TOWARDS A BRANEWORLD ALTERNATIVE TO INFLATIONARY COSMOLOGY

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Abstract. I review some aspects of warped braneworld cosmologies which have some hope of leading to alternatives to inflation. Afterwards, recent work on density perturbations in this context is discussed.

1. Introduction

Inflation is a great paradigm for cosmology. It explains or at least unites several well known initial condition problems such as the "why flat?", "why homogeneous?", "why no relic monopoles?", and "why scale invariant perturbations on large scales?". Its essence in solving these problems is the hypothesis that there was a small patch of the universe that was vacuum energy dominated at an early epoch. One can then show that for some models, the negative pressure of the vacuum energy can stretch a small spatial patch by an exponentially large factor to encompass the observable universe today. The spectacular success of CMB measurements in confirming the standard inflationary cosmology lends significant credence to the picture [1].

Nonetheless, to test the robustness of the inflationary cosmological picture, one needs to compare it against alternative hypotheses. This is particularly important in cosmology for which we cannot create an ensemble of universes to actually test the theory in the usual scientific tradition. Some of the older alternatives to inflation include quantum cosmology, pre-big bang [2] (which is a stringy realization of the general idea of time dependent Planck constant), and multiple lightcone theories [3]. Some of the newer alternatives are Ekpyrotic scenario [4] and other related braneworld scenarios, which purports that brane collision is responsible for the big bang and the fluctuations of the brane before the collision can account for the density perturbations.

In this talk, we look at a braneworld alternative to inflation pioneered in [5], whose general idea is to take time slice events in inflationary cosmology and replace the time label by an extra dimensional coordinate label. We will, for the reasons that will become more clear during the talk, refer to these alternatives as the "short cut" scenario. We warn at the outset, that the model building in this direction that will be presented is in its infancy, and therefore cannot yet come close to the great appeal of inflation. In particular, none of the alternatives to inflation, including the one that we will discuss in this talk, can offer much in ameliorating the flatness problem. The main new result that will be presented in the talk is about how density perturbations are projected using Green's functions in asymmetrically warped spacetimes [6].

2. The Horizon problem and the shortcut scenario

The horizon problem can be succinctly summarized as the fact that at the last scattering surface, there are 10^5 causally disconnected patches that we can observe today, if any

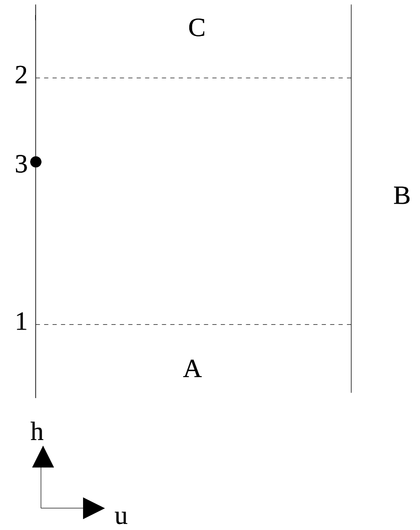

FIGURE 2.1. The dashed segments represent geodesics along the extra dimensional direction $\hat{u}$ and the solid lines represent branes along the $\hat{h}$ direction. A signal can travel from point 1 to point 2 through segments A,B, and C in the bulk. While during the same amount of time, signal on the brane can travel only to point 3.

causal signals cannot travel beyond the spacelike singularity at the initial time of "big bang", thereby defining a horizon. Nonetheless, observations indicate that the mean temperature and its fluctuations are homogeneous in all of the 10^5 causally disconnected patches as if the patches were not really causally disconnected. This is known as the horizon problem.

The braneworld picture assumes that some or all of the standard model fields can be confined to a fewer number of spatial dimensions than the number available for gravity (and possibly some other fields) to propagate in. The simplest realization [5] of this is shown in Fig. 1, where a causal signal (geodesic) can travel from point 1 to point 2 in a shorter time than it would on the brane along the $\hat{h}$ direction. This from a 4D point of view has the effect of broadening the lightcone (i.e. Lorentz violation [7]), and thereby, causally disconnected patches can be connected. Note that this requires that the background spacetime do not possess $SO(1,3)$ isometry in contrast to 4D flat space. The metric, for example, has the form

$$ds^2 = dt^2 - e^{-2bu}d\vec{x}^2 - du^2 \tag{2.1}$$

which obviously violates $SO(1,4)$ isometry, but also $SO(1,3)$ isometry, where the $SO(3)$ subgroup is for the $\vec{x}$ directions. For example, this kind of "short cut" does not work for Randall-Sundrum type of metric [8]. Whether gravity strength interactions can smooth out the inhomogeneities through this kind of shortcut is still an open question.

