Neutralino relic density in SUSY GUTs

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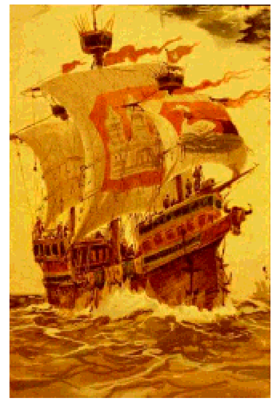
June 17, 2002
SUSY02, the 10th International
Conference on Supersymmetry and
Unification of Fundamental Interactions
DESY Hamburg



OUTLINE

- Status of CDM: experiment and theory
- Strategy and details of calculation
- Results for mSUGRA model
- SO(10) SUSY GUTs and Extra dimensions
- Conclusions

Introduction

- Experimental data:
  - Galactic rotation curves
  - High resolution maps of CMB
  - Large scale structure of the Universe
  - Supernova Type I data

  Combining, the above data we have:

$\Omega_{tot} = \rho/\rho_c \simeq 1$	Total energy
$\Omega_\Lambda \simeq 0.7$	Dark energy
$\Omega_M = 0.3 \pm 0.1$	Matter
$\Omega_{M_b} = 0.020 \pm 0.002$	Barionic matter
$\Omega_c h^2 \simeq 0.2 \pm 0.1$	Cold Dark Matter

- Why Supersymmetry?
  - Solves hierarchy problem of the SM, unifies gauge couplings, EWSB driven by radiative corrections
  - Incorporates gravity
  - Provides with the best CDM candidate: the lightest neutralino !

- Steady improvement in the quality of neutralino relic density in SUSY Goldberg(83),Ellis,Hagelin,Nanopoulos,Srednicki(83) $\to$ 2002
  - Standard procedure is to convolute neutralino annihilation cross sections (CS) with Boltzmann thermal distribution.
    Traditional CS expansion in terms of neutralino velocity – angular and energy integrals can be evaluated analytically
  - Importance of co-annihilation processes/ resonant annihilation — Griest and Seckel(91)
  - Formulae for *relativistic* thermal averaging worked out by Gondolo & Gelmini(91).
    Large $\tan\beta$ neutralino annihilation can be dominated by s-channel A and H resonances — Drees-Nojiri(93)
  - Formalism extended for the co-annihilation case by Edsjö and Gondolo(97)
  - Importance of the neutralino-slepton coannihilation was stressed out in various papers: Mizuta-Yamaguchi(93), Drees-Nojiri(93), Ellis-Falk-Olive(98,99),Gomes-Lazarides-Pallis(00), Baer-Balazs-A.B(02)
  - Large values of A_0 or for non-universal scalar masses: stop or sbottom masses could become degenerate with $\widetilde{Z}_1$ and squark coannihilation processes could become important: Boehm-Djouadi-Drees(00), Ellis-Olive-Santoso(01)

References

- H. Goldberg(83)
- Ellis, Hagelin, Nanopoulos, Olive, Srednicki(84)
- Srednicki, Watkins, Olive; Barbieri, Berezinsky; Griest (88)
- Barbieri, Frigeni and Giudice (89)
- Griest, Kamionkowski and Turner; Jungman (90)
- Griest, Seckel (91)
- Gelmini, Gondolo (91)
- Ellis, Gelmini, Lopez, Nanopoulos , Sarka(92)
- Bottino, de Alfaro, Fornengo, Mignola, Scopel; Berezinsky (92)
- Drees, Nojiri(93)
- Roszkowski, Roberts(93) ; Kane, Kolda, Roszkowski, Wells(94)
- Nath, Arnowitt(93)
- Baer, Brhlik(96); Baer, Brhlik, Diaz, Ferrandis, Mercadante, Quintana, Tata(01)
- Edsjö, Gondolo(97)
- Barger, Kao(98)
- Ellis, Olive, Schmitt(97) ; Ellis, Falk, Olive(98)
- Lahanas, Nanopoulos, Spanos(99,00,01)
- Feng, Matchev, Wilczek (00)
- Gomez,Lazarides, Pallis(00)
- Boehm, Djouadi, Drees (00)
- Arnowitt, Dutta, Santoso (01)
- Roszkowski, Ruiz de Austri, Nihei(01,02)
- Ellis, Olive, Santoso (01)
- Birkedal-Hansen, Nelson(01)
- Belanger, Boudjema, Pukhov, Semenov(01)

Recent studies aim to update CDM results

- Perform the *relativistic* relic density calculation including *all relevant* annihilation and co-annihilation processes.
- Update the relic density consistent with ISAJET set of SUSY models for comparison with collider search/reach results (Baer,Balazs,A.B)
- Relic density informs us about the favored parameter space regions, and likely collider signatures.
- Especially check the large $\tan\beta$ region and consistency of the favored region of parameter space with $g-2$ and $b \to s\gamma$ and $B_s \to \mu^+\mu^-$ data

Details of calculations

- The evolution of the number density of supersymmetric relics (following Griest and Seckel)

$$\frac{dn}{dt} = -3Hn - \langle\sigma_{eff}v\rangle\left(n^2 - n_{eq}^2\right)$$

where $n = \sum_{i=1}^{N} n_i$ and the sum extends over the N particle species contributing to the relic density, n_i – the number density of the ith species. $n_{eq,i}$ given by

$$n_{eq,i} = \frac{g_i m_i^2 T}{2\pi^2} K_2\left(\frac{m_i}{T}\right),$$

Details of calculations (continued)

- $\langle\sigma_{eff}v\rangle$ is the thermally averaged cross section times velocity (Gondolo and Gelmini, Edsjö and Gondolo):

$$\langle\sigma_{eff}v\rangle(x) = \frac{\int_2^\infty K_1\left(\frac{a}{x}\right)\sum_{i,j=1}^N \lambda(a^2,b_i^2,b_j^2)g_i g_j \sigma_{ij}(a)da}{4x\left(\sum_{i=1}^N K_2\left(\frac{b_i}{x}\right)b_i^2 g_i\right)^2},$$

where $x = T/m_{\widetilde{Z}_1}$, σ_{ij} is the cross section for the annihilation reaction $ij \to X$, $a = \sqrt{s}/m_{\widetilde{Z}_1}$ and $b_i = m_i/m_{\widetilde{Z}_1}$

- The relic density of neutralinos is given by

$$\Omega_{\widetilde{Z}_1}h^2 = \frac{\rho(T_0)}{8.1\times 10^{-47}\ \mathrm{GeV}^4}$$

where

$$\rho(T_0) \simeq 1.66\frac{1}{M_{Pl}}\left(\frac{T_{m_{\widetilde{Z}_1}}}{T_\gamma}\right)^3 T_\gamma^3\sqrt{g_*}\frac{1}{\int_0^{x_F}\langle\sigma_{eff}v\rangle dx}.$$

- The freeze-out temperature $x_F = T_F/m_{\widetilde{Z}_1}$ is determined as usual by an iterative solution of the freeze-out relation

$$x_F^{-1} = \log\left[\frac{m_{\widetilde{Z}_1}}{2\pi^3}\frac{g_{eff}}{2}\sqrt{\frac{45}{2g_*G_N}}\langle\sigma_{eff}v\rangle(x_F)\,x_F^{1/2}\right].$$

