Neutral Higgs boson contributions to the decays $\bar{B}_{d,s} \to \mu^+\mu^-$ in the MSSM at large $\tan\beta$

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Abstract

It is well known that the Minimal Supersymmetric Standard Model (MSSM) can enhance $\mathcal{B}\,(\bar{B}_{d,s} \to \mu^+\mu^-)$ by orders of magnitude [1], even if we assume the Cabibbo-Kobayashi-Maskawa (CKM) matrix to be the only source of flavour violation. Of particular interest is the quantity $R \equiv \mathcal{B}\,(\bar{B}_d \to \mu^+\mu^-)/\mathcal{B}\,(\bar{B}_s \to \mu^+\mu^-)$, since (i) the theoretical errors cancel to a large extent, and (ii) it offers a theoretically clean way of extracting the ratio $|V_{td}/V_{ts}|$ in the Standard Model, which predicts $R_{\mathrm{SM}} \sim |V_{td}/V_{ts}|^2 \sim O(10^{-2})$. Exploring three different scenarios of modified minimal flavour violation $(\overline{\mathrm{MFV}})$, we find that part of the MSSM parameter space can accommodate large $\bar{B}_{d,s} \to \mu^+\mu^-$ branching fractions, while being consistent with various experimental constraints. More importantly, we show that the ratio R can be as large as $O(1)$, while the individual branching fractions may be amenable to detection by ongoing experiments. We conclude that within the MSSM with large $\tan\beta$ the decay rates of $\bar{B}_{d,s} \to \mu^+\mu^-$ be of comparable size even in the case where flavour violation is due solely to the CKM matrix.

1 Introduction

The SM predicts the $\bar{B}_s \to \mu^+\mu^-$ branching ratio to be [2,3]

$$\mathcal{B}\,(\bar{B}_s \to \mu^+\mu^-) = (3.2 \pm 1.5) \times 10^{-9}, \tag{1}$$

and the ratio of branching fractions

$$R_{\mathrm{SM}} \equiv \left.\frac{\mathcal{B}\,(\bar{B}_d \to \mu^+\mu^-)}{\mathcal{B}\,(\bar{B}_s \to \mu^+\mu^-)}\right|_{\mathrm{SM}} \approx \frac{\tau_{B_d}}{\tau_{B_s}}\frac{M_{B_d}}{M_{B_s}}\frac{f_{B_d}^2}{f_{B_s}^2}\frac{|V_{td}|^2}{|V_{ts}|^2} \sim O(10^{-2}), \tag{2}$$

where τ_{B_q} is the lifetime of the B_q meson, M_{B_q} and f_{B_q} are the corresponding mass and decay constant. However, given the SM prediction of $\mathcal{B}\,(\bar{B}_d \to \mu^+\mu^-) \sim O(10^{-10})$, the $\bar{B}_d \to \mu^+\mu^-$ decay is experimentally remote unless it is significantly enhanced by new physics. Thus, the purely leptonic decays of neutral B mesons provide an ideal testing ground for physics outside the SM, with the current experimental upper bounds [4] $\mathcal{B}\,(\bar{B}_d \to \mu^+\mu^-) < 2.8 \times 10^{-7}$ and $\mathcal{B}\,(\bar{B}_s \to \mu^+\mu^-) < 2.0 \times 10^{-6}$, both given at 90% C.L..

The main interest in this talk is in a qualitative comparison of the $\bar{B}_{d,s} \to \mu^+\mu^-$ branching fractions in the presence of non-standard interactions, which can be made by using the ratio

$$R \equiv \frac{\mathcal{B}\,(\bar{B}_d \to \mu^+\mu^-)}{\mathcal{B}\,(\bar{B}_s \to \mu^+\mu^-)}. \tag{3}$$

[1]Talk based on collaborations with Christoph Bobeth, Thorsten Ewerth, and Frank Krüger [1,2].

Referring to Eq. (2), it is important to note that the suppression of R in the SM is largely due to the ratio of the CKM elements. This dependence on the CKM factors allegedly pertains to all models in which the quark mixing matrix is the only source of flavour violation. It is therefore interesting to ask if R could be of the order unity in some non-standard models where flavour violation is governed exclusively by the CKM matrix. Working in the framework of the Minimal Supersymmetric Standard Model (MSSM) with a large ratio of Higgs vacuum expectation values, $\tan\beta$ (ranging from 40 to 60), we show that such a scenario does exist, and study its consequences for the $\bar{B}_{d,s} \to \mu^+\mu^-$ branching ratios.

