Bilarge Leptonic Mixing and Lepton Masses from Flavor Symmetries [1]

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Abstract

We present a model for leptonic mixing and the lepton masses based on anomalous $\mathcal{U}(1)$ charges and a non-Abelian flavor symmetry. The model yields bilarge leptonic mixing, an inverted neutrino mass spectrum and hierarchical charged lepton masses. The non-Abelian flavor symmetry enforces an approximately maximal atmospheric mixing angle and predicts the relation $\theta_{12} \simeq \frac{\pi}{4} - \theta_{13}$ between the solar and the reactor mixing angle.

1 Introduction

Atmospheric [1] and solar [2, 3] neutrino data presently prefers the Mikheyev-Smirnov-Wolfenstein (MSW) large mixing angle (LMA) solution of the solar neutrino problem [4]. It is particularly challenging to predict both the bilarge leptonic mixing implied by the MSW LMA solution as well as the lepton mass hierarchies from flavor symmetries. We will consider here an extension of the standard model (SM) where the lepton masses arise from the Froggatt-Nielsen mechanism [5] by the breakdown of Abelian and non-Abelian flavor symmetries. We obtain bilarge leptonic mixing and strictly hierarchical charged lepton masses, where the atmospheric mixing angle is enforced to be exactly maximal (up to subleading corrections) due to the non-Abelian flavor symmetry [6]. Note that we assume in our study that all $\mathcal{CP}$ violation phases are zero.

2 Flavor Symmetries

Let us denote the left-handed lepton doublets as L_α and the right-handed charged leptons as E_α, where $\alpha = e, \mu, \tau$. For simplicity, the electroweak Higgs sector is assumed to consist only of the SM Higgs doublet H, but the standard two-Higgs-doublet model (as required in supersymmetric extensions) is also possible. We will pursue here the idea that an approximately maximal atmospheric mixing angle θ_{23} indicates some non-Abelian flavor symmetry $\mathscr{G}$ which acts on the 2nd and 3rd generation of leptons. As a straightforward realization of this scenario we will respectively combine the $SU(2)$-doublet fields L_μ and L_τ as well as the right-handed charged leptons E_μ and E_τ into the doublet representations $\mathbf{2}_\ell \equiv (L_\mu\ L_\tau)^T$ and $\mathbf{2}_E \equiv (E_\mu\ E_\tau)^T$ of the flavor symmetry group $\mathscr{G}$ which will be specified in more detail further below.

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	(Q_1, Q_2, Q_3)
L_e, E_e	$(1, 0, 0)$
$\mathbf{2}_\ell$, $\mathbf{2}_E$	$(0, 1, 0)$
N_e	$(1, 0, 0)$
$\mathbf{2}_N$	$(0, 1, 0)$
F_1	$(1, 0, 0)$
F_2	$(-1, 0, 1)$
Φ_1	$(1, -1, 2)$
Φ_4	$(-1, -1, 0)$
ϕ_9	$(-2, 0, 1)$
θ	$(0, 0, -1)$

Table 1: Assignment of the $\mathcal{U}(1)$ charges Q_1, Q_2, and Q_3 to the fermionic and scalar fields. The fields not shown here carry zero $\mathcal{U}(1)$ charges.

Recently, replicated representations of non-Abelian groups have attracted attention in deconstructed extra-dimensional gauge theories [7], which can be viewed as viable renormalizable and gauge-invariant UV completions of some fundamental non-perturbative theory, *e.g.*, string theory. It is therefore interesting to add to the SM a number of "copies" of SM singlet scalar fields which transform as replicated doublets under $\mathscr{G}$:

$$\Phi_1 = \begin{pmatrix}\phi_1 & \phi_2\end{pmatrix}^T, \quad \Phi_2 = \begin{pmatrix}\phi_3 & \phi_4\end{pmatrix}^T, \quad \Phi_3 = \begin{pmatrix}\phi_5 & \phi_6\end{pmatrix}^T, \quad \Phi_4 = \begin{pmatrix}\phi_7 & \phi_8\end{pmatrix}^T,$$

$$\Phi_1' = \begin{pmatrix}\phi_1' & \phi_2'\end{pmatrix}^T, \quad \Phi_2' = \begin{pmatrix}\phi_3' & \phi_4'\end{pmatrix}^T, \quad \Phi_3' = \begin{pmatrix}\phi_5' & \phi_6'\end{pmatrix}^T.$$