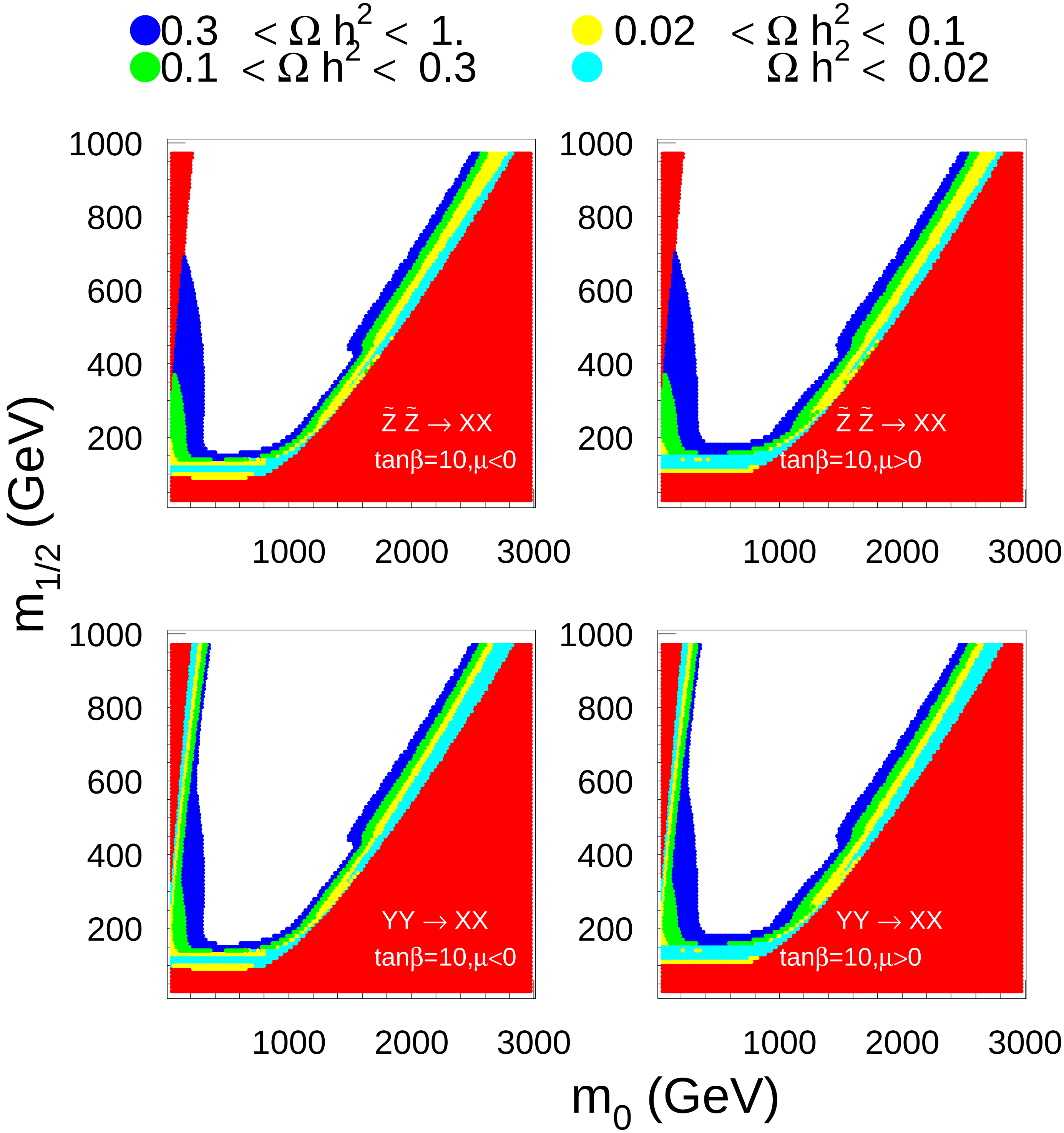
Here, g_{eff} denotes the effective number of degrees of freedom of the co-annihilating particles. The quantity g_* is the parameter of SM effective degrees of freedom. with $\sqrt{g_*}\simeq 9$ over our region of interest.

Details of calculations (continued)

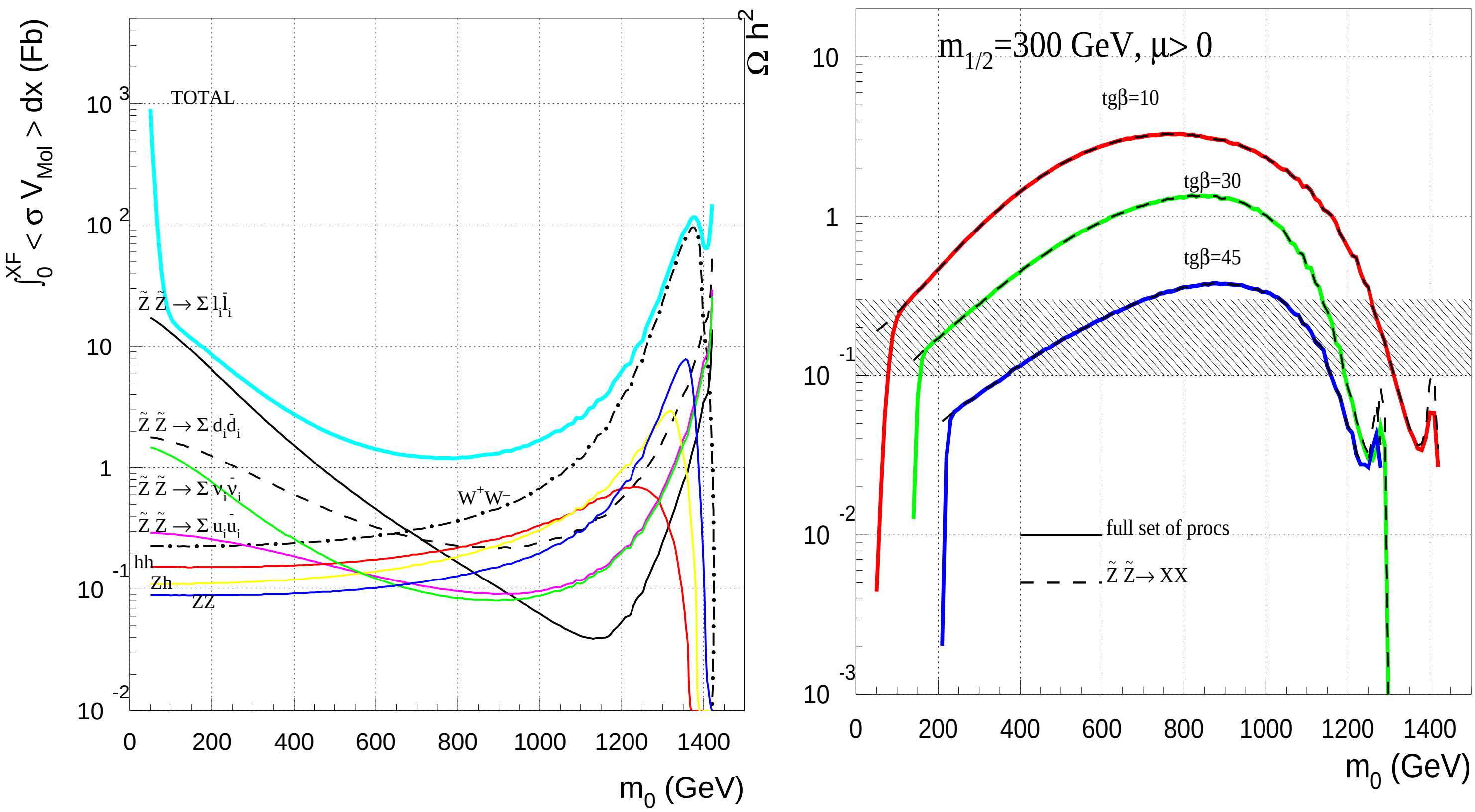
The challenge is to evaluate all possible channels for neutralino annihilation to SM and/or Higgs particles, and also all co-annihilation reactions.