The outline is as follows. First, in Sec. 2, we define modified minimal flavour violation ($\overline{\mathrm{MFV}}$) and discuss briefly three distinct scenarios within the MSSM. Second, the effective Hamiltonian describing the decays $\bar{B}_{d,s} \to \mu^+\mu^-$ in the presence of non-SM interactions is given in Sec. 3. Furthermore, in Sec. 4, numerical results for the branching fractions $\mathcal{B}\,(\bar{B}_{d,s} \to \mu^+\mu^-)$ and the ratio R are presented. Finally, we summarize and conclude.

2 Modified Minimal Flavour Violation

There exists no unique definition of minimal flavour violation (MFV) in the literature (see, e.g., Refs. [5–9]). The common feature of these MFV definitions is that flavour violation and/or flavour-changing neutral current (FCNC) processes are entirely governed by the CKM matrix. On the other hand, they differ, for example, by the following additional assumptions: i) there are no new operators present, in addition to those of the SM [3, 5–7], ii) FCNC processes are proportional to the same combination of CKM elements as in the SM [8] or iii) flavour transitions occur only in charged currents at tree level [9]. While these ad hoc assumptions are useful for certain considerations, such as the construction of the universal unitarity triangle [7], they cannot be justified by symmetry arguments on the level of the Lagrangian. For example, the number of operators with a certain dimension is always fixed by the symmetry of the low-energy effective theory. Whether the Wilson coefficients are negligible or not, depends crucially on the model considered and on the part of the parameter space. Furthermore, the requirement that FCNC processes are proportional to the same combination of CKM elements as in the SM fails, for example, in the MSSM and can be retained only after further simplifying assumptions. The last statement in iii) is of pure phenomenological relevance in order to avoid huge contributions to FCNC processes.

Using symmetry arguments, we propose an approach that relies only on the key ingredient of the MFV definitions in Refs. [5–9], without considering the above mentioned additional assumptions i)–iii). We call an extension of the SM a modified minimal flavour-violating ($\overline{\mathrm{MFV}}$) model if and only if FCNC processes or flavour violation are entirely ruled by the CKM matrix; that is, we require that FCNC processes vanish to all orders in perturbation theory in the limit $V_{\mathrm{CKM}} \to \mathbb{1}$. For a motivation of this definition using symmetry arguments we refer the reader to Ref. [1]. As will become clear, the advantage of $\overline{\mathrm{MFV}}$ is that it is less restrictive than MFV, while the CKM matrix remains the only source of FCNC transitions.

Our definition of $\overline{\mathrm{MFV}}$ is manifest basis independent. However, in order to find a useful classification of different $\overline{\mathrm{MFV}}$ scenarios within the MSSM, we will work in the super-CKM basis. In this basis the quark mass matrices are diagonal, and both quarks and squarks are rotated simultaneously. The scalar quark mass-squared matrices in this basis have the structure

$$\mathcal{M}_U^2 = \begin{pmatrix} \mathcal{M}^2_{U_{LL}} & \mathcal{M}^2_{U_{LR}} \\ \mathcal{M}^{2\dagger}_{U_{LR}} & \mathcal{M}^2_{U_{RR}} \end{pmatrix}, \quad \mathcal{M}_D^2 = \begin{pmatrix} \mathcal{M}^2_{D_{LL}} & \mathcal{M}^2_{D_{LR}} \\ \mathcal{M}^{2\dagger}_{D_{LR}} & \mathcal{M}^2_{D_{RR}} \end{pmatrix}, \tag{4}$$

where the 3×3 submatrices are given in [1]. Here we present the two important entries only:

$$\mathcal{M}^2_{U_{LL}} = M^2_{\tilde{U}_L} + M^2_U + \frac{1}{6} M_Z^2 \cos 2\beta (3 - 4 \sin^2 \theta_W) \mathbb{1}, \tag{5}$$

$$\mathcal{M}^2_{D_{LL}} = M^2_{\tilde{D}_L} + M^2_D - \frac{1}{6} M_Z^2 \cos 2\beta (3 - 2 \sin^2 \theta_W) \mathbb{1}. \tag{6}$$

Because of SU(2) gauge invariance, the mass matrix $M^2_{\tilde{D}_L}$ is intimately connected to $M^2_{\tilde{U}_L}$ via

$$M^2_{\tilde{D}_L} = V^\dagger_{\mathrm{CKM}} M^2_{\tilde{U}_L} V_{\mathrm{CKM}}, \tag{7}$$

which is important for our subsequent discussion.

