CMB constraints on seesaw parameters via leptogenesis[a]

Pasquale DI BARI
IFAE, Universitat Autònoma de Barcelona, 08193 Bellaterra (Barcelona), Spain
e-mail: `dibari@ifae.es`

The possibility to explain the CMB measurement of the baryon asymmetry with leptogenesis results in stringent model independent constraints on some of the see-saw parameters. These include the quadratic mean of the light neutrino masses that has to be lower than $\sim 0.17\,\mathrm{eV}$. Furthermore, for maximal CP asymmetry, the observed value of the baryon asymmetry determines a stringent relation for the temperature of leptogenesis that has to be in the range $(2-6)\times 10^{11}$ GeV for $\eta_{B0}^{CMB}=6\times 10^{-10}$.

1 The baryon asymmetry of the Universe

The observation of the acoustic peaks in the power spectrum of CMB temperature anisotropies confirms that we live in a baryon asymmetric Universe, an important result already inferred from the study of cosmic rays and primordial nuclear abundances. Within standard BBN (SBBN) any measurement of a primordial nuclear abundance leads to a measurement of the baryon asymmetry, conveniently expressed in the form of baryon to photon number ratio. From the measurement of the Deuterium primordial abundance in Quasar absorption systems one finds at 1σ [4]:

$$\eta_{B0}^{SBBN}\big|_{D/H} = (5.6\pm 0.5)\times 10^{-10} \tag{1}$$

A multiple measurement of different primordial abundances represents a test of consistency for SBBN and in principle should lead to a more accurate determination of η_{B0}. However the results from the Helium and Litium primordial abundances are only marginally consistent with the Deuterium abundance and thus it is necessary to account for larger systematic uncertainties and to make some assumptions on their statistical distribution. Thus an acceptable agreement among the abundances leads to a less precise determination of the baryon asymmetry [5]

$$\eta_{B0}^{SBBN} = (2.6-6.2)\times 10^{-10}, \tag{2}$$

valid approximately at the 90% c.l. [6]. The difficulty of SBBN in explaining simultaneously all the current measurements of primordial abundances can also be interpreted as a hint

[a] Based on the reference papers [1], [2], [3].

for the presence of non standard BBN effects. Some of them are well motivated within those models beyond the Standard Model that can incorporate the see-saw and lead to leptogenesis. In any case a determination of the baryon asymmetry from SBBN, at a higher level of accuracy than the range (2), encounters serious obstacles at the present.

Fortunately the recent observation of acoustic peaks in the power spectrum of CMB temperature anisotropies provides a powerful tool to measure the baryon asymmetry and to circumvent the difficulties of SBBN. In this case one has a good consistency of different determinations of the baryon asymmetry from 5 different experiments employing different techniques [7,8]. A recent combined analysis gives [8]

$$\eta_{B0}^{CMB} = (6.0^{+0.8}_{-1.1}) \times 10^{-10}. \quad (3)$$

This determination is in reasonable agreement with that one from the SBBN and at the same level of accuracy. However, in contrast with the SBBN determination, the consistency of the different experimental results so far makes it quite robust and makes possible to expect a reduction of the error in a close future: below the 10% level from the MAP satellite during next years and at the 1% level from the Planck satellite before the end of this decade. We will therefore use the CMB determination of the baryon asymmetry in our following considerations.