- Code of Baer-Balazs-A.B.
  - initial state: $\widetilde{Z}_1$, $\widetilde{Z}_2$, $\widetilde{W}_1$, $\tilde{e}_1$, $\tilde{\mu}_1$, $\tilde{\tau}_1$, $\tilde{t}_1$ and $\tilde{b}_1$
  - relativistic thermal averaging
  - exact calculations: CompHEP – 1722 subprocesses, 7618 diagrams, about 50 MB of $FORTRAN$ code
  - weak scale parameters from supersymmetric models are generated using ISAJET and interfaced with the squared matrix elements from CompHEP
  - for our final result with relativistic thermal averaging, a three-dimensional integral must be performed over
    $i.$) the final state subprocess scattering angle θ
    $ii.$) the subprocess energy parameter $a = \sqrt{s}/m_{\widetilde{Z}_1}$
    $iii.$)the temperature T from freeze-out T_F to the present day temperature of the universe.
    About two orders of magnitude more CPU-time consuming than series expansion approach, which requires just one numerical integration

- Recent package: MicOMEGAs (Belanger,Boudjema,Pukhov,Semenov)
  - all coannihilation channels with neutralinos, charginos, sleptons, squarks and gluinos
  - all matrix elements based on the CompHEP
  - one-loop corrected Higgs width(based on the HDECAY)
  - relic density with any particle as the LSP
  - link with ISASUGRA/ISAJET
  - subroutines for calculation of constraints on the MSSM parameters are included:
    direct limits from colliders, $\Delta\rho$, $b \to s\gamma$ and $(g-2)_\mu$
  - total number of subprocesses is about 2800: creating and linking libraries during the run
  - g^* is tabulated as a function of T – about 5% effect for the relevant range of Ω_c
- Previous publically available packages:
  - Neutdriver(96) (Jungman,Kamionkowski,Griest)
  - DarkSUSY(00) (Gondolo,Edsjö,Bergstrom,Ulio,Baltz)

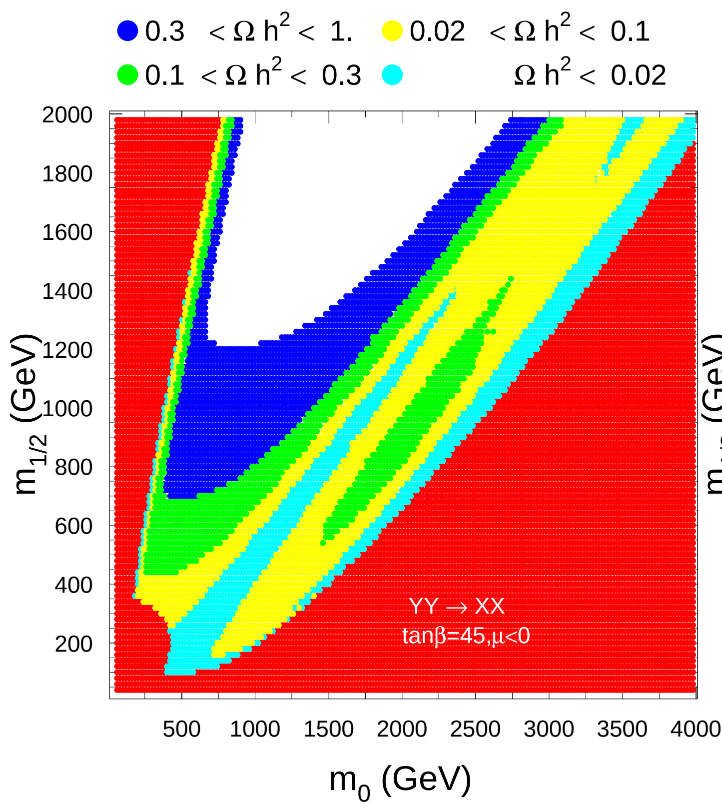


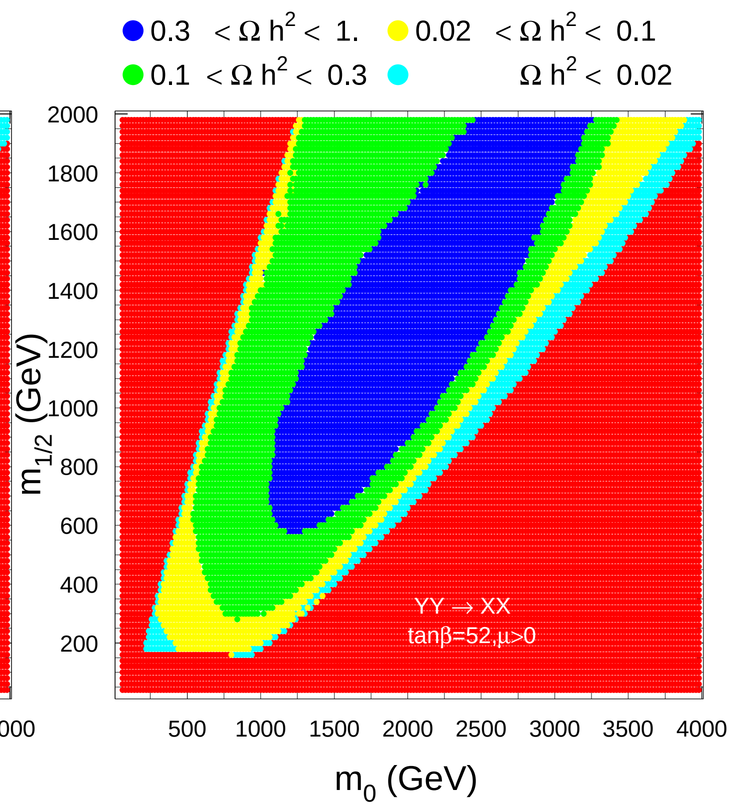
- $\Omega_{\widetilde{Z}_1} h^2$ in the m_0 $vs.$ $m_{1/2}$ plane in the minimal supergravity model for $A_0 = 0$, $\tan\beta = 10$ and for $\mu < (>) 0$
  Red and white regions are excluded:
  red — lack of REWSB, a charged LSP
  white — $\Omega_{\widetilde{Z}_1} h^2 > 1 \to$ the age of universe $< 10 \times 10^9$ years



- Low values of m_0: $\tilde{Z}_1\tilde{Z}_1 \to \ell^+\ell^-$ is dominated via t-channel slepton exchange.
- The lowest values of m_0: the rate is sharply increased by neutralino-stau and stau-stau co-annihilations (Ellis et al.)
- As m_0 increases, the slepton masses also increase, suppressing the annihilation cross section and rising the relic density
- When m_0 increases further — annihilation rate is dominated by scattering into WW, ZZ and Zh.
- $m_{\tilde{Z}_1} \simeq m_{\widetilde{W}_1} \simeq m_{\tilde{Z}_2}$, co-annihilation channels increase even more the annihilation rate.

