Bilarge Leptonic Mixing and Lepton Masses from Flavor Symmetries

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Outline

- Mixing Parameters
- Models for $\theta_{12} \simeq \theta_{23} \simeq 1$
- Extension of the SM
- Yukawa interactions of the Leptons
- Charged Lepton and Neutrino Mass Matrices
- Bilarge Mixing Pattern
- Summary & Conclusions

Mixing Parameters

SM: 13 of 18 parameters in the fermion sector
$\longrightarrow$ understanding necessary

Small quark mixings $\lesssim 0.2$

MSW LMA

U(1) charges,
Irreps for 1st & 2nd generations

solar: $\theta_{12} \approx \pi/6$
atmospheric: $\theta_{23} \approx \pi/4$,
reactor: $\theta_{13} \leq 0.2$

Nearly maximal ν_μ-ν_τ mixing: $|\theta_{23} - \pi/4| \ll 1$

Non-Abelian flavor symmetry acting on 2nd & 3rd generations

$\curvearrowright$ New symmetries and breaking patterns needed

R. N. Mohapatra and S. Nussinov, PRD **60**, 013002 (1999)
C. Wetterich, PLB **451**, 397 (1999)

Models for $\theta_{12} \simeq \theta_{23} \simeq 1$

In the basis where M_ℓ is diagonal:

inverted hierarchy

$$M_\nu = \begin{pmatrix} m_{11} & cM & sM \\ cM & m_{22} & m_{23} \\ sM & m_{23} & m_{33} \end{pmatrix} \longrightarrow \begin{array}{c}\text{VAC}\\ \text{or}\\ \text{LOW}\end{array}$$

where $c = \cos(\theta_{23})$, $s = \sin(\theta_{23})$, and $m_{ij} \ll M$.

R.N.Mohapatra, A.Pérez-Lorenzana, and C.A. de S.Pires, PLB **474**, 355 (2000); L. Lavoura and W. Grimus, JHEP **0009**, 007 (2000); . . .

normal hierarchy

$$M_\nu = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & s^2M & scM \\ m_{13} & scM & c^2M \end{pmatrix} \longrightarrow \theta_{23} \simeq 1$$

S. Davidson and S.L. King, PLB **445**, 191 (1998);

Q. Shafi and Z. Tavartkiladze, PLB **451**, 129 (1999); . . .

Mixing entirely from M_ℓ:

flavor democracy

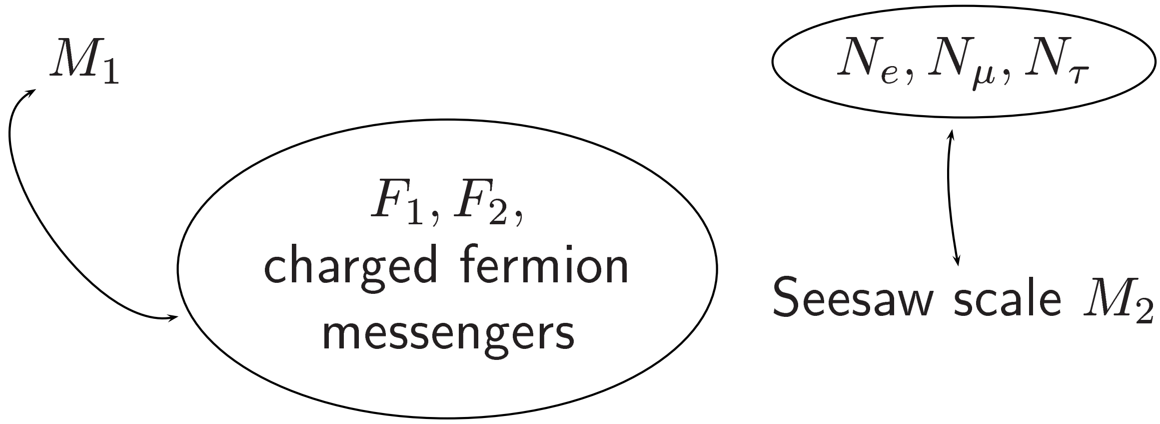
$$M_\ell \simeq \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \longrightarrow \begin{array}{c}\theta_{12} = \pi/4 \\ \sin^2(2\theta_{23}) = 8/9\end{array}$$