2 Basics of leptogenesis

Leptogenesis [9] is the cosmological consequence of the see-saw mechanism. This explains the lightness of neutrino masses by the existence of three RH neutrinos, N_i, much heavier than the electroweak scale. The decay of the heavy neutrinos violates lepton number and, in general, also CP conservation, while the cosmological expansion can yield the necessary departure from thermal equilibrium: all three Sacharov's conditions are satisfied and a lepton number can be generated in the early Universe. The possibility for leptogenesis to explain the observed baryon asymmetry relies crucially on the existence of the non perturbative SM sphaleron processes, that can convert, at temperatures above the electroweak phase transition, about $-1/3$ of the lepton number into a baryon number, while keeping B-L constant. The source of CP violation is naturally provided by the complexity of the neutrino mass matrices in the see-saw. For each of the three N_i one can introduce a CP asymmetry parameter defined as:

$$\varepsilon_i \equiv \frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i}, \quad (4)$$

where Γ_i and $\bar{\Gamma}_i$ are the decay rates of N_i respectively into leptons ($N_i \rightarrow l + \bar{\phi}$) and anti-leptons ($N_i \rightarrow \bar{l} + \phi$).

The problem is greatly simplified if one assumes that only the decays of the lightest RH neutrinos, N_1, can influence the final baryon asymmetry. This is true if the asymmetries generated by the two heavier neutrino decays (with masses M_2 and M_3), even though not negligible, are subsequently washed out by the processes (for example inverse decays) in which the lightest right-handed neutrinos (with mass M_1) are involved, at temperatures $T \sim M_1$. This assumption implies the existence of a mild hierarchy of masses such that $M_{2,3} \gtrsim (2-3)\, M_1$ and also that the wash out N_1-processes are strong enough.

In this way one has to solve a system of only two Boltzmann equations, one for the number of N_1's and one for the $B-L$ asymmetry. Introducing the convenient variable $z \equiv M_1/T$, they can be written in the following simple form [10,11,1]:

$$\frac{dN_{N_1}}{dz} = -(D+S)\,(N_{N_1} - N_{N_1}^{\rm eq})\,, \tag{5}$$

$$\frac{dN_{B-L}}{dz} = -\varepsilon_1\, D\,(N_{N_1} - N_{N_1}^{\rm eq}) - W\, N_{B-L}\,. \tag{6}$$

There are four classes of processes that contribute to the different terms in the equations: decays, inverse decays, $\Delta L = 1$ scatterings and RH neutrino mediated processes. The first three contribute all together to modify the N_1 abundance. Indicating with H the expansion rate, the term $D \equiv \Gamma_D/(H\,z)$ accounts for the decays and inverse decays while the term $S \equiv \Gamma_S/(H\,z)$ accounts for the $\Delta L = 1$ scatterings. The decays are also the source term for the generation of the $B-L$ asymmetry, the first term in the second equation, while all the other processes contribute to the wash out term $W \equiv \Gamma_W/(H\,z)$ that competes with the decay source term.

3 A model independent parameterization

It is simple to see that the dependence on ε_1 is linear in a way that the final baryon asymmetry is given by:

$$N_{B-L}^{\rm fin} = N_{B-L}^{\rm in} - \frac{3}{4}\varepsilon_1\,\kappa_0 \tag{7}$$

Assuming that the wash out processes are strong enough to erase an initial value of N_{B-L}, generated for example by the decays of the two heavier RH neutrinos or by some other unspecified mechanism, we will put $N_{B-L}^{\rm in} = 0$. This assumption is valid under the same conditions for which heavier neutrino decays can be neglected and therefore it does not introduce further restrictions.

The *efficiency factor* κ_0 does not depend on ε_1. It is normalized in a way to be 1 in the limit case that an initial thermal abundance of N_1's decays fully out of equilibrium at the time when all wash out processes are completely frozen. In this limit the wash out term in the kinetic equations is uneffective and can be neglected. Let us introduce the quantity:

$$\bar{m} = \sqrt{m_1^2 + m_2^2 + m_3^2} \tag{8}$$

The *quadratic mean of the light neutrino masses* is simply related to it by $\bar{m}/\sqrt{3}$. A remarkable fact is that for masses $M_1 \ll 10^{14}\,{\rm GeV}\,(0.1\,{\rm eV})/\bar{m})^2$ the three terms D, S, W are proportional to an *effective neutrino mass* $\widetilde{m}_1$ times a function only of z [11]. This means that the final baryon asymmetry will depend only on two parameters: ε_1 and $\widetilde{m}_1$. In this case the *out of equilibrium limit* is obtained for $\widetilde{m}_1 \to 0$. In figure 1 we show the function κ_0 as a function of $\widetilde{m}_1$, for different values of M_1. It can be seen how for small values of M_1 there is no dependence on M_1 itself. We performed the calculations both for an initial thermal abundance (thin lines) and for a zero initial abundance (thick lines). It is evident how there is a critical value of $\widetilde{m}_1$ that separates two different regimes. For $\widetilde{m}_1 \ll 5\times 10^{-4}\,{\rm eV}$ one recovers the limit of out of equilibrium decays and κ_0 is strongly

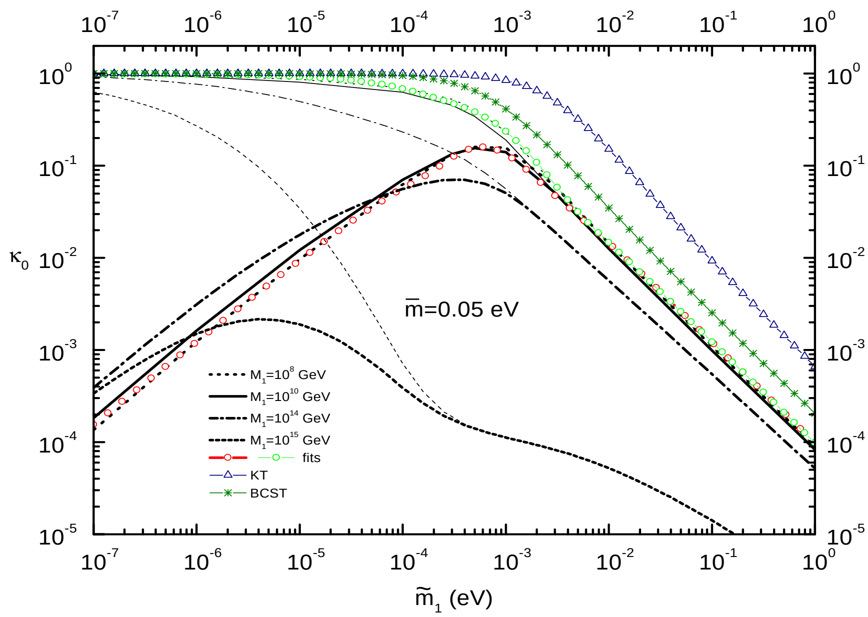

Figure 1: The efficiency factor

dependent on the number of initial N_1's. In the case of zero initial neutrinos, κ_0 is determined by the number of N_1's that are produced by inverse decays and scatterings and this number goes to zero in the limit $\widetilde{m}_1 \to 0$. Therefore in this regime there is a strong dependence on the initial conditions. For $\widetilde{m}_1 \gg 5\times 10^{-4}$ eV there is no dependence on the initial conditions and, even for a zero initial number of N_1's, they are rapidly produced and their number rapidly approaches the thermal value. This means that, in the limit of large values of $\widetilde{m}_1$, the dependence of κ_0 on the N_1 production processes disappears and only a dependence on the wash out processes is left.

In the intermediate regime the value of κ_0 is determined by an interplay between the wash out processes and the number of decaying $N_1's$, determined both by the initial number and by the strength of production processes. It is possible to give a numerical fit of κ_0:

$$\kappa_0 = f_-(x_-)\, e^{-x_-} + f_+(x_+)\, e^{-x_+}\ , \tag{9}$$

with $x_\pm = (\widetilde{m}_1/\widetilde{m}_\pm)^{\alpha_\pm}$. The first term depends on the initial N_1 abundance. For an initial zero abundance:

$$f_-(x_-) = f_+(x_-) = 0.24\, x_-\, e^{-x_-}, \quad \widetilde{m}_- = 3.5\times 10^{-4}\,\text{eV}; \quad \alpha_- = 0.9\,. \tag{10}$$