Although it is not clear whether there exists a stable physical system that gives rise to the stress tensor implied by Eq. (2.1), the presence of $SO(1,3)$ isometry breaking is quite generic. The AdS-Schwarzschild solution has the form

$$ds^2 = -(\Lambda\frac{u^2}{6} + \frac{\mu}{u^2})dt^2 + u^2 d\vec{x}^2 + \frac{du^2}{(\Lambda\frac{u^2}{6} + \frac{\mu}{u^2})} \tag{2.2}$$

where Λ and μ are constants. This background breaks $SO(1,3)$ isometry when $\mu \neq 0$ (leading to apparent Lorentz violations [7, 9]). One can also show that an action of the form

$$S = \int_{M_5} d^5x\sqrt{-g}[\frac{1}{2}R_5 - \frac{3}{2}(\partial\phi)^2 - V_0 e^{\alpha\phi}] + \sum_{i=1}^{2} S_{brane\,i} \tag{2.3}$$

has solutions of the form

$$ds^2 = -(\frac{1-u}{1+u})^{\frac{1-c^2\beta}{1+c^2\beta}}(-dt^2 + \frac{t^2}{(1-u^2)^2}du^2) + (\frac{1-u}{1+u})^{1-\frac{\beta}{\mu}}(\frac{t}{\tau})^{\frac{2(1+\beta)}{\mu}}d\vec{x}^2 \tag{2.4}$$

where c, β, and μ are constants [10]. Clearly, there exist parameter combinations for which the coefficients in front of $d\vec{x}^2$ and dt^2 violate $SO(1,3)$ isometry. As we see in Eq. (2.4), this type of $SO(1,3)$ isometry breaking metrics are more generic than those that do not.

In fact, based on very general embedding of brane worlds into 5D spacetimes, Ishihara [11] has shown that there are no horizons in braneworlds. The most important part of the argument is that in the Friedmann equation on the brane

$$(\frac{1}{a}\frac{da}{d\tau})^2 = \frac{-k}{a^2} + \frac{8\pi}{3}\frac{\kappa^4\sigma}{6}\rho + \frac{\kappa^4}{36}\rho^2 \tag{2.5}$$

where k is the curvature constant, κ is the gravitational constant, σ is the brane tension, and ρ is the energy density on the brane, the ρ^2 term dominates as one approaches the big bang singularity. Since this brane, in particular through a, has a particular embedding in the full 5D metric, Ishihara shows that big bang is a pointlike singularity without a particle horizon for the brane observer. The only problem with this argument is that before ρ^2 can dominate Eq. (2.5), the brane geometry idealization of a delta-function breaks down, and thus the equation itself is no longer valid.

3. Experimental signatures

Braneworld scenarios with the $SO(1,3)$ violating properties of the shortcut metric generically have experimental signatures [7, 12, 13]. Any residual $SO(1,3)$ isometry breaking leads to a 4D effective field theory that has Lorentz violations. It should be stressed that there is no fundamental Lorentz violation in the higher dimensional theory. The effect is only an apparent one resulting from the fact that 4D effective theories of higher dimensional theories are generically nonlocal.

One simple but robust experimental signature [7] is the fact that the photon speed v_γ and the gravitational wave speeds v_g are not equal:

$$\frac{v_\gamma}{v_g} \approx 1 + c\frac{L}{R} \tag{3.1}$$

where c is a numerical constant of order 1, L is the distance between the branes or the characteristic size of the extra dimensions, and R is the radius of curvature of $SO(1,3)$ isometry breaking. This means that if gravitational waves are detected from a supernova or a pulsar which is a distance D away, and if there is no detection of a time difference with resolution Δt, one can put a constraint on L/R as

$$\frac{L}{R} < \frac{\Delta t}{D} \tag{3.2}$$

which can be small as 10^{-19} for an extra-galactic pulsar gravitational wave source with $D = 100$ Mpc if $\Delta t = 10^{-3}$ sec. Note that for extra-galactic sources, the optimistic event rate is around 30/yr. The ν-gravitational wave time arrival correlation measurements can also put a bound on L/R, but this measurement is limited by statistics.

4. Density perturbations

One of the most important "predictions" of the inflationary paradigm is the generation of scale invariant density perturbations from which the collapse of large scale structures and galaxies began. In [6], we investigate the question of what the density perturbations on our brane would look like if there was a density perturbation on another brane. Explicitly, we assume the usual toy model of asymmetrically warped brane

$$ds^2 = -dt^2 + e^{-2bu} a^2(t) d\vec{x}^2 + du^2 \tag{4.1}$$

and the existence of scale invariant causal density perturbations in the bulk (say from a second order phase transition) on the other brane at $u = L$ or $u = 0$.