H. Fritzsch and Z. Z. Xing, PLB **372**, 265 (1996);

M. Tanimoto, T. Watari, and T. Yanagida, PLB **461**, 345 (1999); . . .

Extension of the SM

Fermions:



SM singlet scalars:

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}, \begin{pmatrix}\phi_3\\ \phi_4\end{pmatrix}, \begin{pmatrix}\phi_5\\ \phi_6\end{pmatrix}, \begin{pmatrix}\phi_7\\ \phi_8\end{pmatrix}, \phi_9, \phi_{10}, \theta$$

$$\begin{pmatrix}\phi'_1\\ \phi'_2\end{pmatrix}, \begin{pmatrix}\phi'_3\\ \phi'_4\end{pmatrix}, \begin{pmatrix}\phi'_5\\ \phi'_6\end{pmatrix}$$

Leptonic mass and mixing parameters from approximately conserved flavor symmetries:

Symmetry breaking parameter $\epsilon \simeq \frac{\langle\theta\rangle}{M_1} \simeq \frac{\langle\phi_i\rangle}{M_1} \simeq \frac{\langle\phi'_j\rangle}{M_1} \simeq 10^{-1}$

C. D. Froggatt and H.B. Nielsen, NPB **147**, 277 (1979);
S. Dimopoulos, PLB **129**, 417 (1983); J. Bagger, S. Dimopoulos, H. Georgi and S. Raby, *5th workshop on grand unification*, Providence, 95 (1984); E. Witten, PLB **105**, 267 (1981)

U(1) Charges

Assignment of anomaly-free U(1) charges:

Fermions	(Q_1, Q_2, Q_3)
L_e,E_e	$(1, 0, 0)$
L_μ,L_τ,E_μ,E_τ	$(0, 1, 0)$
N_e	$(1, 0, 0)$
N_μ, N_τ	$(0, 1, 0)$
F_1	$(1, 0, 0)$
F_2	$(-1, 0, 1)$

Scalars	(Q_1, Q_2, Q_3)
H_1, H_2	$(0, 0, 0)$
ϕ_1, ϕ_2	$(1, -1, 2)$
ϕ_3, ϕ_4	$(0, 0, 0)$
ϕ_5, ϕ_6	$(0, 0, 1)$
$\phi'_1, \phi'_2, \phi'_3, \phi'_4, \phi'_5, \phi'_6$	$(0, 0, 0)$
ϕ_7, ϕ_8	$(-1, -1, 0)$
ϕ_9	$(-2, 0, 1)$
ϕ_{10}	$(0, 0, 0)$
θ	$(0, 0, -1)$

Note: SSB close to TeV-scale possible

M. Leurer, Y. Nir and N. Seiberg, NPB **398**, 319 (1993); NPB **420**, 468 (1994)

Discrete Symmetries

Distinguishing the first generation with $\mathbb{Z}_n$ symmetry:

$$\mathcal{D}_1 \quad : \quad \begin{cases} E_e \to P^{-4} E_e, & E_\mu \to P^{-1} E_\mu, & E_\tau \to P^{-1} E_\tau, \\ \phi_1 \to P\phi_1, & \phi_2 \to P\phi_2, & \phi_3 \to P\phi_3, \\ \phi_4 \to P\phi_4, & \phi_5 \to P\phi_5, & \phi_6 \to P\phi_6, \end{cases}$$

where $P \equiv \exp(2\pi i/n)$, $n \geq 5$ & RH νs $\mathcal{D}_1$-singlets.

