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The SM prediction of g-2 of the muon

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I. Why a reanalysis?

- Current situation: originally quoted $2.6\,\sigma$ discrepancy between TH and EXP down to $1.x\,\sigma$ (after 'fixing' of the Light-by-Light scattering contributions), but:

▶ No Lose: either find sign for *New Physics* or *constrain* models beyond the SM. And:

▶ Both EXP and TH will improve, hence more potential for the future!

Note: The biggest contribution to the TH error comes from the LO hadronic vacuum-polarization. To improve this, we need both:

⋆ new, more precise data as input in the dispersion integral

$$a_\mu^{had} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty \mathrm{d}s\, \sigma_{had}^0(s) K(s)\,,$$

with $K(s) = \frac{m_\mu^2}{3s} \cdot (0.63 \ldots 1)\,,$

⋆ convergence of the many different TH analyses..
$\longrightarrow$ careful, data-driven reanalysis to be compared with

see Table ▶

and to address the 'puzzle' inclusive $\leftrightarrow$ exclusive (shown up in the similar case of $\Delta\alpha_{QED}$) and/or data $\leftrightarrow$ pQCD, see below.

Evaluations of the LO hadronic contribution $a_\mu^{\rm had}$:

authors(year)[Ref]	$a_\mu^{\rm had} \times 10^{10}$	comments
Barkov *et al.*(85)[1]	684.0 ± 11.0	primarily e^+e^- data
Kinoshita *et al.*(85)[2]	707.0 ± 18.0	primarily e^+e^- data
Casas *et al.*(85)[3]	710.0 ± 11.5	QCD, theory and some e^+e^-
Martinovič+Dubnička(90)[4]	704.8 ± 11.5	e^+e^- and QCD
Martinovič+Dubnička(90)[4]	705.2 ± 7.6	e^+e^- and QCD
Eidelman+Jegerlehner(95)[5]	702.4 ± 15.3	primarily e^+e^- data
Adel+Ynduráin(95)[6]	711.3 ± 10.3	QCD, theory and some e^+e^-
Brown+Worstell(96)[7]	702.6 ± 16.0	primarily e^+e^- data
Alemany *et al.*(98)[8]	695.0 ± 15.0	primarily e^+e^- data
Alemany *et al.*(98)[8]	701.1 ± 9.4	$e^+e^- + \tau$ data
Davier+Höcker(98)[9]	695.1 ± 7.5	e^+e^-, τ and pQCD
Davier+Höcker(98)[10]	692.4 ± 6.2	e^+e^-, τ, and QCD sum rules
Eidelman+Jegerlehner(98)[11]	696.7 ± 11.9	(update of their result in 1995)
Jegerlehner(01)[12]	697.4 ± 10.5	primarily e^+e^- data
Narison(01)[13]	702.1 ± 7.6	e^+e^-, τ and QCD sum rules
de Trocóniz+Ynduráin(01)[14]	690.9 ± 6.4	e^+e^- and τ data
Cvetič *et al.*(01)[15]	702.9 ± 9.6	QCD sum rules
Cvetič *et al.*(01)[15]	699.5 ± 9.1	QCD sum rules
Hoefer+Gluza+Jegerl.(01)[16]	697.4 ± 10.5	addr. rad. corrs. for $\pi\pi(\gamma)$
HMNT [this analysis]	689.50 ± 5.85	primarily e^+e^- data; 'excl.'
	683.51 ± 5.70	'incl.'

Martinovic and Dubnicka have got two values "depending on the way in which the experimental systematic errors of the dominant two-pion contribution to $a_\mu^{\rm vac}$ are taken into account".

Cvetič *et al.* cites two values depending on the values of the gluon condensate.

II. Processing the hadronic data: Clustering

A fit that's not a fit:

- Need to combine data from different experiments (for the same channel) in one way or another:

  $\longrightarrow$ $\overline{\sum}_{exp} \int \ldots$ leads in general to an overestimate of the errors.