High $\tan\beta$ results:



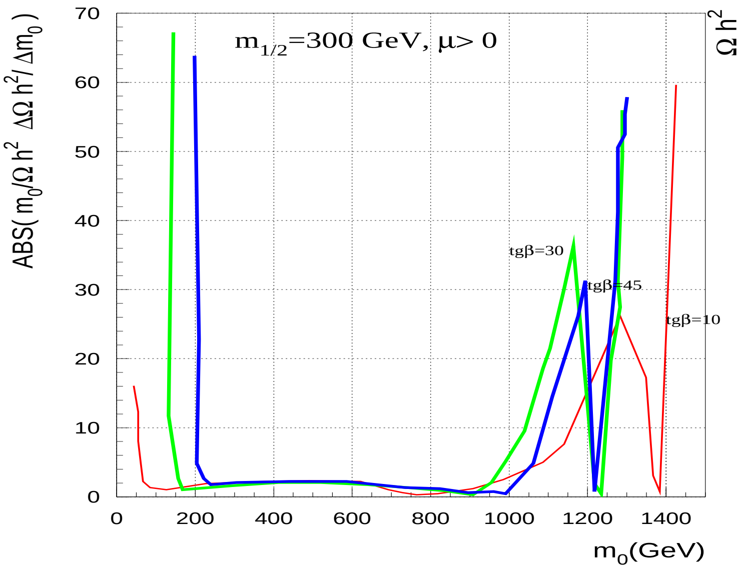


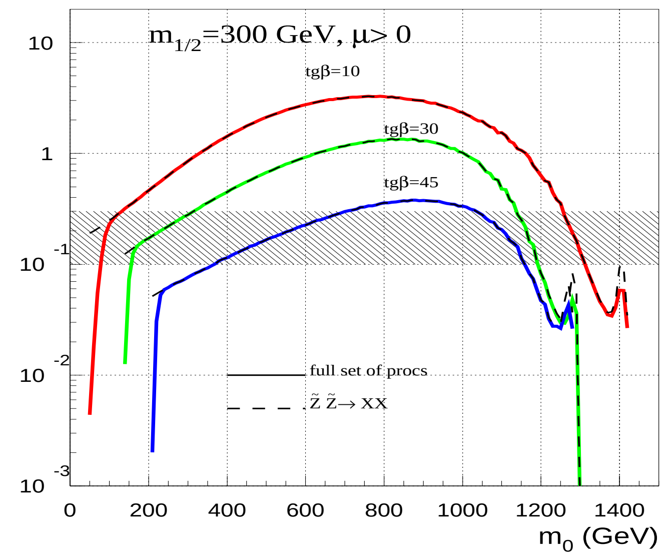
relic density regions are qualitatively different from the lower $\tan\beta$ plots

- $\mu < 0, \tan\beta = 45$:
  - diagonal strip – annihilation through s-channel A and H
  - the A and H widths are typically $20-60$ GeV. Even if $|2m_{\widetilde{Z}_1} - m_{A(H)}|$ is relatively large, efficient annihilation can still take place.
  - the relic density changes slowly on the flanks of the annihilation corridor $\Rightarrow$ little fine tuning
- $\mu > 0, \tan\beta = 52$:
  - relic density annihilation corridor occurs near the boundary of the excluded $\tilde{\tau}_1$ LSP region
  - the width of the A and H Higgs ranges from 30 GeV for $m_{1/2} \sim 400$ GeV, to 110-130 GeV for $m_{1/2} \sim 2000$ GeV
  - *none* of the entire parameter plane is excluded by $\Omega_{\widetilde{Z}_1} h^2 > 1$!

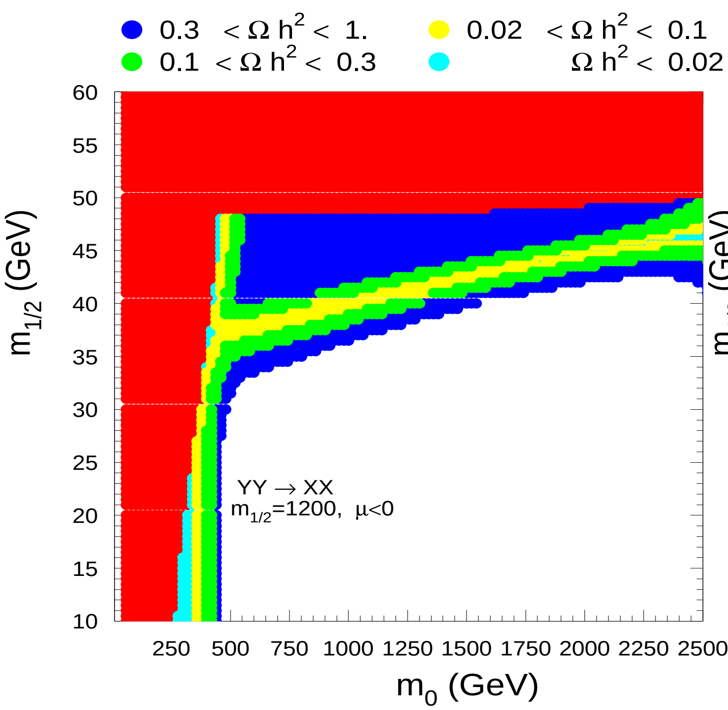
Fine-tuning measure

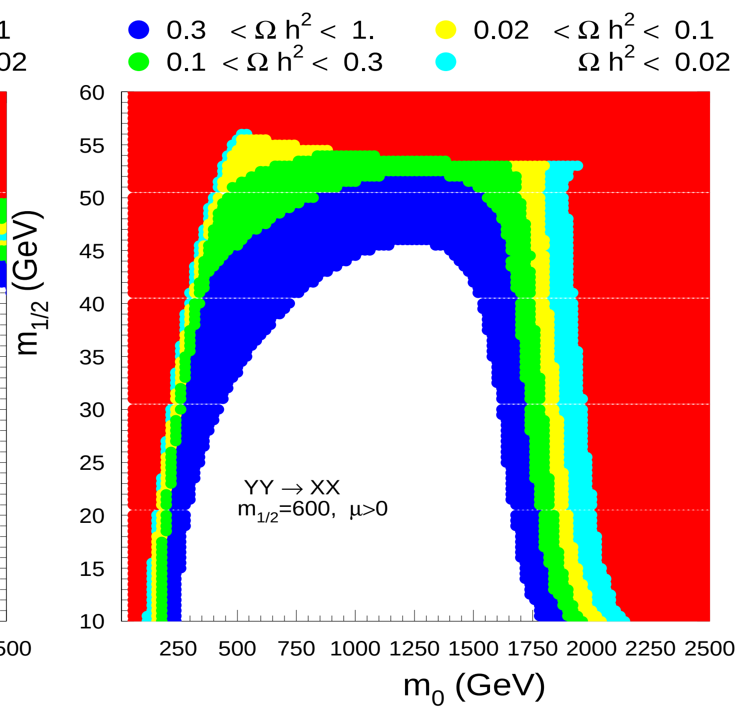
- $\Delta(m_0) = \left| \frac{m_0}{\Omega_{\widetilde{Z}_1} h^2} \frac{\partial \Omega_{\widetilde{Z}_1} h^2}{\partial m_0} \right|$ (Ellis and Olive)