For an initial thermal abundance:

$$f_-(x_-) = e^{-x_-}, \quad \widetilde{m}_- = 4.0\times 10^{-4}\,\text{eV}, \quad \alpha_- = 0.7\,. \tag{11}$$

The second term is independent on the initial conditions:

$$f_+(x_+) = 0.24\, x_+, \quad \widetilde{m}_+ = 8.3\times 10^{-4}\,\text{eV}, \quad \alpha_+ = -1.1. \tag{12}$$

In the limit of weak coupling ($\widetilde{m}_1 \ll 5 \times 10^{-4}\,\mathrm{eV}$) one has $\kappa_0 \simeq f_-(x_-)$, while in the limit of strong coupling ($\widetilde{m}_1 \gg 5 \times 10^{-4}\,\mathrm{eV}$) one has:

$$\kappa_0 \simeq f_+(x_+) \simeq 10^{-4} \left(\frac{\mathrm{eV}}{\widetilde{m}_1}\right)^{1.1} . \tag{13}$$

The two fits are optimal for $M_1 = 10^8\,\mathrm{GeV}$ and are represented in figure 1 with circled lines. It is interesting to compare these results with an analytical approximation for κ_0 originally derived in the context of GUT baryogenesis [12] but also adapted to the case of leptogenesis (see for example [13]):

$$\kappa_0 = 1 \quad \text{for} \quad K \ll 1 \tag{14}$$

$$\kappa_0 = \frac{0.3}{K\,(\ln K)^{0.6}} \quad \text{for} \quad K \gg 1 \tag{15}$$

As a simple interpolating expression we can use:

$$\kappa_0 = \frac{0.3}{K\,[\ln(1+K)]^{0.6}} \left(1 + \frac{0.3}{K\,[\ln(1+K)]^{0.6}}\right)^{-1} \tag{16}$$

The quantity K is related to $\widetilde{m}_1$ simply by:

$$K = \frac{1}{2}\, D|_{z=1} \simeq 170\, \frac{\widetilde{m}_1}{\mathrm{eV}} \tag{17}$$

The expression (16) is represented in the figure with the triangle line. One can see that it overestimates the efficiency factor by ~ 7. This is not surprising because this analytical approximation takes into account only the inverse decays in the wash out term and it neglects the other processes that are equivalently important. A more specific analytical approach was described in [14] and the result for $\kappa_0(\widetilde{m}_1)$ is represented in the figure with the starry line [b]. One can see that it better agrees with the numerical results but it still overestimates them by a factor $2-3$.

For large values of M_1 the efficiency factor depends also on M_1 itself and actually for $M_1 \gtrsim 10^{14}\,\mathrm{GeV}\,(0.1\,\mathrm{eV})/\bar{m})^2$ there is a suppression in the regime for large $\widetilde{m}_1$. The suppression is due to a term $\Delta W \propto M_1\,\bar{m}^2/z^2$, originating from the RH neutrinos mediated processes, that, for large M_1, dominates in the total wash-out term W [1]. This term suppresses exponentially the baryon asymmetry yielding a term $\exp\left(-\mathrm{const}\, M_1\,\bar{m}^2/\bar{z}\right)$ in the efficiency factor, where $\bar{z}$ is that value of z, larger than 1, at which ΔW starts to dominate and it can depend only on M_1, $\bar{m}$ and $\widetilde{m}_1$. Thus , in the most general case, the final baryon asymmetry can be described in terms of only four parameters: ε_1, $\widetilde{m}_1$, M_1 and $\bar{m}$ [c].

[b] We deduced it from the figure 1 in [14] interpolating the points for $M_1 = 10^8\,\mathrm{GeV}$. Strangely the result does not correspond to the analytical expression that is given in the text and numbered as Eq. (4.3).