Furthermore, we introduce three scalar fields ϕ_9, ϕ_{10}, and θ, which are trivial representations of $\mathscr{G}$. The neutrino sector is augmented by five extra SM singlet Dirac neutrinos N_e, N_μ, N_τ, F_1, and F_2 among which the fields N_μ and N_τ actually transform as a $\mathscr{G}$-doublet representation $\mathbf{2}_N \equiv (N_\mu \;\; N_\tau)^T$. The neutrinos F_1 and F_2 are assumed to have masses of the order of some relevant high mass scale M_1 (which can be as low as several TeV) whereas N_e, N_μ, and N_τ all have masses of the order of some seesaw scale $M_2 \simeq 10^{14}$ GeV which is responsible for the smallness of the neutrino masses in comparison with the charged lepton masses.

The Yukawa matrix patterns arise partly also from the approximate conservation of additional replicated $\mathcal{U}(1)$ gauge symmetries with anomalous charges Q_1, Q_2, and Q_3. The corresponding $\mathcal{U}(1)$ charge assignment is shown in table 1. The first generation is furthermore distinguished from the second and third generations by the Z_n symmetry

$$\mathcal{D}_1 \quad : \qquad E_e \to P^{-4} E_e, \quad \mathbf{2}_E \to P^{-1}\mathbf{2}_E, \quad \Phi_i \to P\,\Phi_i \quad (i = 1, 2, 3),$$

where $P \equiv e^{2\pi \mathrm{i}/n}$ for some integer $n \geq 5$. With the $\mathscr{G}$-doublet representation content given above, we can now define $\mathscr{G}$ as the group which is generated by the following set of discrete symmetry transformations

$$\mathcal{D}_2 \quad : \quad \begin{cases} \mathbf{2}_\ell \to D(C_b)\,\mathbf{2}_\ell, & \mathbf{2}_E \to D(C_b)\,\mathbf{2}_E, \quad \mathbf{2}_N \to D(C_b)\,\mathbf{2}_N, \\ \Phi_1 \to D(C_{b'})\,\Phi_1, & \Phi'_1 \to D(C_{b'})\,\Phi'_1 \quad \Phi_4 \to D(C_b)\,\Phi_4, \end{cases}$$

$$\mathcal{D}_3 \quad : \quad \begin{cases} \mathbf{2}_\ell \to D(C_b)\,\mathbf{2}_\ell, & \mathbf{2}_N \to D(C_b)\,\mathbf{2}_N, \\ \Phi_i \to D(C_{b'})\,\Phi_i, & \Phi'_i \to D(C_{b'})\,\Phi'_i \quad (i=2,3), \\ \Phi_4 \to D(C_b)\,\Phi_4, & \end{cases}$$

$$\mathcal{D}_4 \quad : \quad \begin{cases} \mathbf{2}_\ell \to D(C_{b'})\,\mathbf{2}_\ell, & \mathbf{2}_E \to D(C_{b'})\,\mathbf{2}_E, \quad \mathbf{2}_N \to D(C_{b'})\,\mathbf{2}_N, \\ \Phi_i \to D(C_a)\,\Phi_i & (i=1,2,3), \\ \Phi_4 \to D(C_{b'})\,\Phi_4, & \end{cases}$$

$$\mathcal{D}_5 \quad : \quad \Phi_i \to D(C_b)\,\Phi_i, \quad \Phi'_i \to D(C_b)\,\Phi'_i \quad (i=1,2,3),$$

where $D(C_a), D(C_b)$, and $D(C_{b'})$ denote generators of the two-dimensional vector representation of the non-Abelian dihedral group $\mathscr{D}_4$ and can be explicitly written as follows

$$D(C_a) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad D(C_b) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad D(C_{b'}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Inspection of the discrete symmetry transformations shows that $\mathscr{G}$ can be considered as an *n-valued* representation of $\mathscr{D}_4$, where[3] $n = 8$. An interesting vacuum structure will be possible, if some terms in the multi-scalar potential are forbidden by the cyclic symmetry

