MSSM with Yukawa Quasi-Unification

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ABSTRACT

We consider the constrained minimal supersymmetric standard model which emerges from one theory with a small deviation from Yukawa unification which is adequate for $\mu > 0$. We show that this model possesses a wide and natural range of parameters which is consistent with the data on $b \to s\gamma$, the muon anomalous magnetic moment, the cold dark matter abundance in the universe, and the Higgs boson masses.

1 Introduction

We study the phenomenological consequences of imposing on the constrained MSSM (CMSSM) an asymptotic relation for the Yukawa couplings at the GUT scale. This assumption (Yukawa unification) naturally restricts [1] the t-quark mass to large values compatible with the data. Also, the emerging model is highly predictive [2]. Despite of its appealing, the simple scheme of a single Yukawa for the three third generations at the GUT scale leads to an unacceptable b–quark mass. This fact excludes minimal versions of GUT groups with this property, such as Pati–Salam unification ($G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$), $SO(10)$ or E_6.

We consider the SUSY GUT model described in Ref.[3] which is based in the G_{PS} as described in Refs.[4] and establish an 'asymptotic' relation for the Yukawa couplings that depends on a single complex parameter c:

$$h_t : h_b : h_\tau = |1 + c| : |1 - c| : |1 + 3c|, \qquad (1)$$

For simplicity, we will restrict our analysis to real values of $0 < c < 1$. The relative splitting of the Yukawa couplings becomes: $\delta h \equiv -(h_b - h_t)/h_t = (h_\tau - h_t)/h_t = 2c/(1+c)$. This means that the bottom and tau Yukawa couplings split from the top Yukawa coupling by the same amount but in opposite directions, with h_b becoming smaller than h_t.

2 The MSSM with Quasi-Yukawa Unification

This model, below M_{GUT}, reduces to the MSSM supplemented by the 'asymptotic' Yukawa coupling quasi-unification condition in Eq.(1). We will assume universal soft SUSY breaking terms at M_{GUT}, i.e., a common mass for all scalar fields m_0, a common gaugino mass $M_{1/2}$ and a common trilinear scalar coupling A_0. In the present work, we will concentrate on the $\mu > 0$. The case $\mu < 0$ is phenomenologically less interesting, it will be presented in [5]. We follow the notation as well as the RG and radiative electroweak breaking analysis of Ref.[6] for the CMSSM with the improvements of Refs.[7, 3] (recall that the sign of μ in Refs.[6, 7] is opposite to Ref.[3], which is the one adopted here).

For any given $m_b(M_Z)$ in its 95% c.l. range ($2.684 - 3.092$ GeV for $\alpha_s(M_Z) = 0.1185$), we can determine the parameters c and $\tan\beta$ at $M_{SUSY} = (m_{\tilde{t}_1} m_{\tilde{t}_2})^{1/2}$ ($\tilde{t}_{1,2}$ are the stop mass eigenstates) so that the 'asymptotic' condition in Eq.(1) is satisfied. We use fixed values for the running top quark mass $m_t(m_t) = 166$ GeV and the running tau lepton mass $m_\tau(M_Z) = 1.746$ GeV and incorporate not only the SUSY correction to the bottom quark mass but also the SUSY threshold correction to $m_\tau(M_{SUSY})$ from the approximate formula of Ref.[8]. After imposing the conditions of gauge coupling unification, successful electroweak breaking and Yukawa quasi-unification in Eq.(1), we are left with three free input parameters m_0, $M_{1/2}$ and A_0. In order to make the notation physically more transparent, we replace m_0 and $M_{1/2}$ equivalently by the mass m_{LSP} (or $m_{\tilde{\chi}}$) of the lightest supersymmetric particle (LSP), which turns out to be the lightest neutralino ($\tilde{\chi}$), and the relative mass splitting $\Delta_{\tilde{\tau}_2} = (m_{\tilde{\tau}_2} - m_{\tilde{\chi}})/m_{\tilde{\chi}}$ between the lightest stau mass eigenstate ($\tilde{\tau}_2$) and the LSP. In Fig.1 we display the changes on M_{SUSY} and the mass of the pseudo-scalar Higgs, m_A, for several values of $\Delta_{\tilde{\tau}_2}$, and $m_b(M_Z)$. These changes will help us to understand the corresponding predictions for $\Omega_{LSP} h^2$ in the presence of resonant annihilation channels for values of $m_A \approx 2 \cdot m_{LSP}$.