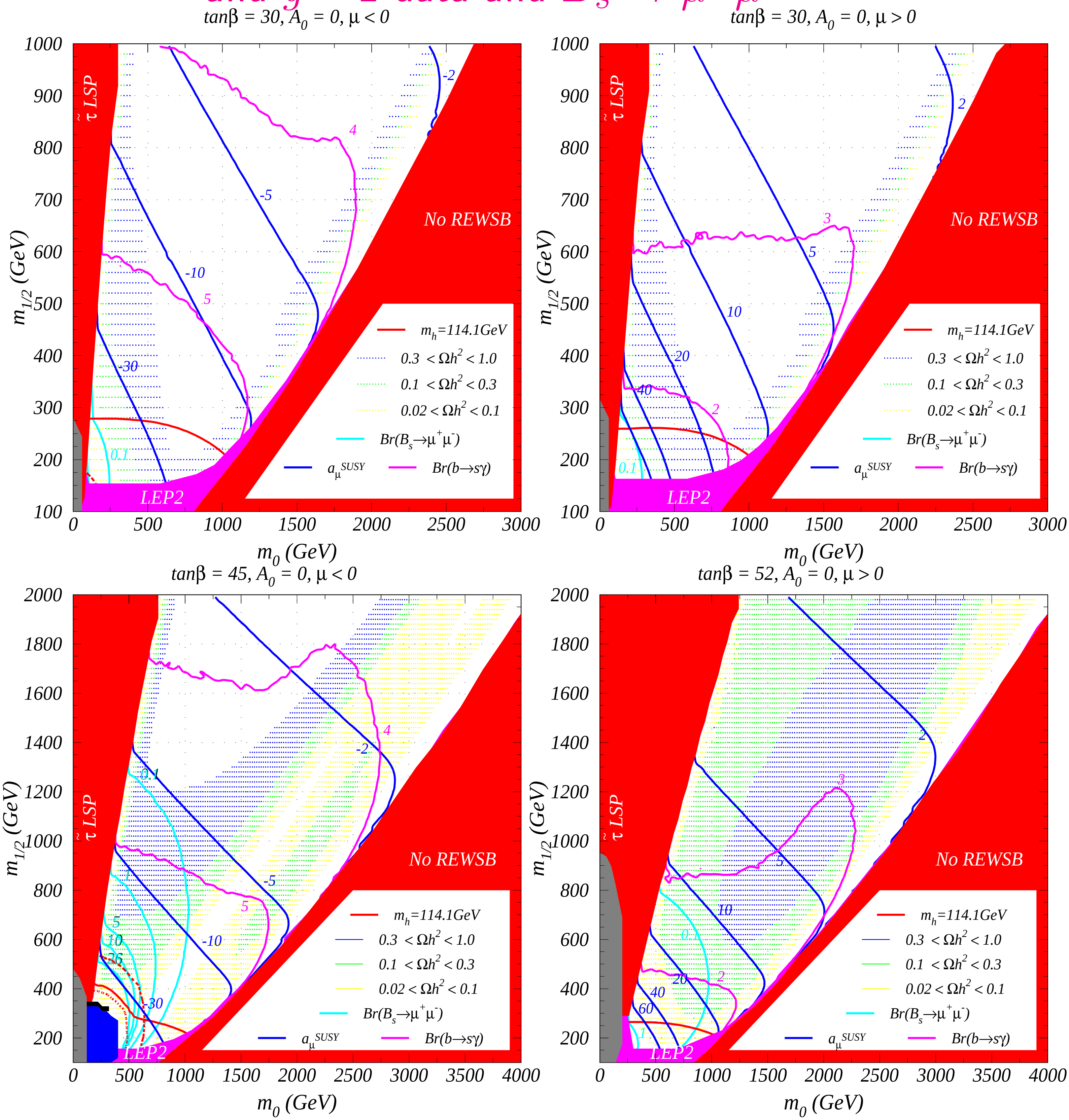


- $\tan\beta - m_0$ plane





LEPII constraints and regions favored by $b \to s\gamma$, and $g-2$ data and $B_s \to \mu^+\mu^-$



weak scale sparticle masses $A_0 = 0$ and $m_t = 175$ GeV.

parameter	value				
$point$	1	2	3	4a	4b
m_0	100	165	1200	2750	800
$m_{1/2}$	300	550	250	1800	800
$\tan(\beta)$	10	10	10	45	52
$sgn(\mu)$	1	1	1	-1	1
$m_{\tilde{g}}$	701.4	1225.6	658.0	3810.8	1757.6
$m_{\tilde{u}_L}$	630.7	1099.1	1271.1	4185.0	1715.3
$m_{\tilde{u}_R}$	611.1	1060.0	1269.1	4080.7	1662.4
$m_{\tilde{d}_L}$	635.6	1102.0	1273.6	4185.8	1717.1
$m_{\tilde{d}_R}$	610.0	1055.7	1269.6	4067.4	1656.5
$m_{\tilde{b}_1}$	584.9	1020.8	1072.4	3501.7	1484.4
$m_{\tilde{b}_2}$	610.7	1053.4	1260.8	3537.6	1539.4
$m_{\tilde{t}_1}$	471.7	858.5	825.2	3213.2	1328.2
$m_{\tilde{t}_2}$	648.1	1064.0	1084.3	3529.6	1533.6
$m_{\tilde{\nu}_e}$	216.4	396.7	1203.1	2972.3	952.4
$m_{\tilde{e}_L}$	230.4	404.5	1205.7	2973.4	955.8
$m_{\tilde{e}_R}$	155.5	264.8	1201.7	2822.1	851.7
$m_{\tilde{\nu}_\tau}$	215.6	395.4	1198.0	2750.8	834.0
$m_{\tilde{\tau}_1}$	147.5	257.6	1191.0	2320.8	524.2
$m_{\tilde{\tau}_2}$	233.4	405.2	1200.8	2752.6	847.9
$m_{\tilde{Z}_1}$	117.5	225.1	88.6	785.0	336.2
$m_{\tilde{Z}_2}$	215.1	416.9	144.1	1235.1	620.2
$m_{\tilde{Z}_3}$	398.5	668.1	198.2	1249.2	848.7
$m_{\tilde{Z}_4}$	417.8	682.8	260.9	1461.7	862.3
$m_{\widetilde{W}_1}$	214.7	416.9	136.5	1234.5	620.3
$m_{\widetilde{W}_2}$	418.0	682.6	260.3	1461.7	862.5
m_h	114.7	119.0	114.4	123.9	121.5
m_H	443.9	766.5	1204.9	1495.2	810.1
m_A	443.3	765.7	1203.9	1494.2	809.5
m_{H^+}	450.7	770.4	1207.4	1498.4	816.5
μ	392.0	664.9	188.7	-1247.2	845.8
Ωh^2	0.232	0.218	0.262	0.210	0.181
$BF(b \to s\gamma) \times 10^4$	3.12	3.46	3.20	3.92	2.85
$a_\mu^{SUSY} \times 10^{10}$	22.6	7.13	2.65	-1.48	10.2
$BF(B_s \to \mu^+\mu^-) \times 10^7$	0.0399	0.0389	0.0384	0.0306	0.0870