$$\mathcal{D}_6 \quad : \quad \begin{cases} E_e \to P^{-(4l+1)}\,E_e, & N_e \to P\,N_e, & \\ \Phi_1 \to P^k\,\Phi_1, & \Phi_2 \to P^l\,\Phi_2, & \Phi_3 \to P^m\,\Phi_3, \\ \Phi'_1 \to P^{-k}\,\Phi'_1, & \Phi'_2 \to P^{-l}\,\Phi'_2, & \Phi'_3 \to P^{-m}\,\Phi'_3, \\ \phi_9 \to P^{-1}\,\phi_9, & \phi_{10} \to P\,\phi_{10}, & \end{cases}$$

where k, l, and m are integers. In addition, we require some general constraints on the $\mathcal{D}_6$-charges of the Froggatt-Nielsen states with non-zero hypercharge. Careful analysis of the multi-scalar potential then yields [6] that there exists a range of parameters for the scalar couplings where the vacuum expectation values (VEVs) of the component fields of the scalar $\mathscr{G}$-doublets are (up to a possible relative sign) pairwise *exactly* degenerate [4], enforced by the symmetry $\mathscr{G}$. Specifically, one obtains the exact relations

$$\langle\phi_1\rangle = \langle\phi_2\rangle, \quad \langle\phi_3\rangle = -\langle\phi_4\rangle, \quad \langle\phi_5\rangle = \langle\phi_6\rangle, \quad \langle\phi_7\rangle = \pm\langle\phi_8\rangle. \tag{1}$$

We will account for the hierarchical lepton masses by the assumption that the SM singlet scalar fields break the flavor symmetries when they aquire their VEVs at a high mass scale and thereby give rise to a small expansion parameter

$$\epsilon \simeq \langle\phi_i\rangle/M_1 \simeq \langle\phi'_j\rangle/M_1 \simeq \langle\theta\rangle/M_1 \simeq 10^{-1},$$

where $i = 1, \ldots, 10$, and $j = 1, \ldots, 6$.

[3]Note that $\mathscr{G}$ is a subgroup of the n-fold replicated dihedral group $(\mathscr{D}_4)^n \equiv \mathscr{D}_4 \times \ldots \times \mathscr{D}_4$.

[4]We consider here only the tree-level approximation.

3 Lepton masses and mixing angles

Let us now consider the effective Yukawa coupling operators $\mathcal{O}^{\ell}_{\alpha\beta}$ and $\mathcal{O}^{\nu}_{\alpha\beta}$ which respectively generate the entries in the charged lepton mass matrix $\mathcal{M}_\ell$ and the neutrino mass matrix $\mathcal{M}_\nu$ via the mass terms

$$\mathscr{L}^{\ell}_{Y} = \overline{L_\alpha} H \mathcal{O}^{\ell}_{\alpha\beta} E_\beta + \text{h.c.} \quad \text{and} \quad \mathscr{L}^{\nu}_{Y} = \overline{L^c_\alpha} \frac{H^2}{M_2} \mathcal{O}^{\nu}_{\alpha\beta} L_\beta + \text{h.c.}$$

Taking the $\mathcal{U}(1)$ symmetries into account, the leading order $\mathscr{G}$-invariant effective Yukawa coupling matrix of the charged leptons is

$$(\mathcal{O}^{\ell}_{\alpha\beta}) = \begin{pmatrix} A_1[(\phi_3)^4+(\phi_4)^4] & B_1[\phi_1\phi_1'-\phi_2\phi_2'] & B_1[\phi_1\phi_1'+\phi_2\phi_2'] \\ 0 & C(\phi_3'\phi_3-\phi_4'\phi_4)+D(\phi_5'\phi_5-\phi_6'\phi_6) & 0 \\ 0 & 0 & C(\phi_3'\phi_3+\phi_4'\phi_4)+D(\phi_5'\phi_5+\phi_6'\phi_6) \end{pmatrix},$$

where the dimensionful couplings A_1, B_1, C, and D are given by

$$A_1 = Y^{\ell}_a \phi_{10}/(M_1)^5, \quad B_1 = Y^{\ell}_b \theta^2/(M_1)^4, \quad C = Y^{\ell}_c/(M_1)^2, \quad D = Y^{\ell}_d \theta/(M_1)^3,$$