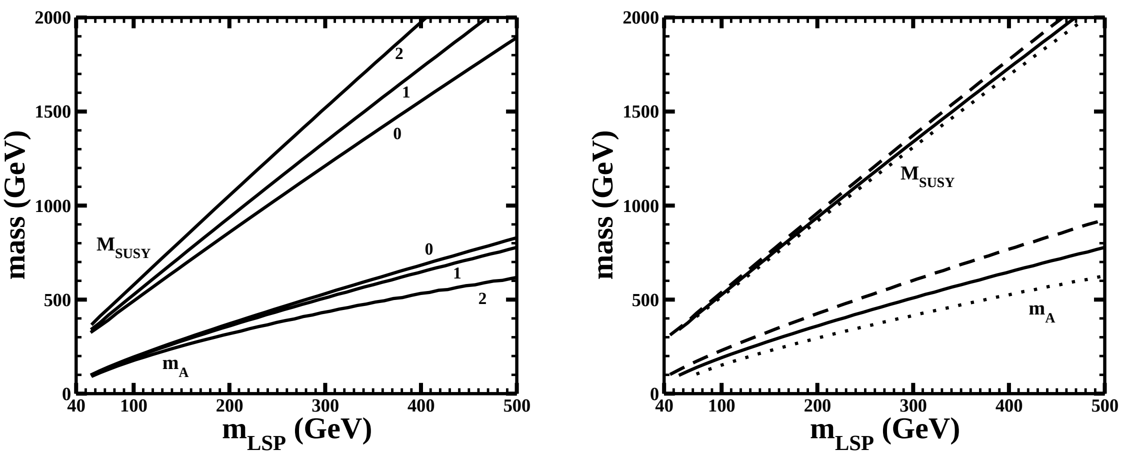

Figure 1: *The mass parameters m_A and M_{SUSY} as functions of m_{LSP}, with $\alpha_s(M_Z) = 0.1185$ and $A_0 = 0$. On the panel of the left we display various values of $\Delta_{\tilde{\tau}_2}$ (indicated on the curves) for $m_b(M_Z) =$* 2.888 GeV. *On the right panel we fix $\Delta_{\tilde{\tau}_2} = 1$ and show the curves for $m_b(M_Z) =$* 2.684 GeV *(dashed lines),* 3.092 GeV *(dotted lines) or* 2.888 GeV *(solid lines).*

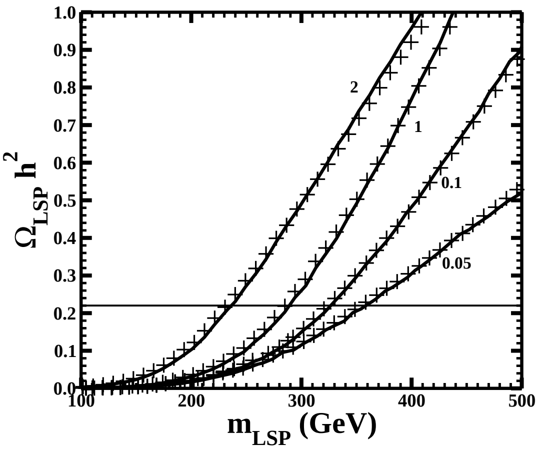

Figure 2: *The LSP relic abundance $\Omega_{LSP}\ h^2$ versus m_{LSP} for various $\Delta_{\tilde{\tau}_2}$'s (indicated on the curves) and with $A_0 = 0$, $m_b(M_Z) = 2.888$ GeV, $\alpha_s(M_Z) = 0.1185$. The solid lines (crosses) are obtained by* `micrOMEGAs` *(our alternative method). The upper bound on $\Omega_{LSP}\ h^2$ (=0.22) is also depicted.*

Here, the LSP ($\tilde{\chi}$) is an almost pure bino. Its relic abundance will be calculated by `micrOMEGAs` [9], which is the most complete code available. It includes all the coannihilations [10] of neutralinos, charginos, sleptons, squarks and gluinos since it incorporates automatically all possible channels by using `COMPHEP` [11] (A similar calculation has appeared in Ref.[12].) Also, poles and thresholds are properly handled and one-loop QCD corrected Higgs decay widths [13] are used, which is the main improvement provided by Ref.[9]. The SUSY corrections [14] to these widths are, however, not included. Fortunately, in our case, their effect is much smaller than that of the QCD corrections. From the recent results of DASI [15], one finds that the 95% c.l. range of $\Omega_{CDM}\ h^2$ is $0.06 - 0.22$. Therefore, we require that $\Omega_{LSP}\ h^2$ does not exceed 0.22.