Note that this question is trivial for $SO(1,3)$ isometric metrics since in that case, one can resort to a low energy $4D$ effective field theory analysis. On the other hand, for the shortcut metrics, the effective field theory has Lorentz violation and is nonlocal. Hence, it is easier to carry out a local, covariant 5D analysis of the shortcut metrics.

The 5D analysis can be carried out using the Green's function formalism.

$$\partial^2 G(x, u; x_0, u_0) = \delta^{(4)}(x - x_0)\delta(u - u_0) \tag{4.2}$$

where the ∂^2 correspond to the d'Alembertian operator in the background of Eq. (4.1). One can defined the apparent projected stress $\underline{S_{\mu\nu}}$ as

$$\bar{h}_{\mu\nu}(x, u = L_1) = \int d^4x_0 du_0 G(x, u = L_1; x_0 u_0) S_{\mu\nu}(u = L_2) \tag{4.3}$$

$$= \int d^4x_0 du_0 G(x, u = L_1; x_0 u_0) \underline{S_{\mu\nu}}(u = L_1) \tag{4.4}$$

where $\bar{h}_{\mu\nu}$ is a metric perturbation of the induced metric at $u = L_1$ and $S_{\mu\nu}$ is the stress tensor having the energy density perturbations on $u = L_2$. Intuitively, $\underline{S_{\mu\nu}}$is the density perturbation seen by the observer on brane at L_1 due to perturbations on brane at $L_2 \equiv L_1 + L$.

Although the final horizon arguments must be made by noting a time dependence of $a(t)$, one can carry out an adiabatic approximation and neglect the time dependence of $a(t)$ to leading approximation. Partially Fourier transforming as

$$G = \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-x_0)} H(p, u, u_0) \tag{4.5}$$

one can obtain a one dimensional differential equation for $H(p, u, u_0)$. Solving with the proper boundary conditions, one can solve for G, and hence $\underline{S_{\mu\nu}}$. The results are shown in table 1, where for comparison, the trivial $SO(1,3)$ isometric counterpart (the Randall-Sundrum metric) case is also shown.

Each factor of $\exp(-bL)$ can be understood from the number of dimensions that get stretched by the warp factor. Note that there is not much qualitative difference in terms amplitude between the two scenarios:

	shortcut ($SO(1,3)$ violating)	AdS(RS-Type) ($SO(1,3)$ isometric)
long wavelength	$\underline{\hat{\rho}_2}(\vec{p}) \approx e^{-3bL}\hat{\rho}_2(\vec{p})$	$\underline{\hat{\rho}_2}(\vec{p}) \approx e^{-4bL}\hat{\rho}_2(\vec{p})$
short wavelength	$\underline{\hat{\rho}_2}(\vec{p}) \approx e^{-2bL}e^{\frac{-p}{b}(e^{bL}-1)}\hat{\rho}_2(\vec{p})$	$\underline{\hat{\rho}_2}(\vec{p}) \approx 2e^{-\frac{5bL}{2}}e^{\frac{-p}{b}(e^{bL}-1)}\hat{\rho}_2(\vec{p}_2)$

TABLE 1. Amplitude of the projected density perturbations in the shortcut metric compared to that in the Randall-Sundrum slice of the AdS metric. $\hat{\rho}_2$ is the density perturbation on brane 2 located at $u = L_2 = L_1 + L$. $\underline{\hat{\rho}_2}$ is the energy density projected (perceived gravitationally) onto brane 1 at $u = L_1$.

	shortcut ($SO(1,3)$ violating)	AdS(RS-Type) ($SO(1,3)$ isometric)
physical momenta	$p_{\text{phys2}} = e^{bL}p_{\text{phys1}}$	$p_{\text{phys2}} = e^{bL}p_{\text{phys1}}$
horizon length	$d_H(u,t) = a(t)\int_0^t \frac{dt'}{a(t')}$	$d_H(u,t) = e^{-b(u-L_1)}a(t)\int_0^t \frac{dt'}{a(t')}$

TABLE 2. Physical wavelengths on each brane for a fixed comoving momentum p is given in row 1. The horizon length scale on each brane ($u = L_1$ or $L_1 + L$) is given in row 2.

- In the long wavelength limit, the observer on brane 1 at $u = L_1$ sees density perturbations from brane 2 at $u = L_2 = L_1 + L$ as an effective density perturbations on brane 1 with an amplitude reduced by an exponential factor.
- In the short wavelength limit, the observer on brane 1 does not see the density perturbation from brane 2.