  $\longrightarrow$ Data-points with small errors should dominate nearby data of poor quality, but sparse high quality data should not suppress shape information from other experiments with larger errors.

- Aim: preserve as much exp. information as possible and impose as few theor. constraints as possible on the shape of R (data vs. pQCD) ('minimum bias').

▶ Our fit-model: piecewise constant R within a *Cluster* of a given (min.) size.

▶ Realization: Non-linear χ^2-minimalization of:

$$\chi^2(\bar{R}_m, f_k) = \sum_{k=1}^{\#ofExp.} \left[(1 - f_k)\,/\mathrm{d}f_k\right]^2$$
$$+ \sum_{m=1}^{\#ofCl.} \sum_{i=1}^{N_{\{k,m\}}} \left[\left(R_i^{\{k,m\}} - f_k \bar{R}_m\right) /\mathrm{d}R_i^{\{k,m\}}\right]^2 .$$

Advantages. Stability.

- All data contribute, weighted by their uncertainty^{-1}, hence no loss of information.

- Error estimate using the complete covariance matrix, taking into account statistical and systematic (p.t.p. and overall) errors from all different experiments
  $\longrightarrow$ correlations over different energies taken into account!

- Data-sets with a large (overall-) normalization uncertainty can still contribute shape information without leading to artificial pile-up of (asymmetric) fluctuations.

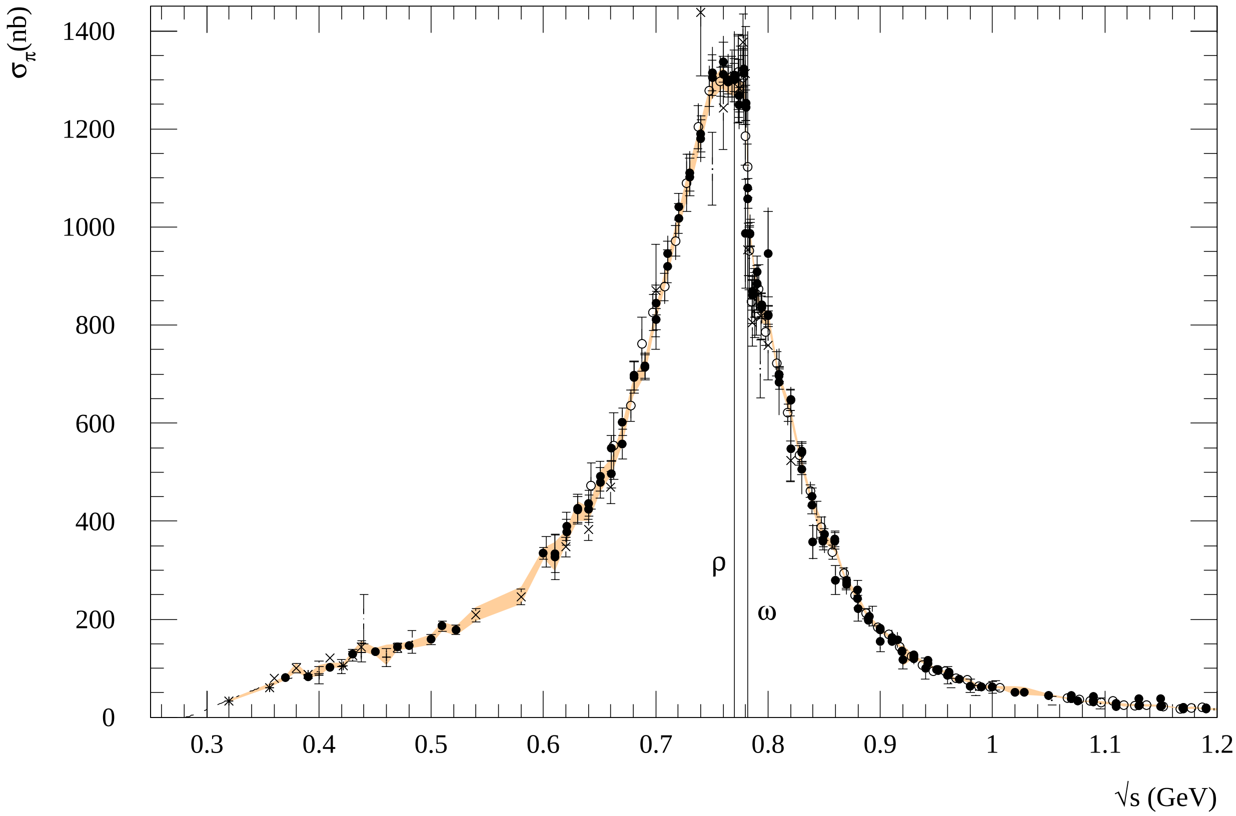
- Easy check of data-consistency and fit-quality via χ^2, and:

▶ we always find a stable a_μ^{had} with stable error TOGETHER with a minimal χ^2 when varying the cluster-size.

How does it work? A few examples:

$\longrightarrow$

The most important channel: $\pi^+\pi^-$



Cluster-size: min. 5 MeV. $\rho - \omega$ interference: zoom $\longrightarrow$

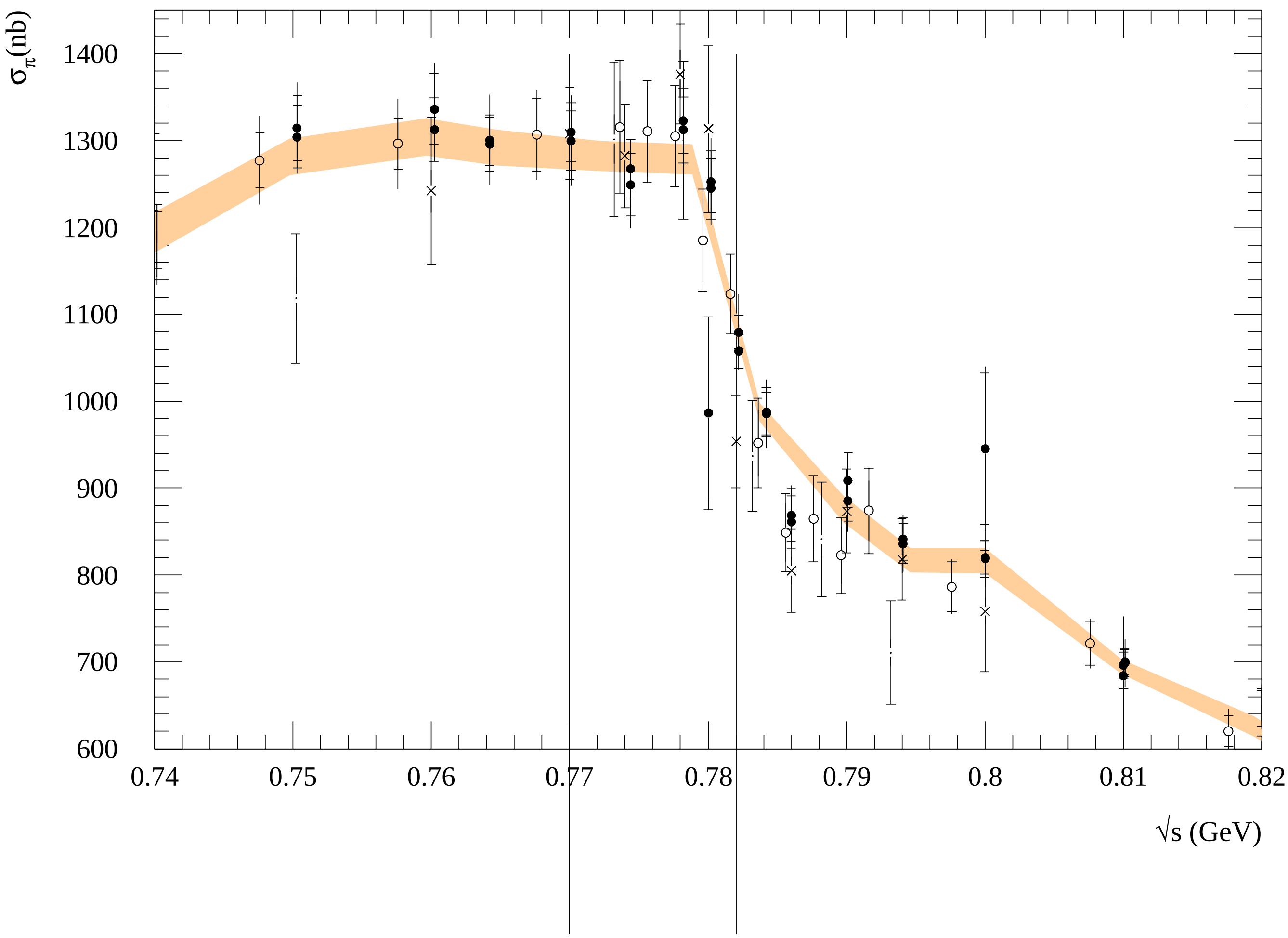
Contribution to a_μ if integrated from 0.32 up to 2 GeV: $(490.94 \pm 4.75) \cdot 10^{-10}$
Further improvement will be possible soon:

- Radiative return analysis at KLOE aims at an accuracy comparable with the recent CMD-2 measurement from Novosibirsk (but probably cannot near threshold).
- In the tail below 0.6 GeV the data is still poor, but the analysis of CMD-2 is underway, from which alone we estimate[1] an improvement of the error of a_μ^{had} of about $1 \cdot 10^{-10}$!

[1] using preliminary CMD-2 data with an estimated systematic error of 2% included already in the plot for illustration

$\rho - \omega$ interference:

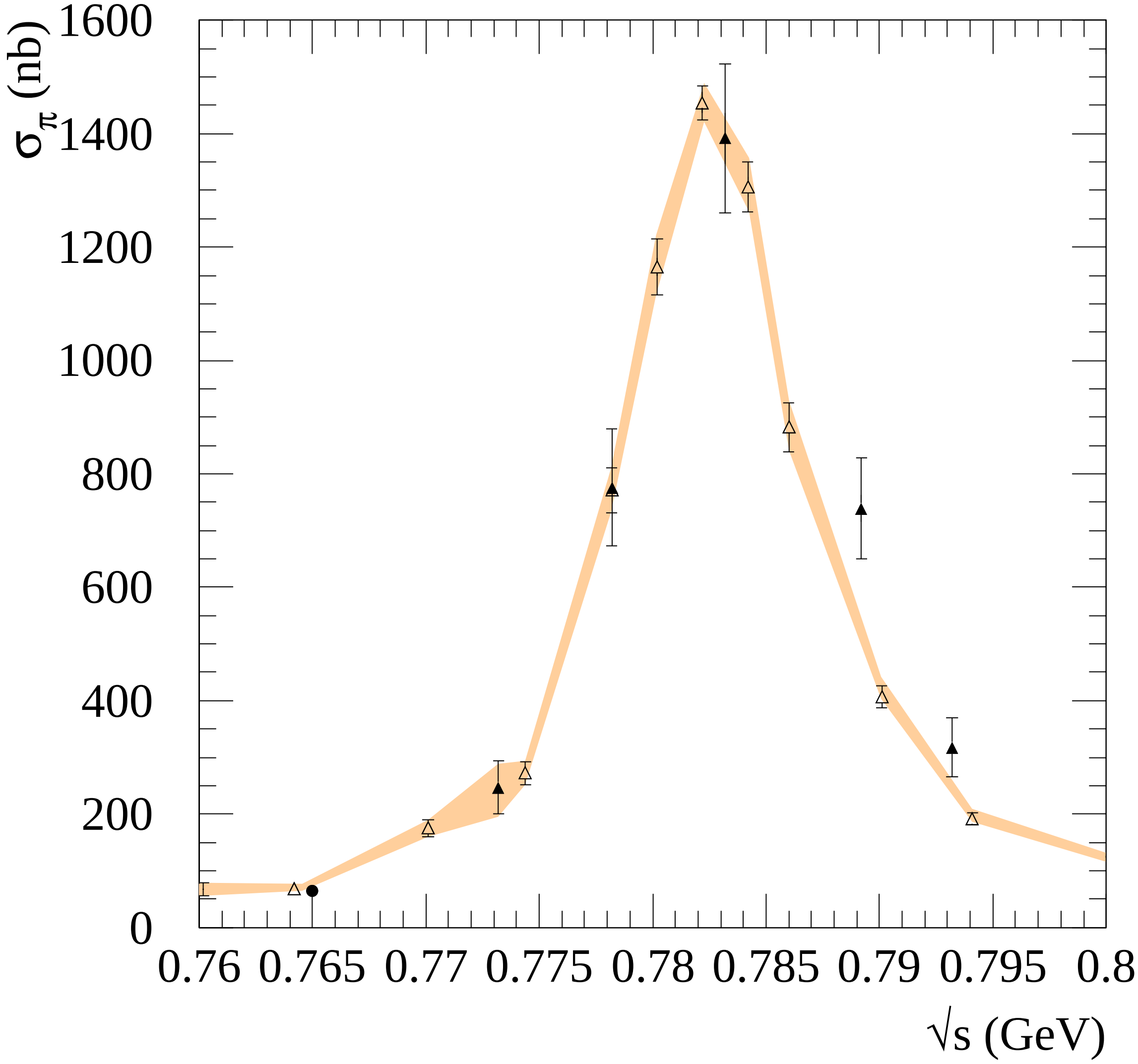
The result for a_μ and it's error are stable w.r.t. the variation of the clustering size; no significant improvement of $\chi^2/d.o.f. \approx 1$ of the overall-fit of the $\pi^+\pi^-$ data for clustering finer than 5 MeV.



ω in the 3π channel:

For the (narrow!) ω and ϕ the same data-driven approach is applied: integration of the $e^+e^- \rightarrow hadrons$ data without using parametrizations of the resonance-shapes.

Clustering size: 0.5 MeV. Note the energy scale!