and where $Y^{\ell}_a, Y^{\ell}_b, Y^{\ell}_c$, and Y^{ℓ}_d are coefficients of order unity. Correspondingly, the most general leading-order effective Yukawa coupling matrix $\mathcal{O}^{\nu}_{\alpha\beta}$ of the neutrinos is

$$(\mathcal{O}^{\nu}_{\alpha\beta}) = \frac{1}{M_1}\begin{pmatrix} Y^{\nu}_a \frac{\theta}{(M_1)^2}\phi_9\phi_{10} & Y^{\nu}_b\phi_7 & Y^{\nu}_b\phi_8 \\ Y^{\nu}_b\phi_7 & 0 & 0 \\ Y^{\nu}_b\phi_8 & 0 & 0 \end{pmatrix},$$

where Y^{ν}_a and Y^{ν}_b are coefficients of order unity. Inserting the VEVs of Eq. (1) into $(\mathcal{O}^{\ell}_{\alpha\beta})$ and $(\mathcal{O}^{\nu}_{\alpha\beta})$ leads (up to a possible $\mu \leftrightarrow \tau$ permutation) to the lepton mass matrices

$$\mathcal{M}_\ell \simeq m_\tau \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon^4 \\ \epsilon^3 & \epsilon & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}, \quad \mathcal{M}_\nu \simeq m_\nu \begin{pmatrix} \epsilon^2 & 1 & -1 \\ 1 & \epsilon^4 & \epsilon^4 \\ -1 & \epsilon^4 & \epsilon^4 \end{pmatrix}, \tag{2}$$

where m_τ is the tau mass and m_ν denotes the absolute neutrino mass scale. In Eq. (2) it is important to note, that the entries "1" and "-1" in the neutrino mass matrix are enforced to be exactly degenerate due to the discrete flavor symmetry. On the other hand, the degeneracy of the VEVs of the SM singlet scalar fields ensures that relevant components of the effective charged lepton Yukawa tensor operator exactly cancel, which leads to a hierarchical pattern of $\mathcal{M}_\ell$. Denoting by m_e and m_μ respectively the electron and the muon mass, we obtain the hierarchical charged lepton mass ratios $m_e/m_\mu \simeq \epsilon^2$ and $m_\mu/m_\tau \simeq \epsilon$. The neutrino masses exhibit an inverse hierarchy, where the solar and atmospheric mass squared differences are $\Delta m^2_\odot \simeq 2\sqrt{2}\epsilon^2 m^2_\nu$ and $\Delta m^2_{\text{atm}} \simeq 2m^2_\nu$ [6]. Taking $m_\nu \simeq 0.04$ eV and $\epsilon \simeq 0.1$ we arrive at the presently preferred values for $\Delta m^2_\odot$ and Δm^2_{atm} of the MSW LMA solution. Note that the leptonic mixing angles receive a significant contribution from the charged lepton sector. In standard parameterization, the leptonic mixing angles are given by [6]

$$\theta_{12} = \pi/4 - \theta_{13} + \mathcal{O}(\epsilon^2), \quad \theta_{13} = \epsilon/\sqrt{2} + \mathcal{O}(\epsilon^3), \quad \theta_{23} = \pi/4 + \mathcal{O}(\epsilon^2), \tag{3}$$

where the predictions $\theta_{12} \simeq \pi/4 - \theta_{13}$ and $|\theta_{23} - \pi/4| \ll 1$ are enforced by the symmetry $\mathscr{G}$. Without tuning of parameters, the leptonic mixing angles are

$$\theta_{12} \simeq 41^\circ, \quad \theta_{13} \simeq 4^\circ, \quad \text{and} \quad \theta_{23} \simeq 44^\circ, \tag{4}$$

i.e., we have a large but not necessarily maximal solar mixing angle, a small reactor mixing angle which is below the CHOOZ bound [8], and an approximately maximal atmospheric mixing angle as required by the MSW LMA solution. Since the reactor angle obeys $\theta_{13} \lesssim 9^\circ$ [8], the solar mixing angle is bounded from below by $37^\circ \lesssim \theta_{12}$ in our model.

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