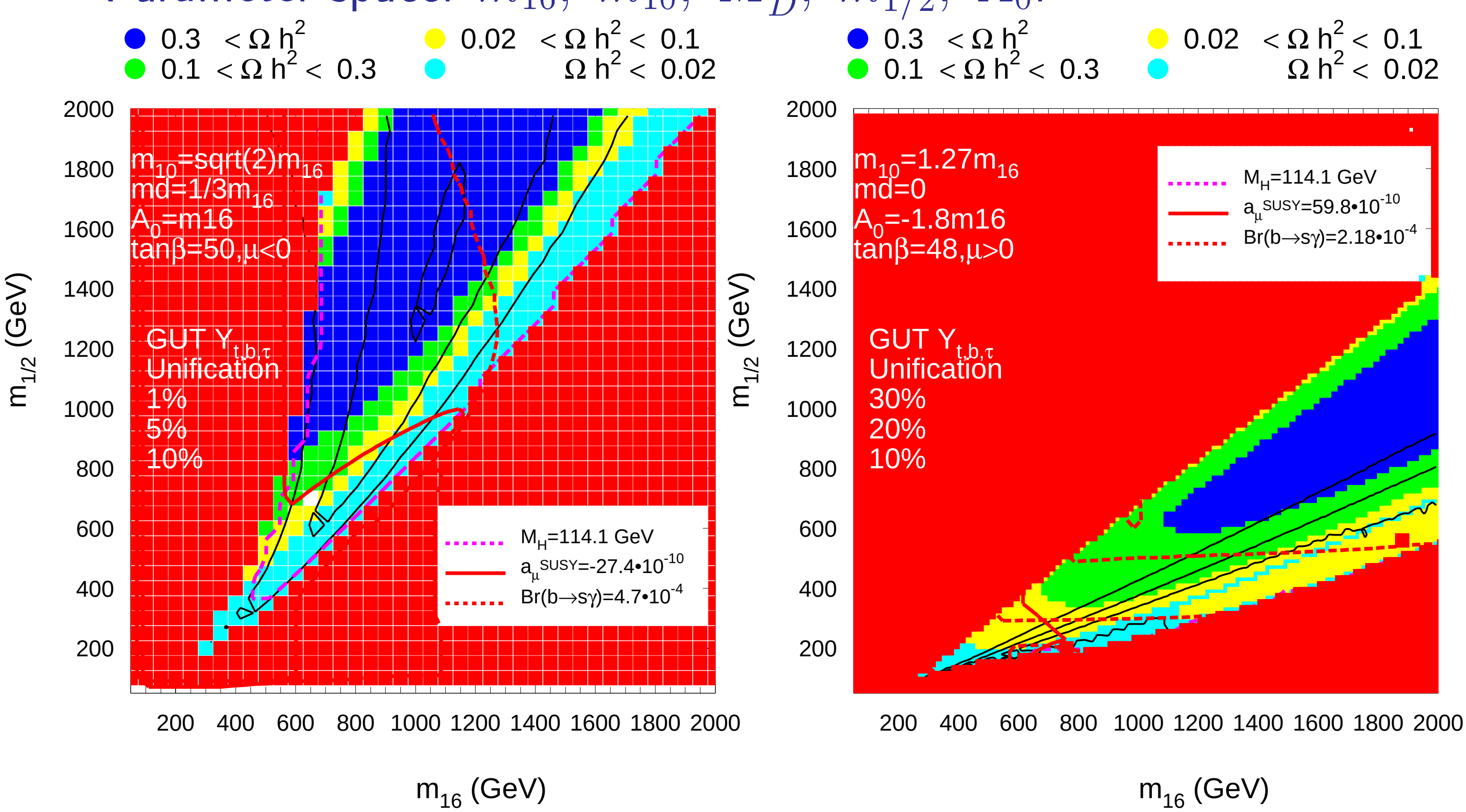
Relic density in SO(10) SUSY GUT model

(Baer et al.)

- $(\psi(\mathbf{16}),\ \phi(\mathbf{10}))$ – unifies matter and interactions
- Yukawa coupling unification
- R-parity is conserved
- REWSB — regained by D-term splitting in scalar masses due to the breakdown of $SO(10)$:

$$
\begin{aligned}
m_Q^2 &= m_E^2 = m_U^2 = m_{16}^2 + M_D^2 \\
m_D^2 &= m_L^2 = m_{16}^2 - 3M_D^2 \\
m_{H_{u,d}}^2 &= m_{10}^2 \mp 2M_D^2
\end{aligned}
$$

- Parameter space: $m_{16},\ m_{10},\ M_D^2,\ m_{1/2},\ A_0$.



SUSY GUTs in Extra dimensions and relic density

- Problems in 4D: mechanism of SUSY breaking, flavor violation, doublet-triplet splitting, proton decay, CP and μ problems

- Solution in XD: orbifold compactification — preserve positive features of 4D GUT and solves its problems (Kawamura; Hall,Nomura; Altarelli,Feruglio; Kobakhidze; Hebecker,March-Russel; Asaka,Buchmuller,Covi; Dermisek,Mafi; Hall,Nomura,Okui,Smith):
  existence of brane(s) at orbifold fixed points on which the gauge symmetries the subgroup of the GUT symmetry

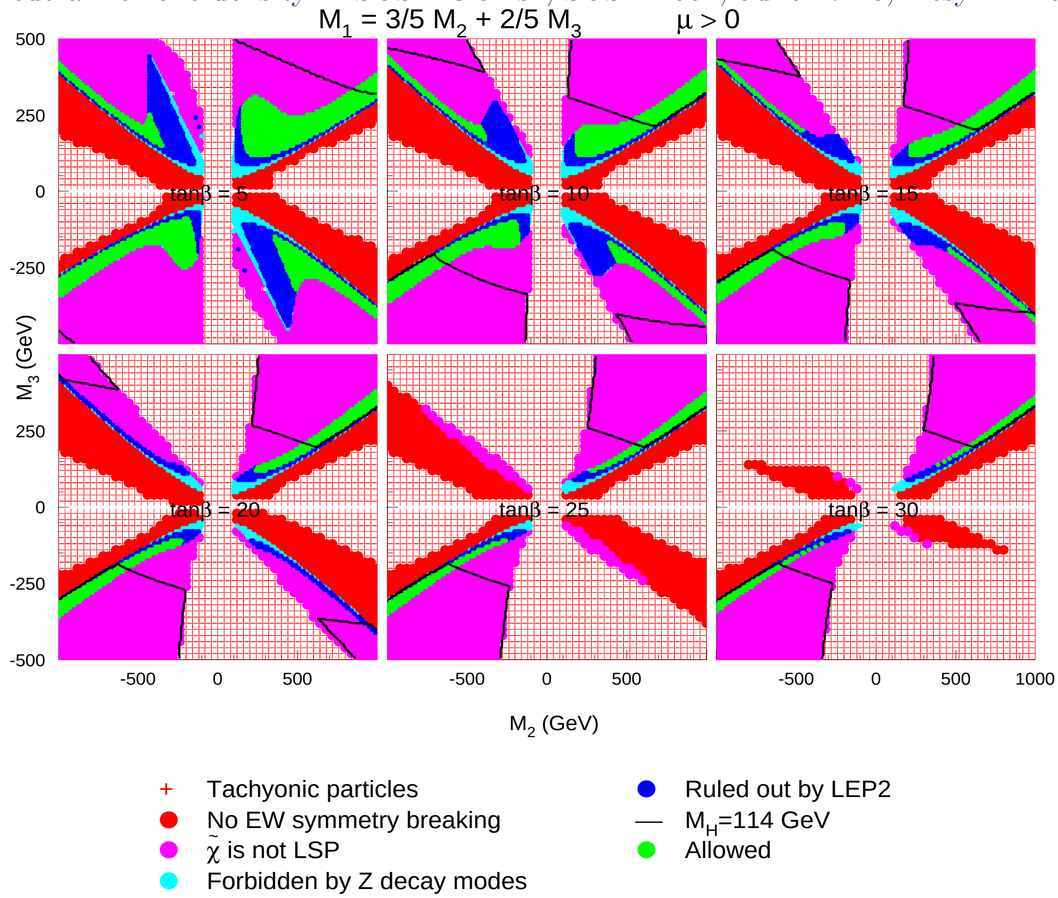
- Gaugino-mediated SUSY breaking
  - universal case $\Rightarrow \tilde{\tau}$ is LSP or small B-term problem
  - in XD scenario: SUSY is broken on a brane with the restricted symmetry – non-universal gaugino masses are generated $\Rightarrow$ solution