[c] Actually, considering that the dependence on M_1 and on $\bar{m}$ occurs only through the product $M_1\,\bar{m}^2$, one could use just three parameters. However, from a physical point of view, it is better to distinguish the dependence on M_1 and $\bar{m}$.

4 The surface of maximum baryon asymmetry

In order to obtain a prediction for η_{B0}, to be compared with the measured value η_{B0}^{CMB} in the Eq.(3), one has to multiply $N_{B-L}^{\rm fin}$ for the fraction of the $B-L$ asymmetry that is converted into baryons by the sphaleron processes, given by a factor $28/79 \simeq 1/3$, and divide for the *dilution factor* $f = N_\gamma^\star/N_\gamma^0$. This takes into account that the generated baryon asymmetry gets diluted compared to the number of photons that are produced in the annihilations of all standard model particle species. If one assumes a standard thermal history of the early Universe then $f \simeq 28$ and in the end one gets the simple relation

$$\eta_{B0} \simeq -10^{-2}\,\varepsilon_1\,\kappa_0 \tag{18}$$

A first trivial model independent bound on η_{B0} is obtained considering that $|\varepsilon_1| \leq 1$ and thus $\eta_{B0} \lesssim 10^{-2}\,\kappa_0$. It is however possible to show a more stringent bound on $|\varepsilon_1|$ [15,14,16,1,2]:

$$|\varepsilon_1| \leq \frac{3}{16\pi}\,\frac{M_1}{v^2}\,\frac{m_3^2-m_1^2}{m_3} = 10^{-6}\left(\frac{M_1}{10^{10}\,{\rm GeV}}\right)\beta \tag{19}$$

From neutrino mixing experiments $m_3^2 - m_1^2 = \Delta m^2_{\rm atm} + \Delta m^2_{\rm sol}$ and one can write:

$$\beta \simeq \frac{\Delta m^2_{\rm atm} + \Delta m^2_{\rm sol}}{0.051\,{\rm eV}\,m_3} \tag{20}$$

For example in the case of fully hierarchical neutrinos, for $m_1 = 0$, one has $m_3 = \sqrt{\Delta m^2_{\rm atm} + \Delta m^2_{\rm sol}}$ and $\beta \simeq 1$. While in the case of quasi degenerate neutrinos with $\bar{m} \simeq 1\,{\rm eV}$, one has $m_3 \simeq 0.58\,{\rm eV}$ and $\beta \simeq 0.1$. From the CP bound one can see that, given the atmospheric neutrino mass scale, the mass of the lightest RH neutrino cannot be higher than $10^{16}\,{\rm GeV}$ otherwise $|\varepsilon_1|$ would be, absurdly, higher than 1. This has to be also consistently derived within the see-saw formula. It is certainly true in the oversimplified case of one generation see-saw formula: if $m \gtrsim 0.05\,{\rm eV}$ then $M \lesssim 10^{15}\,{\rm GeV}$. For three generations the result is analogous and again consistent with the CP bound (see for example [17]) as it has to be. Thus in the end one can express the maximum baryon asymmetry just in terms of the three parameters $\widetilde{m}_1$, M_1 and $\bar{m}$:

$$\eta_{B0}^{\rm max} \simeq 10^{-8}\,\beta(\bar{m})\left(\frac{M_1}{10^{10}\,{\rm GeV}}\right)\kappa_0(\widetilde{m}_1, M_1, \bar{m}) \tag{21}$$