The above given definition of $\overline{\mathrm{MFV}}$ requires that the soft SUSY trilinear couplings A_U, A_D and the soft SUSY breaking squark masses $M_{\tilde{U}_R}$, $M_{\tilde{D}_R}$ are diagonal.

Taking into account the relation in Eq. (7), one encounters three cases of $\overline{\mathrm{MFV}}$.

- **Scenario (A):**
  $M^2_{\tilde{U}_L}$ is proportional to the unit matrix, and so $M^2_{\tilde{U}_L} = M^2_{\tilde{D}_L}$. As a result, there are no gluino and neutralino contributions to flavour-changing transitions at one-loop level. This scenario of $\overline{\mathrm{MFV}}$ coincides with the MFV scenario at low $\tan\beta$ as defined in Refs. [7,9].

- **Scenario (B):**
  $M^2_{\tilde{D}_L}$ is diagonal but not proportional to the unit matrix and, in consequence, $M^2_{\tilde{U}_L}$ has non-diagonal entries. In such a case, there are again no gluino and neutralino contributions to flavour-changing one-loop transitions involving only external down-type quarks and leptons. However, additional chargino contributions show up, due to non-diagonal entries of $M^2_{\tilde{U}_L}$.

- **Scenario (C):**
  $M^2_{\tilde{U}_L}$ is diagonal but not proportional to the unit matrix, which gives rise to off-diagonal entries in $M^2_{\tilde{D}_L}$. Accordingly, gluino and neutralino exchange diagrams (in addition to those involving $W^\pm$, $\tilde{\chi}^\pm$, charged and neutral Higgs bosons) contribute to flavour-changing transitions at one-loop level that involve external down-type quarks.

The common feature of all these scenarios is that the CKM matrix is the only source of flavour violation.

3 Effective Hamiltonian

The effective Hamiltonian responsible for the processes $\bar{B}_q \to l^+l^-$, with $q = d, s$ and $l = e, \mu, \tau$, in the presence of non-standard interactions is given by

$$H_{\text{eff}} = -\frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{tq}^* \sum_{i=10,S,P} [c_i(\mu)\mathcal{O}_i(\mu) + c_i'(\mu)\mathcal{O}_i'(\mu)], \tag{8}$$

with the short-distance coefficients $c_i^{(\prime)}(\mu)$ and the local operators

$$\mathcal{O}_{10} = (\bar{q}\gamma^\mu P_L b)(\bar{l}\gamma_\mu \gamma_5 l), \quad \mathcal{O}_S = m_b(\bar{q}P_R b)(\bar{l}l), \quad \mathcal{O}_P = m_b(\bar{q}P_R b)(\bar{l}\gamma_5 l), \tag{9}$$

where $P_{L,R} = (1 \mp \gamma_5)/2$. The primed operators can be obtained by $P_L \leftrightarrow P_R$ and $m_b \to m_q$. It turns out that the primed Wilson coefficients are negligibly small, and hence can be safely neglected.

At high $\tan\beta$ the scalar and pseudoscalar operators $\mathcal{O}_S$ and $\mathcal{O}_P$ become important in addition to the so called SM-operator $\mathcal{O}_{10}$. In this region of the parameter space an expansion according to $\tan\beta$ is possible and a general expression for $c_i\mathcal{O}_i$ looks like

$$c_i\mathcal{O}_i = \sum_n A_n \tan^{n+1}\beta \left(\frac{m}{M}\right)^n + \sum_n B_n \tan^n\beta \left(\frac{m}{M}\right)^n + \ldots, \tag{10}$$

where we call the first term leading and the second term subleading. m denotes lepton and light quark masses while M stands for masses of particles that have been integrated out. Explicit expressions for the Wilson-coefficients can be found in Refs. [1, 2, 10].

4 Numerical analysis

The experimental bounds used in this numerical analysis as well as the ranges of the MSSM parameters can be found in Ref. [1].

Scenario (A) and (B)

Recall that in scenario (A) the matrices $M^2_{\tilde{U}_L}$ and $M^2_{\tilde{D}_L}$ are equal and proportional to the unit matrix. Therefore, the gluinos and neutralinos do not contribute at one-loop level. The scan over the parameter region shows that the ratio R is approximately constant and close to $R_{\text{SM}} \approx 0.03$.