$|\theta_{23} - \pi/4| \ll 1$ from $\mathbb{Z}_2$ permutation symmetries:

$$\mathcal{D}_2 \quad : \quad \begin{cases} L_\mu \to -L_\mu, & E_\mu \to -E_\mu, & N_\mu \to -N_\mu, \\ \phi_1' \leftrightarrow \phi_2', & \phi_1 \leftrightarrow \phi_2, & \phi_7 \to -\phi_7, \end{cases}$$

$$\mathcal{D}_3 \quad : \quad \begin{cases} L_\mu \to -L_\mu, & N_\mu \to -N_\mu, \\ \phi_3' \leftrightarrow \phi_4', & \phi_3 \leftrightarrow \phi_4, \\ \phi_5' \leftrightarrow \phi_6', & \phi_5 \leftrightarrow \phi_6, \\ \phi_7 \to -\phi_7, \end{cases}$$

$$\mathcal{D}_4 \quad : \quad \begin{cases} L_\mu \leftrightarrow L_\tau, & E_\mu \leftrightarrow E_\tau, & N_\mu \leftrightarrow N_\tau, \\ \phi_2 \to -\phi_2, & \phi_4 \to -\phi_4, & \phi_6 \to -\phi_6, \\ \phi_7 \leftrightarrow \phi_8. \end{cases}$$

- Degenerate Yukawa couplings in E_α and ν_α sector
- Degenerate scalar-scalar couplings

Forbidding Terms

Terms spoiling the vacuum alignment are forbidden by

$$\mathcal{D}_5 \quad : \quad \begin{cases} \phi_1' \to -\phi_1', & \phi_3' \to -\phi_3', & \phi_5' \to -\phi_5' \\ \phi_1 \to -\phi_1, & \phi_3 \to -\phi_3, & \phi_5 \to -\phi_5, \end{cases}$$

$$\mathcal{D}_6 \quad : \quad \begin{cases} E_e \to P^{-(4l+1)} E_e, & N_e \to P N_e, \\ \phi_1' \to P^{-k} \phi_1', & \phi_2' \to P^{-k} \phi_2', \\ \phi_3' \to P^{-l} \phi_3', & \phi_4' \to P^{-l} \phi_4', \\ \phi_5' \to P^{-m} \phi_5', & \phi_6' \to P^{-m} \phi_6' \\ \phi_1 \to P^{k} \phi_1, & \phi_2 \to P^{k} \phi_2, \\ \phi_3 \to P^{l} \phi_3, & \phi_4 \to P^{l} \phi_4, \\ \phi_5 \to P^{m} \phi_5, & \phi_6 \to P^{m} \phi_6, \\ \phi_9 \to P^{-1} \phi_9, & \phi_{10} \to P \phi_{10}. \end{cases}$$

- k, l, m: integers with $|k-l|$, $|k-m|, |l-m| \gg 1$
- $\mathcal{D}_6$-charges of charged fermion messengers: $\mathbb{N} \times \{2\pi k/n, 2\pi l/n, 2\pi m/n\}$

Charged Lepton Yukawa Interactions

Charged lepton mass terms:

$$\mathcal{L}_Y^\ell = \overline{L_\alpha} H_2 \left[(Y_{\text{eff}}^1)_{\alpha\beta} + (Y_{\text{eff}}^2)_{\alpha\beta} \right] E_\beta + \text{h.c.}$$

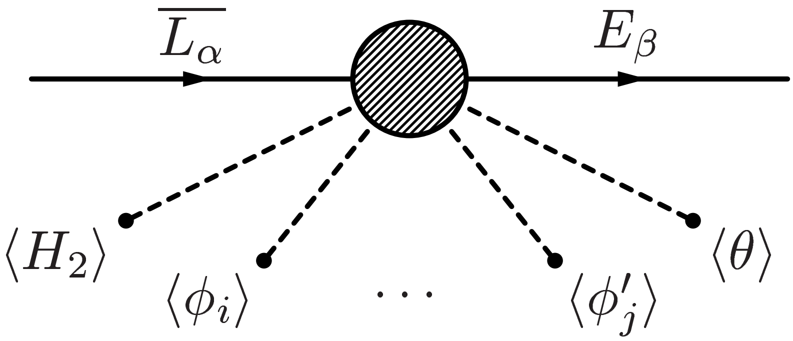


Fig. 1: Non-renormalizable terms generating Y_{eff}^1 and Y_{eff}^2.