  Non-universal gaugino-mediated SUSY breaking

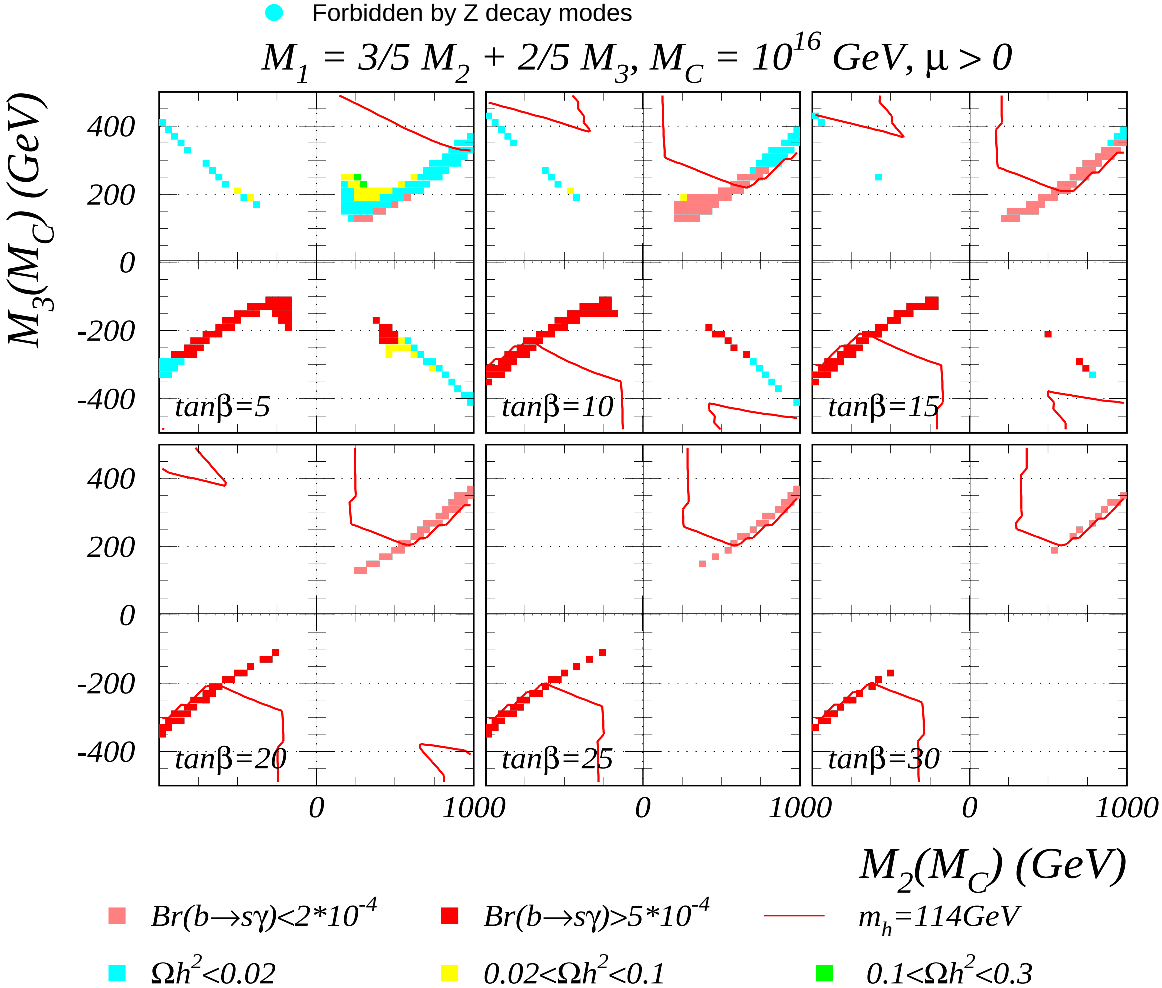
  (Baer,Balazs,A.B.,Dermisek,Mafi,Mustafayev)

- Pati-Salam case: $SU(4) \times SU(2)_L \times SU(2)_R$ symmetry on a brane $\Rightarrow$
  $M_1 = 3/5 M_2 + 2/3 M_3$ at $M_c \lesssim M_{GUT}$ with $m_0 \approx A_0 \approx 0$
  model parameters: M_2, M_3, $\tan\beta$, $sing(\mu)$

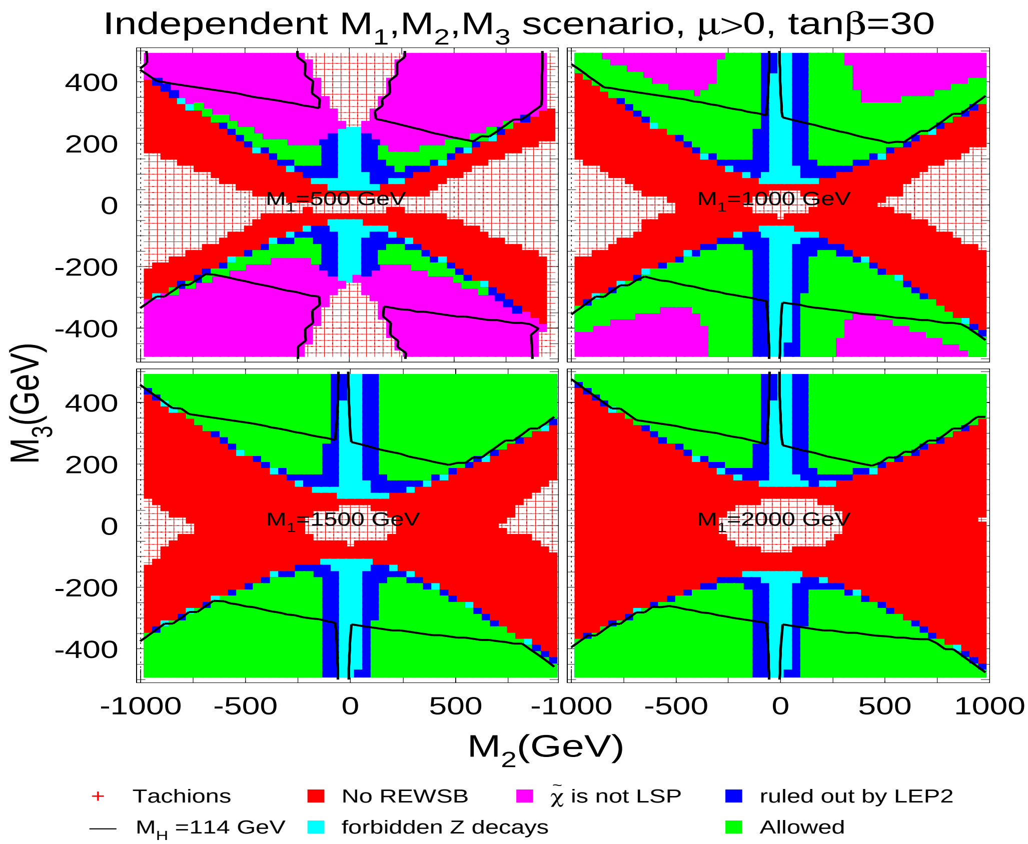
- Independent gaugino masses: $SU(3)_c \times SU(2)_L \times U(1)_Y$ symmetry on a brane $\Rightarrow$
  model parameters: M_1, M_2, M_3, $\tan\beta$, $sing(\mu) \Rightarrow$ large viable regions for all parameters (provided M_1 is large enough)