This is the surface of maximum baryon asymmetry and the CMB constraint is given by the requirement that $\eta_{B0}^{\rm max} \geq \eta_{B0}^{CMB}$. Since κ_0 and $\beta \leq 1$, one immediately derives a bound on M_1 and thus on the leptogenesis temperature [1]:

$$T_L \simeq M_1 \geq 10^8\,{\rm GeV}\,\frac{\eta_B^{CMB}}{10^{-10}} \gtrsim 4\times 10^8\,{\rm GeV}, \tag{22}$$

at $\sim 2\,\sigma$ from the Eq. (3). In the case of a zero initial abundance one can see from the figure that $\kappa_0 \leq 0.16$ and thus a more stringent constraint follows:

$$T_L \simeq M_1 \geq 6.25\times 10^8\,{\rm GeV}\,\frac{\eta_B^{CMB}}{10^{-10}} \gtrsim 2.5\times 10^9\,{\rm GeV}. \tag{23}$$

These bounds are saturated in the case of a normal hierarchy such that $m_1 = 0$, $\bar{m} \simeq \sqrt{\Delta m^2_{\rm atm}}$ and $\beta \simeq 1$. Larger values of $\bar{m}$ give $\beta > 1$ and the constraints become more restrictive. There is also an upper bound on M_1 deriving from the exponential suppression of κ_0 at large M_1. Also this constraint becomes more restrictive when the absolute neutrino mass scale $\bar{m}$ increases. More generally, the allowed region in the plane $(\widetilde{m}_1, M_1)$, determined by the constraint $\eta_{B0}^{\rm max} \geq \eta_{B0}^{CMB}$, shrinks when $\bar{m}$ increases. Moreover one can show [18] that $\widetilde{m}_1 > m_1$ and this, for $m_1 > 0$, introduces a second constraint. Increasing $\bar{m}$ there will be a value for which the two constraints together cannot be simultaneously satisfied. This represents an upper limit on $\bar{m}$ given by $\bar{m} \leq 0.30\,{\rm eV}$ [2]. In terms of light neutrino masses it corresponds to have $m_3 \lesssim 0.18\,{\rm eV}$ and $m_1 \simeq m_2 \lesssim 0.17\,{\rm eV}$ meaning that *leptogenesis is incompatible with quasi-degenerate light neutrinos.*

For maximal CP asymmetry it is possible to show that $\widetilde{m}_1 = m_3$ [3]. In this case, using the Eq.(21) and imposing $\eta_B^{\rm max} = \eta_B^{CMB}$, it is possible to derive an expression for $T_L \simeq M_1$. For small M_1 one can use the Eq.(13) for the efficiency factor and in the end one arrives to the following stringent relation between the leptogenesis temperature and the absolute neutrino mass scale [3]:

$$T_L \simeq M_1 \simeq 2\,\times 10^{11}\left(\frac{\eta_B^{CMB}}{6\times 10^{-10}}\right)\left(\frac{m_3}{0.05\,{\rm eV}}\right)\,{\rm GeV} \lesssim 6\times 10^{11}\left(\frac{\eta_B^{CMB}}{6\times 10^{-10}}\right)\,{\rm GeV}\,. \quad (24)$$

5 Conclusions

It is remarkable that the leptogenesis predictions of the final baryon asymmetry can be expressed in terms of just 4 parameters, in a model independent way. This result relies on two main assumptions: the existence of a mild hierarchy in the masses of the RH neutrinos ($M_{2,3} \gtrsim (2-3)\,M_1$) and that the initial temperature can be assumed to be larger than M_1. With this parameterization one can easily describe the requirements for a successful leptogenesis and the most striking result is that the light neutrino masses cannot be too larger than the atmospheric neutrino mass scale $\sim 0.05\,{\rm eV}$, thus ruling out the class of quasi degenerate neutrino models. This implies a strong, though negative, prediction on the possibility of future experiments to detect a sub-eV neutrino mass scale, unless their sensitivity can be pushed below $\mathcal{O}(0.1\,{\rm eV})$. It is also remarkable that a quite precise temperature for leptogenesis seems to emerge to explain the observed baryon asymmetry. In conclusion leptogenesis seems to be a testable model to explain the observed baryon asymmetry, closely related to the absolute neutrino mass scale.

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