On the other hand, the causal properties of each scenario is very different. In the first row of Table 2, one sees that physical wavelength on brane 2 for a fixed coordinate momentum p is exponentially shorter than the physical wavelength on brane 1. For there to be an acausal projection of the perturbation, we must have

$$d_H(u = L_1, t)p_{\text{phys1}} < 1 \tag{4.6}$$

while causality on brane 2 forces

$$d_H(u = L_2, t)p_{\text{phys2}} > 1. \tag{4.7}$$

The relationship between the p_{phys1} and p_{phys2} gives in Eq. (4.7), the result

$$a(t)\int_0^t \frac{dt'}{a(t')}p_{\text{phys1}} > 1 \tag{4.8}$$

for the AdS slice metric. Hence, this does not satisfy Eq. (4.6) required for apparent acausal projection. On the other hand, for the shortcut metric, we have Eq. (4.7) implying

$$a(t)\int_0^t \frac{dt'}{a(t')}e^{bL}p_{\text{phys1}} > 1 \tag{4.9}$$

which does satisfy Eq. (4.6) for a range of p_{phys1}. Hence, causal perturbations on brane 2 can look acausal on brane 1.

5. Summary and conclusion

We have demonstrated in [6] that using the warp factor in $SO(1,3)$ isometry violating metrics, one can make large perturbations with short wavelengths on one brane look like small perturbations with long wavelengths on the other brane. Hence, if we can have a causal process generating larger perturbations on one brane, we can use that to generate small perturbations on our brane on acausal scales without resorting to inflation. This is a type of lensing special to braneworld. Of course, both the generation of the perturbations and the protection of generated perturbations from other scales creeping into them on acausal scales are not easy problems to solve.

Finally, we note that even with inflation, this may be a useful way to dilute large perturbations if they are localized on a brane. For example, suppose brane 1 has negligible perturbations while brane 2 has large perturbations $\delta\rho_2$. Then, the total perturbations on brane 1 will look like

$$\frac{\delta\rho_{\text{tot}}}{\rho_{\text{tot}}} = \frac{\delta\underline{\rho_2}}{\rho_1 + \underline{\rho_2}} \ll \frac{\delta\rho_2}{\rho_2} \tag{5.1}$$

if $\underline{\rho_2} \ll \rho_1$where the underlined quantities, as before, refer to quantities projected onto brane 1 (what a brane 1 observer sees). For example, this may save some 4D inflationary models with unacceptably large density perturbations $\delta\rho_2$, if the models are transplanted into the shortcut 5D scenario.

It is encouraging that the shortcut scenarios not only offer some hope of finding an alternative to inflation, but are testable in future experiments [7, 12, 13].

References

[1] M. Tegmark and M. Zaldarriaga, "Current cosmological constraints from a 10 parameter CMB analysis," Astrophys. J. **544**, 30 (2000) [arXiv:astro-ph/0002091].

[2] M. Gasperini and G. Veneziano, "The pre-big bang scenario in string cosmology," arXiv:hep-th/0207130.

[3] M. A. Clayton and J. W. Moffat, "Dynamical Mechanism for Varying Light Velocity as a Solution to Cosmological Problems," Phys. Lett. B **460**, 263 (1999) [arXiv:astro-ph/9812481].

[4] J. Khoury, B. A. Ovrut, P. J. Steinhardt, and N. Turok, "The ekpyrotic universe: Colliding branes and the origin of the hot big bang," Phys. Rev. D **64**, 123522 (2001) [arXiv:hep-th/0103239].

[5] D. J. Chung and K. Freese, "Can geodesics in extra dimensions solve the cosmological horizon problem?," Phys. Rev. D **62**, 063513 (2000) [arXiv:hep-ph/9910235].

[6] D. J. Chung and K. Freese, "Lensed density perturbations in braneworlds: An alternative to perturbations from inflation," arXiv:astro-ph/0202066.

[7] D. J. Chung, E. W. Kolb, and A. Riotto, "Extra dimensions present a new flatness problem," Phys. Rev. D **65**, 083516 (2002) [arXiv:hep-ph/0008126].

[8] L. Randall and R. Sundrum, "A large mass hierarchy from a small extra dimension," Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].

[9] C. Csaki, J. Erlich, and C. Grojean, "Gravitational Lorentz violations and adjustment of the cosmological constant in asymmetrically warped spacetimes," Nucl. Phys. B **604**, 312 (2001) [arXiv:hep-th/0012143].

[10] D.J.H. Chung,J. Khoury,R. Zhang, (2002) work in progress.

[11] H. Ishihara, "Big bang of the brane universe," Phys. Rev. D **66**, 023513 (2002) [arXiv:gr-qc/0107085].

[12] G. D. Moore and A. E. Nelson, "Lower bound on the propagation speed of gravity from gravitational Cherenkov radiation," JHEP **0109**, 023 (2001) [arXiv:hep-ph/0106220].

[13] C. P. Burgess, J. Cline, E. Filotas, J. Matias, and G. D. Moore, "Loop-generated bounds on changes to the graviton dispersion relation," JHEP **0203**, 043 (2002) [arXiv:hep-ph/0201082].