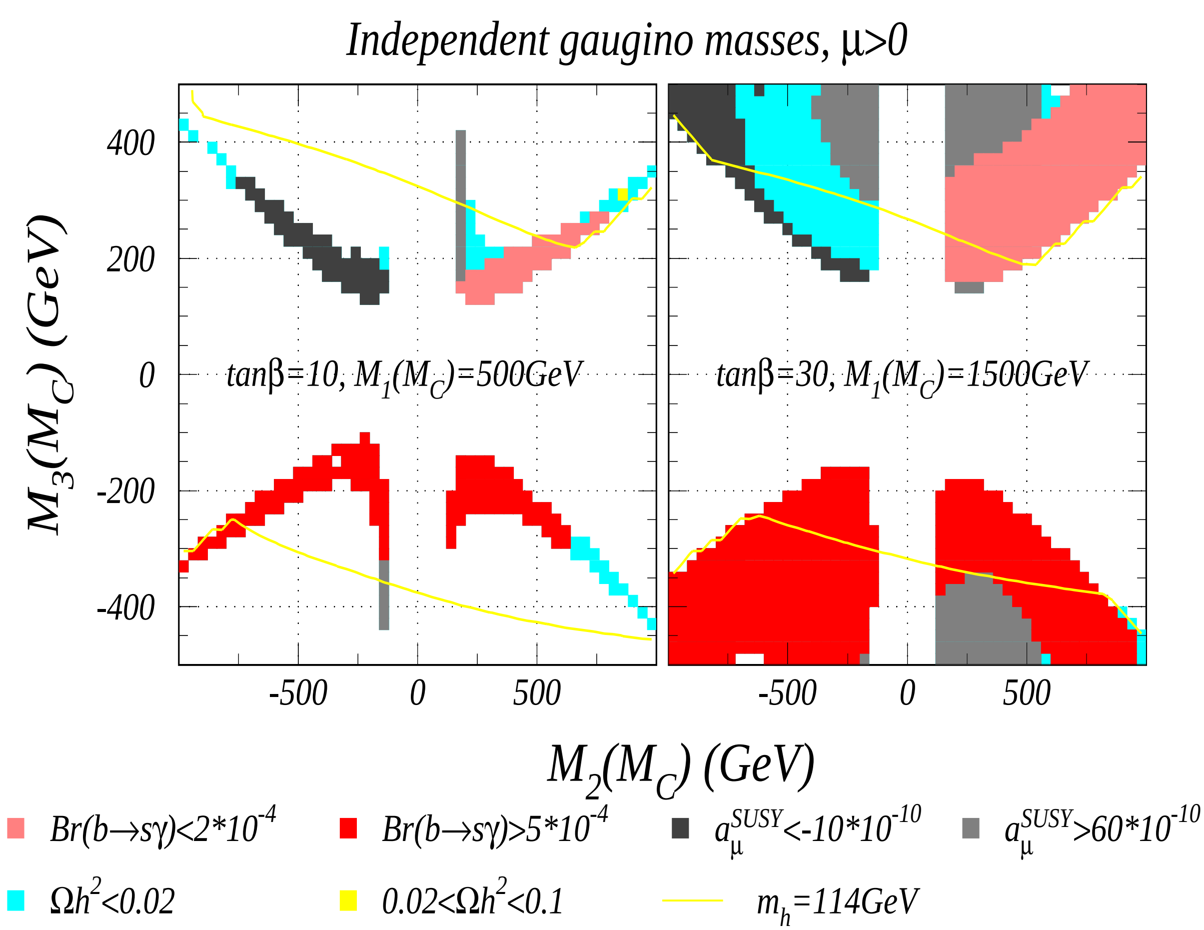
M1 = 3/5 M2 + 2/5 M3 μ > 0
tanβ = 5
tanβ = 10
tanβ = 15
tanβ = 20
tanβ = 25
tanβ = 30
M3 (GeV)
M2 (GeV)
Tachyonic particles
No EW symmetry breaking
χ̃ is not LSP
Forbidden by Z decay modes
Ruled out by LEP2
MH=114 GeV
Allowed



M1 = 3/5 M2 + 2/5 M3, MC = 10^16 GeV, μ > 0
M3(MC) (GeV)
tanβ=5
tanβ=10
tanβ=15
tanβ=20
tanβ=25
tanβ=30
M2(MC) (GeV)
Br(b→sγ)<2*10^-4
Br(b→sγ)>5*10^-4
mh=114GeV
Ωh^2<0.02
0.02<Ωh^2<0.1
0.1<Ωh^2<0.3



Independent M1,M2,M3 scenario, μ>0, tanβ=30
M1=500 GeV
M1=1000 GeV
M1=1500 GeV
M1=2000 GeV
M3(GeV)
M2(GeV)
Tachions
No REWSB
χ̃ is not LSP
ruled out by LEP2
MH =114 GeV
forbidden Z decays
Allowed



Independent gaugino masses, μ>0
tanβ=10, M1(MC)=500GeV
tanβ=30, M1(MC)=1500GeV
M3(MC) (GeV)
M2(MC) (GeV)
Br(b→sγ)<2*10^-4
Br(b→sγ)>5*10^-4
aμSUSY<-10*10^-10
aμSUSY>60*10^-10
Ωh2<0.02
0.02<Ωh2<0.1
mh=114GeV

Summary

- Neutralino relic density plays crucial role in the restricting parameter space of SUSY models
- Importance of including all $2 \to 2$ neutralino annihilation and co-annihilation processes including relativistic thermal averaging
- SUGRA case: there are four regions of parameter space that lead to relic densities satisfying cosmological measurements ($0.1 < \Omega_{\tilde{Z}_1} h^2 < 0.3$)
  - 1. annihilation through t-channel slepton (low m_0 and $m_1/2$)
  - 2. stau co-annihilation region (very low m_0 but large $m_1/2$)
  - 3. focus point region (large m_0 but low to intermediate $m_1/2$)
  - 4. flanks of neutralino s-cannel annihilation via A and H corridor at large $\tan\beta$ when Γ_A and Γ_H are very large
- Co-annihilations are on the edges of the model parameter space, where some amount of fine-tuning is necessary to obtain a reasonable relic density. Alternatively, at high $\tan\beta$, annihilation through very broad Higgs resonances gives rise to an acceptable neutralino density.
- In Yukawa unified $SO(10)$ SUSY GUT models, relic density is reasonable over wide regions of parameter space.
- XD $SO(10)$ SUSY GUT
  - solves problems of 4D $SO(10)$
  - non-universal gaugino scenario leads to viable parameter space
  - Pati-Salam scenario $\Rightarrow$ parameter space is tightly restricted
  - Independent $M1, M_2, M_3$ scenario $\Rightarrow$ viable region of parameter space is large, provided M_1 is sufficiently large
  - relic density is below expectations from cosmological measurements (non-negligible higgsino component to the LSP). One may expect additional CDM states associated,for instance, with the hidden brane.