$$Y_{\text{eff}}^1 = \begin{pmatrix} A_1 & B_1 - B_2 & B_1 + B_2 \\ 0 & C_1 - C_2 & 0 \\ 0 & 0 & C_1 + C_2 \end{pmatrix},$$

$$Y_{\text{eff}}^2 = \text{diag}\left(0, \quad D_1 - D_2, \quad D_2 + D_2\right)$$

$$A_1 = Y_a^\ell \frac{\phi_{10}}{(M_1)^5} [(\phi_3)^4 + (\phi_4)^4],$$

$$B_1 = Y_b^\ell \frac{\theta^2}{(M_1)^4} \phi_1 \phi'_1, \; C_1 = Y_c^\ell \frac{\phi'_3 \phi_3}{(M_1)^2}, \; D_1 = Y_d^\ell \frac{\phi'_5 \phi_5}{(M_1)^3},$$

$$B_2 = Y_b^\ell \frac{\theta^2}{(M_1)^4} \phi_2 \phi'_2, \; C_2 = Y_c^\ell \frac{\phi'_4 \phi_4}{(M_1)^2}, \; D_2 = Y_d^\ell \frac{\phi'_6 \phi_6}{(M_1)^3}$$

Neutrino Yukawa Interactions

$$\mathcal{L}_Y^\nu = \overline{L_\alpha^c}\frac{(H_1)^2}{M_2}(Y_{\rm eff}^3)_{\alpha\beta}L_\beta + \text{h.c.}, \quad Y_{\rm eff}^3 = \begin{pmatrix} A_2 & B_3 & B_4 \\ B_3 & 0 & 0 \\ B_4 & 0 & 0 \end{pmatrix},$$

Dimensionful Yukawa couplings:

$$A_2 = Y_a^\nu \frac{\phi_9\phi_{10}\theta}{(M_1)^3}, \quad B_3 = Y_b^\nu \frac{\phi_7}{M_1}, \quad B_4 = Y_b^\nu \frac{\phi_8}{M_1},$$

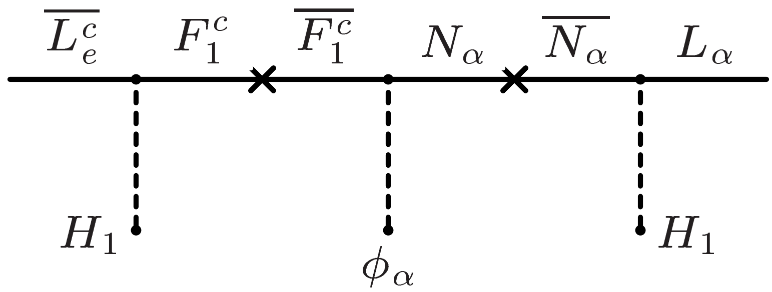
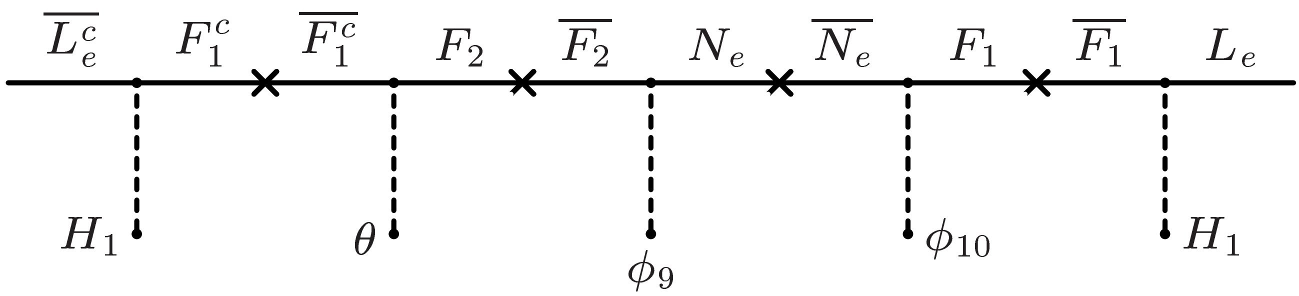