In scenario (B), the matrix $M^2_{\tilde{D}_L}$ is diagonal, $M^2_{\tilde{D}_L} = \text{diag}(m^2_{\tilde{d}_L}, m^2_{\tilde{s}_L}, m^2_{\tilde{b}_L})$, with at least two different entries. Hence, there are no gluino and neutralino contributions at one-loop level. Employing the relation in Eq. (7), the matrix $M^2_{\tilde{U}_L} = V_{\text{CKM}} M^2_{\tilde{D}_L} V^\dagger_{\text{CKM}}$ becomes non-diagonal. These off-diagonal flavour-changing entries are constrained by experimental data on K^0–$\bar{K}^0$, B^0–$\bar{B}^0$, D^0–$\bar{D}^0$ oscillations, and the $b \to s\gamma$ decay [11–13]. It is important to note that the bounds on these flavour-changing entries [11] severely constrain the additional chargino contributions in scenario (B), hence we end up with a result similar to scenario (A). R varies between 0.026 and 0.030. Neglecting the constraints we could have found R in the range $0.002 \lesssim R \lesssim 0.115$.

Scenario (C)

In this case, the matrix $M^2_{\tilde{U}_L}$ is diagonal, $M^2_{\tilde{U}_L} = \mathrm{diag}(m^2_{\tilde{u}_L}, m^2_{\tilde{c}_L}, m^2_{\tilde{t}_L})$, with at least two different entries. According to the relation in Eq. (7), this implies that $M^2_{\tilde{D}_L}$ has non-diagonal entries, so that gluinos and neutralinos contribute to the $b \to q l^+ l^-$ transition already at one-loop level. As before, we take the constraints of Refs. [11–13] on these off-diagonal elements.

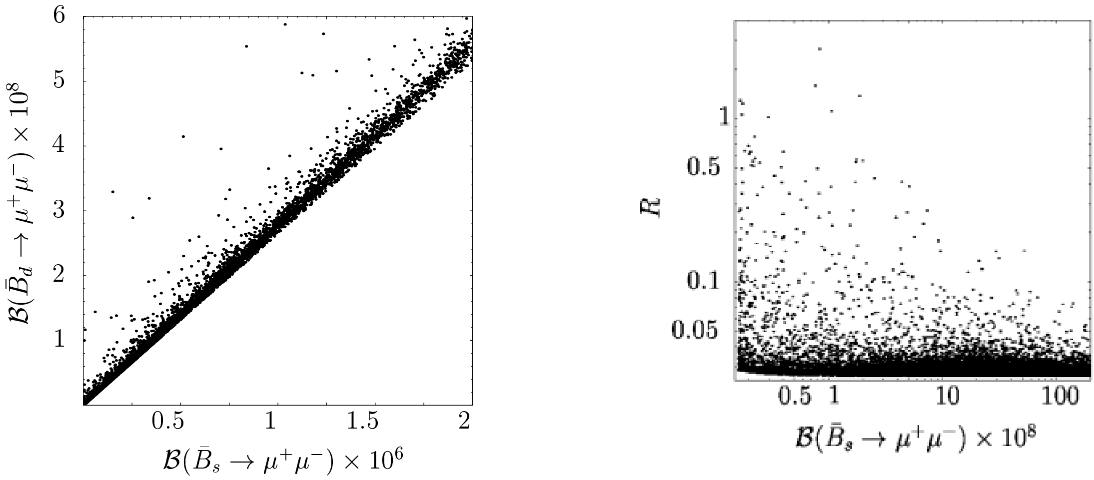

Figure 1: Predictions for the branching ratios $\mathcal{B}\,(\bar{B}_d \to \mu^+\mu^-)$ vs. $\mathcal{B}\,(\bar{B}_s \to \mu^+\mu^-)$ (left plot) and R vs. $\mathcal{B}\,(\bar{B}_s \to \mu^+\mu^-)$ (right plot) in scenario (C). R varies between 0.026 and 2.863.

The scatter plots in Fig. 1 exhibit an order-of-magnitude deviation from $R_{\mathrm{SM}} \approx 0.03$. We find $0.026 \lesssim R \lesssim 2.863$. A noticeable feature of scenario (C) is that there exists a lower bound on R, i.e. $R \gtrsim 0.95\,R_{\mathrm{SM}}$ (see left plot of Fig. 1), which is due to the structure of the CKM matrix. We stress that this bound is valid only within scenario (C) and does not apply to scenario (B) or scenarios with new sources of flavour violation (see Sec. 2).