In order to have an independent check of `micrOMEGAs`, we also use the following alternative method for calculating $\Omega_{LSP}\ h^2$ in our model. In most of the parameter space where coannihilations are unimportant, $\Omega_{LSP}\ h^2$ can be calculated by using `DarkSUSY` [16]. Its neutralino annihilation part is in excellent numerical agreement with the recent exact analytic calculation of Ref.[17], the main defect of its current version is that it uses the tree-level Higgs decay widths. This can be approximately corrected if, in evaluating the Higgs decay widths, we replace $m_b(m_b)$ by m_b at the mass of the appropriate Higgs boson in the couplings of the b-quark to the Higgs bosons (see Ref.[9]). In the region of the parameter space where coannihilations come into play, the next-to-lightest supersymmetric particle (NLSP) turns out to be the $\tilde{\tau}_2$ and the only relevant coannihilations are the bino-stau ones [6, 18]. In this region, which is given by $\Delta_{\tilde{\tau}_2} < 0.25$, we calculate $\Omega_{LSP}\ h^2$ by using an improved version of the analysis of Ref.[6, 7, 19, 21]. The list of bino-stau coannihilation channels appropriate for all $\tan\beta$'s given Ref. [6] has been completed with some additional channels as described in [3](see also Refs.[18, 20]). Their corresponding cross sections are combined with the results of `DarkSUSY` as described in [3]. The results presented in Fig. 2 show an impressive agreement of the two methods.

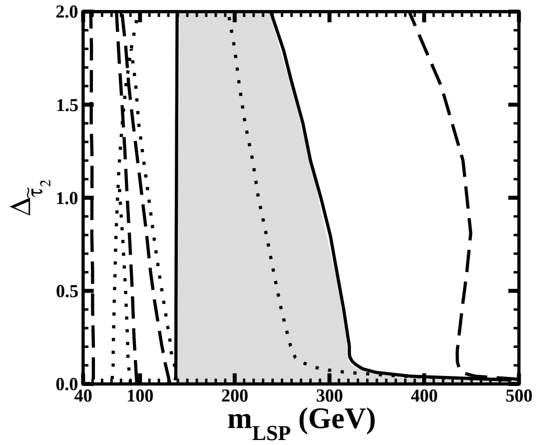

Figure 3: *Restrictions on the $m_{LSP} - \Delta_{\tilde{\tau}_2}$ plane for $A_0 = 0$, $\alpha_s(M_Z) = 0.1185$. From left to right, the dashed (dotted) lines depict the lower bounds on m_{LSP} from $m_A > 110$ GeV, $\mathrm{BR}(b \to s\gamma) > 1.9 \times 10^{-4}$ and $\delta a_\mu < 58 \times 10^{-10}$, and the upper bound on m_{LSP} from $\Omega_{LSP}\ h^2 < 0.22$ for $m_b(M_Z) = 2.684$ GeV (3.092 GeV). The left (right) solid line depicts the lower (upper) bound on m_{LSP} from $m_h > 114.1$ GeV ($\Omega_{LSP}\ h^2 < 0.22$) for $m_b(M_Z) = 2.888$ GeV. The allowed area for $m_b(M_Z) = 2.888$ GeV is shaded.*

We calculate $\mathrm{BR}(b \to s\gamma)$ using the formalism of Ref.[22], where the SM contribution is factorized out. This contribution includes the next-to-leading order (NLO) QCD and the leading order (LO) QED corrections. The charged Higgs boson contribution to $\mathrm{BR}(b \to s\gamma)$ is evaluated by including the NLO QCD corrections from Ref.[23]. The dominant SUSY contribution includes the NLO QCD corrections from Ref.[24], which hold for large $\tan\beta$. With the considerations of [3] the 95% c.l. range of this branching ratio then turns out to be about $(1.9 - 4.6) \times 10^{-4}$.

According with the latest measurement [25] of the anomalous magnetic moment of the muon $a_\mu \equiv (g_\mu - 2)/2$, the deviation of from its predicted value in the SM [26], δa_μ, is found to lie, at 95% c.l., in the range from -4.7×10^{-10} to 56×10^{-10} when the SM calculations based in e^+e^- data and in τ data are both taken into account. The calculation of δa_μ in the CMSSM is performed here by using the analysis of Ref.[27], the updating of the experimental bounds does not introduce significant differences respect the results presented in [3].

