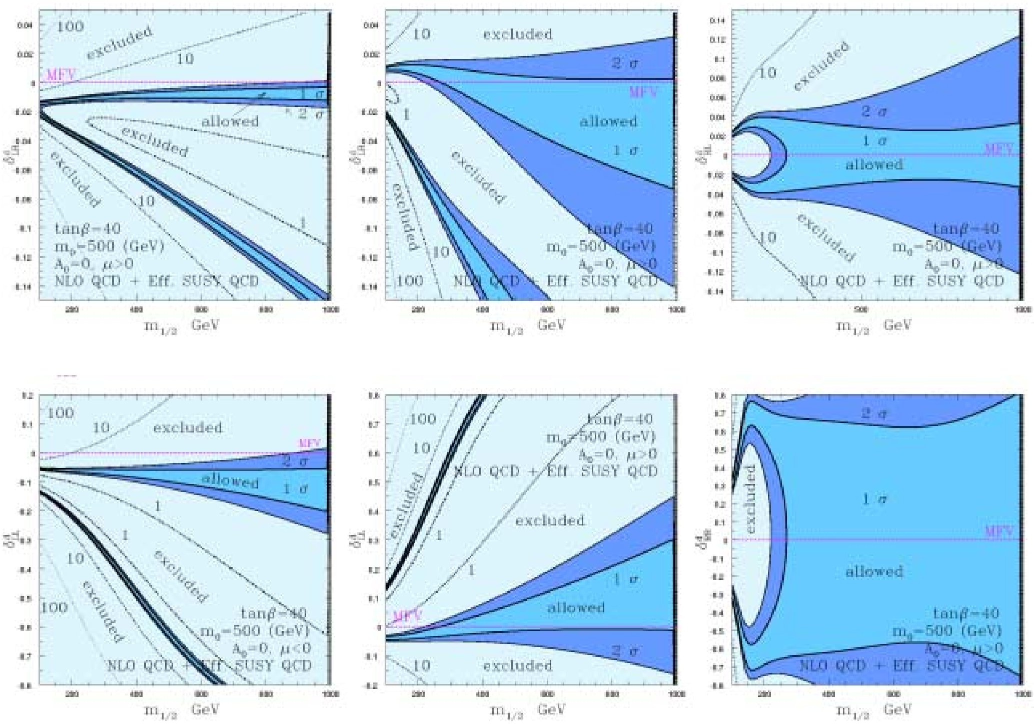

FIG. 3: We plot contours of $BR(B \to X_s\gamma)$ *vs.* $m_{1/2}$ and various flavor violating terms in the Constrained MSSM. Regions beyond the 1σ (light blue) and 2σ CL (dark blue) agreement with experiment (Eq. (1)) are marked “excluded”.

VI Conclusions

We have analysed $BR(B \to X_s\gamma)$ in MSSM with general flavor mixing including leading NLO corrections within the SM operator basis, assuming the common mass scale, $\overline{\mu}_{\rm SUSY}$ for squarks and gluino. We have identified the NLO focusing effect, which reduces the gluino contribution to $BR(B \to X_s\gamma)$ relative to that with LO matching condition. This effect leads to a considerable relaxation of the constraints on flavor mixing terms, especially for δ^d_{LR}. Including the focusing effect, we have shown in CMSSM that $BR(B \to X_s\gamma)$ is still very sensitive to δ^d_{LL} and δ^d_{LR}. The lower bound for $m_{1/2}$ obtained in MFV can be easily removed by them even at $\mu < 0$, while it is rather stable against δ^d_{RR} and δ^d_{RL}. NLO-level calculation appears to be essential in analysis of SUSY models beyond MFV with large $\tan\beta$. This calls for a future complete NLO calculation of $BR(B \to X_s\gamma)$ in supersymmetry which could be applicable in more general circumstance.

Acknowledgments

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