III. Results

Adding up the channels. Excl. vs. inclusive

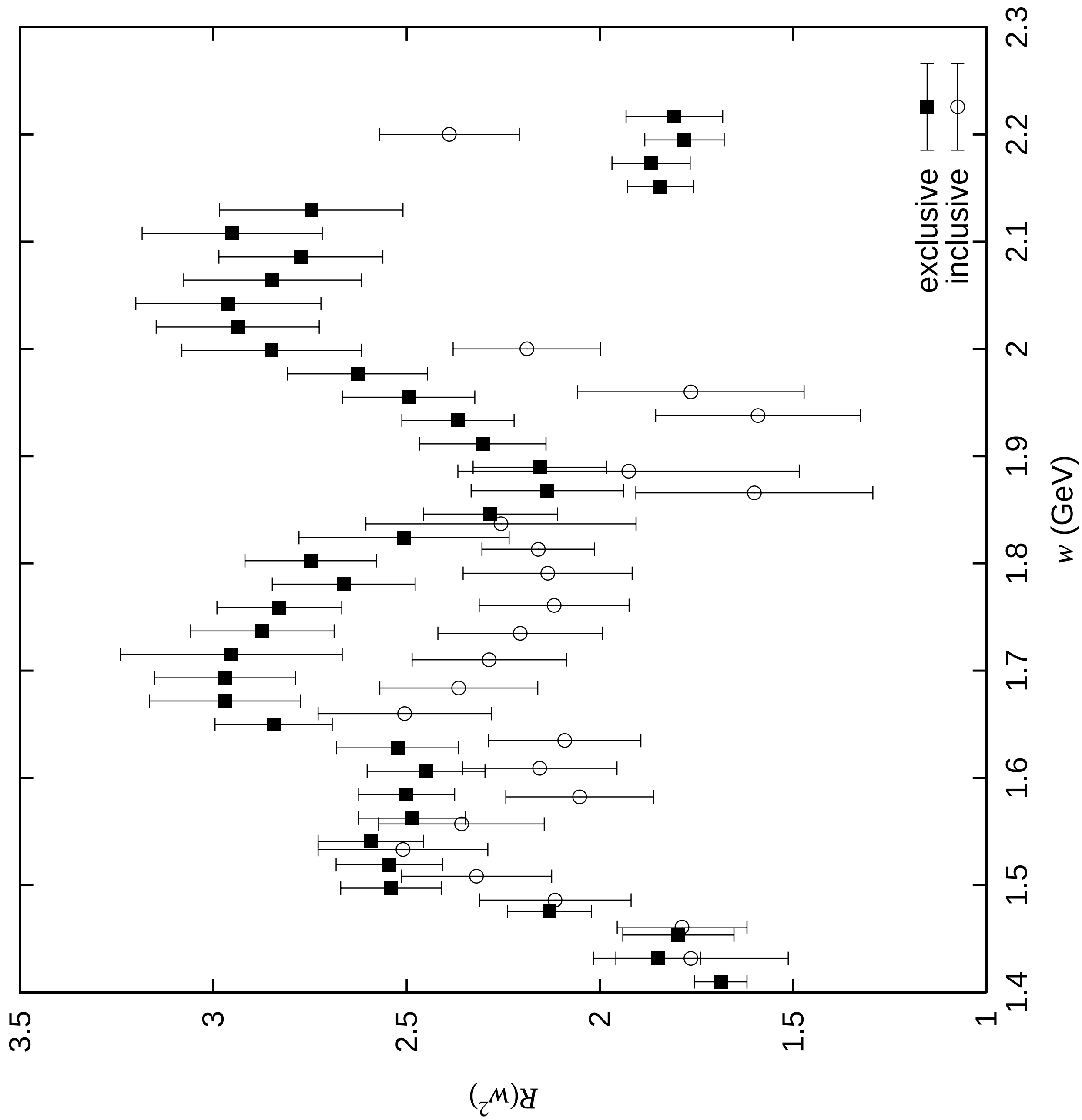
energy range	comments	$a_\mu^{\rm had} \times 10^{10}$
$2m_\pi \ldots 0.32$	chiral PT	2.30 ± 0.05
$0.32 \ldots 1.43$	excl. only	597.55 ± 4.97
$1.43 \ldots 2.00$	excl. only	38.14 ± 1.68
	incl. only	32.15 ± 2.46
$2.00 \ldots 11.09$	incl. only	42.06 ± 1.25
J/ψ and $\psi(2S)$	NW	7.31 ± 0.43
$\Upsilon(1-6S)$	NW	0.10 ± 0.00
$11.09 \ldots \infty$	pQCD	2.03 ± 0.01
$\sum$ of all	ex-ex-in	689.50 ± 5.85
	ex-in-in	683.51 ± 5.70

⋆ Note: pure data-integration without using parametrizations over the ρ, ω and ϕ resonances, hence no problem with double counting, tails, interferences!

♢ Extended use of pQCD from $3.00 \ldots 3.73$ and $5.50 \ldots \infty$ gives the slightly (but not significantly) smaller values 688.52 ± 5.79 (682.53 ± 5.64).