Fig. 2: The dimension six operators for $\alpha = e, \mu$ and $\phi_\alpha = \phi_7, \phi_8$ generating $M_{e\mu}^\nu$ and $M_{e\tau}^\nu$.



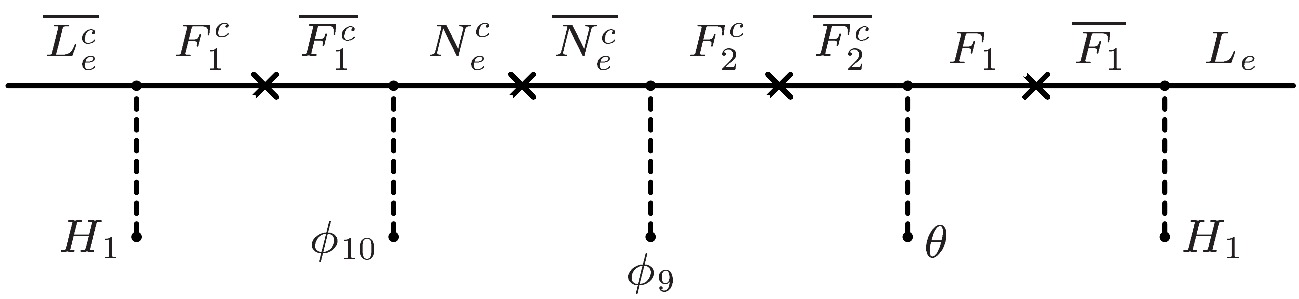


Fig. 3: The dimension eight operators generating M_{ee}^ν.

Charged Lepton Masses

For a range of parameters vacuum alignment mechanism :

$$\begin{pmatrix} \frac{\langle\phi_1\rangle}{\langle\phi_2\rangle} & \frac{\langle\phi_3\rangle}{\langle\phi_4\rangle} & \frac{\langle\phi_5\rangle}{\langle\phi_6\rangle} \end{pmatrix} = \pm\begin{pmatrix} +1 & -1 & +1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{\langle\phi'_1\rangle}{\langle\phi'_2\rangle} & \frac{\langle\phi'_3\rangle}{\langle\phi'_4\rangle} & \frac{\langle\phi'_5\rangle}{\langle\phi'_6\rangle} \end{pmatrix} = \pm\begin{pmatrix} +1 & +1 & +1 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} \frac{B_1}{B_2} & \frac{C_1}{C_2} & \frac{D_1}{D_2} \end{pmatrix} = \pm\begin{pmatrix} +1 & -1 & +1 \end{pmatrix}$$

$\curvearrowright$ Leading order effective Yukawa couplings:
Exactly degenerate with relative sign

Strictly hierarchical charged lepton mass matrix patterns:

$$M_\ell \simeq m_\tau \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon^4 \\ \epsilon^3 & \epsilon & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \quad \leftrightarrow \quad m_\tau \begin{pmatrix} \epsilon^3 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \end{pmatrix}$$

Mass ratios:

$$m_e/m_\mu \simeq \epsilon^2 \simeq 10^{-2} \quad \text{and} \quad m_\mu/m_\tau \simeq \epsilon \simeq 10^{-1}$$

Mixing parameters:

$$\theta^\ell_{12} \simeq 6^\circ \quad \text{and} \quad \theta^\ell_{13} \approx \theta^\ell_{23} \approx 0^\circ$$