An interesting subset of parameter points was considered in Fig. 2 (for details see Ref. [1]) Interestingly, in the left (right) plot, the ratio R ranges between $0.15 \lesssim R \lesssim 0.81$ ($0.44 \lesssim R \lesssim 1.78$), while the magnitude of the individual branching fractions decreases drastically with increasing charged Higgs boson mass, M_H. Note that for small M_H and $\tan\beta$ close to 60 both branching ratios are in a region that can be probed experimentally, in Run II of the Fermilab Tevatron, BABAR and Belle.

The neutralino Wilson coefficients are numerically smaller than those coming from the chargino and gluino contributions. However, we have found that in certain regions of the MSSM parameter space cancellations between the chargino and gluino coefficients occur, in which case the neutralino contributions become important. As a matter of fact, for the SUSY parameter sets examined, we found that a large value of $R \equiv \mathcal{B}\,(\bar{B}_d \to \mu^+\mu^-)/\mathcal{B}\,(\bar{B}_s \to \mu^+\mu^-)$ always involves such a cancellation.

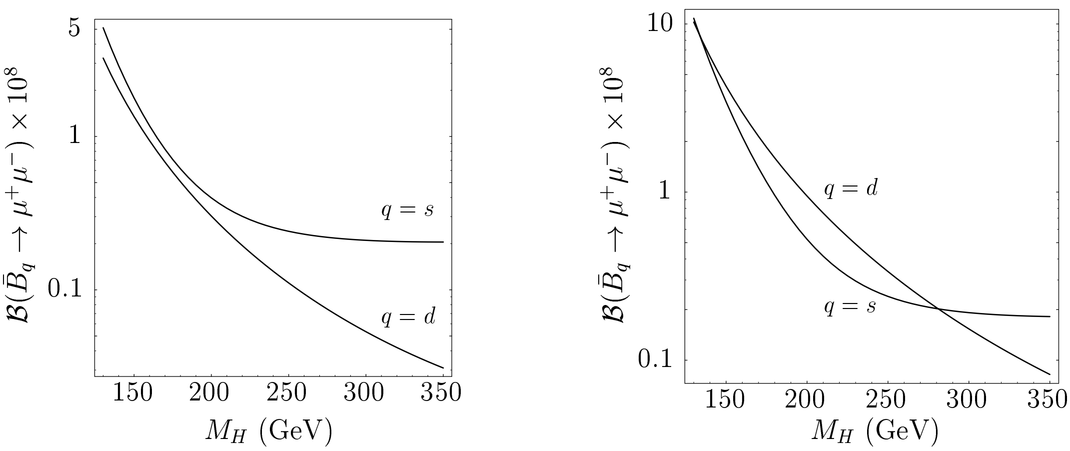

Figure 2: $\mathcal{B}\,(\bar{B}_{d,s} \to \mu^+\mu^-)$ as function of the charged Higgs boson mass, M_H, for $\tan\beta = 50$ (left plot) and $\tan\beta = 60$ (right plot), taking into account the experimental constraints on the rare B decays and on the flavour-changing entries. For the remaining parameters see [1].

5 Summary and conclusions

We have defined $\overline{\mathrm{MFV}}$ using symmetry arguments and have shown that $\overline{\mathrm{MFV}}$ is less restrictive than MFV, while the CKM matrix remains the only source of flavour violation. Within the MSSM we have investigated three scenarios that are possible within the context of $\overline{\mathrm{MFV}}$. In particular, we have studied the case where the chargino exchange diagrams [2] as well as the gluino and neutralino exchange diagrams [1] contribute besides $W^\pm, H^\pm, \tilde{\chi}^\pm$ [scenario (C)].

Including current experimental data on rare B decays, as well as on K, B, D meson mixing, we found that in certain regions of the SUSY parameter space the branching ratios $\mathcal{B}\,(\bar{B}_d \to \mu^+\mu^-)$ and $\mathcal{B}\,(\bar{B}_s \to \mu^+\mu^-)$ can be up to the order of 10^{-7} and 10^{-6} respectively. Specifically, we showed that there exist regions in which the branching fractions of both decay modes are comparable in size, and may well be accessible to Run II of the Fermilab Tevatron as well as R can deviate from R_{SM} by orders of magnitude.

We wish to stress that a measurement of the branching ratios $\mathcal{B}\,(\bar{B}_{d,s} \to \mu^+\mu^-)$, or equivalently, a ratio R of $O(1)$, does not necessarily imply the existence of new flavour violation outside the CKM matrix. Nevertheless, any observation of these decay modes in ongoing and forthcoming experiments would be an unambiguous signal of new physics.

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