♢ 'Puzzle' exclusive vs. inclusive analysis! Are there 'missing' channels (e.g. neutral modes in incl.) and/or radiative corrections (e.g. vac-pol. in excl.)? ⟶

Compilation of inclusive and $\sum$ over exclusive data after 'clustering':



Comparison with selected other analyses

Group	Comments	$a_\mu^{\rm had} \times 10^{10}$
this analysis	ex-ex-in	689.50 ± 5.85
	ex-in-in	683.51 ± 5.70
E+J '95		702.4 ± 15.3
Jegerlehner '01	(update of E+J '98)	697.4 ± 10.5
J. (Marseille, March '02)	(prel., w. new CMD-2 data)	688.9 ± 5.8
this analysis	with their E-'binning' etc.	683.38 ± 6.06
A+D+H '98	primarily e^+e^- data	695.0 ± 15.0
D+H '98	e^+e^-, τ and pQCD	695.1 ± 7.5
D+H '98	e^+e^-, τ and QCD $\sum$-rules	692.4 ± 6.2
this analysis	with their E-'binning' etc.	690.68 ± 5.91

Note: comparing single channels/energy regimes the biggest discrepancy comes from the $\pi^+\pi^-$ channel.

► Here we use the latest (published) results from CMD-2 for $\sigma^0_{\pi\pi(\gamma)}$ (not π-formfactor values, sometimes not suitably defined for the use in the dispersion integral), which include a *corrected* treatment of radiative corrections, very good systematics (0.6%!) and an improved data-analysis. This is partly responsible for the reduction of the contribution to a_μ in this channel.

- Important improvement through new data from Beijing (BES, inclusive) and from Novosibirsk (CMD-2 and SND, exclusive channels).

- ? Newer experiments $\longrightarrow$ smaller x-sections ?

The complete SM prediction of g-2 of the muon

source of	contribution to $a_\mu \times 10^{11}$	Ref.
QED	116584705.7 ± 2.9	
EW	152 ± 4	Knecht et al., `hep-ph/0205102`
LO hadronic	6895 ± 59	this analysis, ‘excl.’
	6835 ± 57	this analysis, ‘incl.’
NLO hadronic	-100 ± 6	A+D+H ’98 in agreement with Krause ’97
Light-by-Light	80 ± 40	A. Nyffeler, `hep-ph/0203243`
$\sum$	116591732.7 ± 72	‘excl.’, errors added in quadrature
$\sum$	116591672.7 ± 70	‘incl.’, errors added in quadrature
Brookhaven Exp.	116592023 ± 151	Brown et al., PRL86(2001)2227
EXP-TH	290 ± 167	‘excl.’
	350 ± 166	‘incl.’

IV. Summary/Outlook

- The biggest uncertainty in g-2 still comes from the LO hadronic (vac.-pol.) contributions and is about $\pm 60 \cdot 10^{-11}$, but the error from LbL is conservatively estimated to be as high as $\pm 40 \cdot 10^{-11}$.
- We have presented a new, *data-driven* analysis for the LO hadr. contr., including new data from BES, CMD-2, SND.
- 'Non-linear Clustering' allows for a thorough treatment of stat. and sys. errors in the combination of data from different experiments.
- (New) Data in the ω and ϕ resonance regions are integrated over on the same footing (no resonance-parametrizations). Distortions from underlying background are therefore included automatically.
- τ data are not used as it is hard to quantify the associated uncertainty.

▶ For the future:

⋆ Further improved treatment of 'missing channels' and radiative corrections; inclusion of correlations between different experiments and/or channels.

⋆ Reanalysis of $\Delta\alpha_{\rm QED}$ using the same data-driven approach.

⋆ Use of QCD $\sum$-rules to disentangle the pQCD $\leftrightarrow$ incl. $\leftrightarrow$ excl. puzzle.

♢ Waiting for more precise data from direct and radiative return measurements (KLOE, BABAR, BELLE).

♣ Compare to the forthcoming new g-2 measurement.

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