Neutrino Masses

For a range of parameters: vacuum alignment $\curvearrowright$

$$\frac{\langle\phi_7\rangle}{\langle\phi_8\rangle} = \frac{B_3}{B_4} = \pm 1$$

$\curvearrowright$ Effective neutrino mass matrix:

$$M_\nu = \frac{\langle H_1\rangle^2}{M_2}\begin{pmatrix} Y_a^\nu\epsilon^3 & Y_b^\nu\epsilon & -Y_b^\nu\epsilon \\ Y_b^\nu\epsilon & \epsilon^5 & \epsilon^5 \\ -Y_b^\nu\epsilon & \epsilon^5 & \epsilon^5 \end{pmatrix} = m_\nu \begin{pmatrix} \epsilon^2 & 1 & -1 \\ 1 & \epsilon^4 & \epsilon^4 \\ -1 & \epsilon^4 & \epsilon^4 \end{pmatrix}$$

$\curvearrowright$ M_ν on bimaximal mixing form

V. Barger *et al.*, PLB **437**, 107 (1998);

R. N. Mohapatra and S. Nussinov, PRD **60**, 013002 (1999); . . .

$$\theta_{12}^\nu = \pi/4 + \mathcal{O}(\epsilon^2), \quad \theta_{13}^\nu \approx 0, \quad \text{and} \quad \theta_{23}^\nu = \pi/4,$$

$$m_1 \simeq \sqrt{2}m_\nu, \quad m_2 \simeq \sqrt{2}m_\nu, \quad \text{and} \quad m_3 \simeq 2\epsilon^4 m_\nu \approx 0$$

Inverse hierarchical spectrum

Mass squared differences:

$$\begin{aligned} \Delta m_\odot^2 &\equiv |\Delta m_{21}^2| \simeq 2\sqrt{2}\epsilon^2 m_\nu^2 \\ \Delta m_{\rm atm}^2 &\equiv |\Delta m_{32}^2| \simeq 2 m_\nu^2 \end{aligned}$$

MSW LMA values $\curvearrowright$ $m_\nu \simeq 0.04\,\text{eV}$ and $M_2 \simeq 10^{14}\,\text{GeV}$

Bilarge Mixing

Charged lepton & neutrino mixing $\curvearrowright$

$$\begin{aligned}\theta_{12} &= \pi/4 - \epsilon/\sqrt{2} + \mathcal{O}(\epsilon^2),\\ \theta_{13} &= \epsilon/\sqrt{2} + \mathcal{O}(\epsilon^3),\\ \theta_{23} &= \pi/4 + \mathcal{O}(\epsilon^2)\end{aligned}$$

For $\epsilon \simeq 10^{-1}$:

$$\theta_{12} \simeq 41^\circ, \quad \theta_{13} \simeq 4^\circ, \quad \text{and} \quad \theta_{23} \simeq 44^\circ$$

Mild fine-tuning $Y_b^\ell / Y_c^\ell \simeq 2$:

$$\theta_{12} \simeq 37^\circ, \quad \theta_{13} \simeq 8^\circ, \quad \text{and} \quad \theta_{23} \simeq 44^\circ$$

$\curvearrowright$ Compatible with MSW LMA and CHOOZ upper bound

Summary and Conclusions

- Strict hierarchical charged lepton mass pattern
- Large, but not necessarily close to maximal θ_{12}
- Relation $\theta_{12} \simeq \pi/4 - \theta_{13}$
- $|\theta_{23} - \pi/4| \ll 1$ due to symmetries
- Inverted neutrino mass hierarchy
- Can give the MSW LMA solution

References:

T .Ohlsson and G. Seidl, Phys. Lett. **B** 537, 95 (2002), hep-ph/0203117;
T. Ohlsson and G. Seidl, *A Flavor Symmetry Model for Bilarge Leptonic Mixing and the Lepton Masses*, hep-ph/0206087