We will also impose the 95% c.l. LEP bound on the lightest CP-even neutral Higgs boson mass $m_h > 114.1$ GeV. In the CMSSM, this bound holds almost always for all $\tan\beta$'s, at least as long as CP is conserved. The CP-even neutral Higgs boson mass matrix by using `FeynHiggsFast` [28]. Finally, for the values of $\tan\beta$ which appear here (about 60), the CDF results yield the 95% c.l. bound $m_A > 110$ GeV [29].

3 The Allowed Parameter Space

The restrictions on the $m_{LSP} - \Delta_{\tilde{\tau}_2}$ plane, for $A_0 = 0$ and with the central value of $\alpha_s(M_Z) = 0.1185$, are shown in Fig.3. The dashed (dotted) lines correspond to the 95%

c.l. lower (upper) experimental bound on $m_b(M_Z)$ which is 2.684 GeV (3.092 GeV), while the solid lines correspond to the central experimental value of $m_b(M_Z) = 2.888$ GeV. From left to right, the dashed (dotted) lines depict the lower bounds on m_{LSP} from the constraints $m_A > 110$ GeV, BR$(b \to s\gamma) > 1.9 \times 10^{-4}$ and $\delta a_\mu < 58 \times 10^{-10}$, and the 95% c.l. upper bound on m_{LSP} from $\Omega_{LSP}\, h^2 < 0.22$. The constraints BR$(b \to s\gamma) < 4.6 \times 10^{-4}$ and $\delta a_\mu > -6 \times 10^{-10}$ do not restrict the parameters since they are always satisfied for $\mu > 0$. The left solid line depicts the lower bound on m_{LSP} from $m_h > 114.1$ GeV which does not depend much on $m_b(M_Z)$, while the right solid line corresponds to $\Omega_{LSP}\, h^2 = 0.22$ for the central value of $m_b(M_Z)$. We see that m_A is always smaller than $2m_{LSP}$ but close to it. Thus, generally, the neutralino annihilation via the s-channel exchange of an A-boson is by far the dominant (co)annihilation process. We also observe that, as m_{LSP} or $\Delta_{\tilde{\tau}_2}$ increase, we move away from the A-pole, which thus becomes less efficient. As a consequence, $\Omega_{LSP}\, h^2$ increases with m_{LSP} or $\Delta_{\tilde{\tau}_2}$ (see Fig.2).

In the allowed (shaded) area of Fig. 3 which corresponds to the central value of $m_b(M_Z)$, the parameter c ($\tan\beta$) varies between about 0.15 and 0.20 (58 and 59). For $m_b(M_Z)$ fixed to its lower or upper bound, we find that, in the corresponding allowed area, the parameter c ($\tan\beta$) ranges between about 0.17 and 0.23 (59 and 61) or 0.13 and 0.17 (56 and 58). We observe that, as we increase $m_b(M_Z)$, the parameter c decreases and we get closer to exact Yukawa unification. This behavior is certainly consistent with the fact that the value of $m_b(M_Z)$ which corresponds to exact Yukawa unification lies well above its 95% c.l. range. The LSP mass is restricted to be higher than about 138 GeV for $A_0 = 0$ and $\alpha_s(M_Z) = 0.1185$, with the minimum being practically $\Delta_{\tilde{\tau}_2}$-independent. At this minimum, $c \approx 0.16 - 0.20$ ($c \approx 0.13 - 0.23$) and $\tan\beta \approx 59$ ($\tan\beta \approx 58 - 61$) for $m_b(M_Z) = 2.888$ GeV ($m_b(M_Z) = 2.684 - 3.092$ GeV).

In Fig. 3, we present the restrictions on the $m_{LSP} - A_0/M_{1/2}$ plane for $m_b(M_Z) = 2.888$ GeV, $\alpha_s(M_Z) = 0.1185$ and fixed values of $\Delta_{\tilde{\tau}_2}$. The most significant restriction on the allowed area are due to the displacement of the $\Omega_{LSP}\, h^2 = 0.22$ line as $\Delta_{\tilde{\tau}_2}$ increases, showing clearly the effect of the bino-stau coannihilations on the left panel. On the right panel we can observe that the allowed area becomes narrower as $|A_0/M_{1/2}| \neq 0$.

4 Conclusions

We showed that, in the particular model with Yukawa quasi-unification considered, there exists a wide and natural range of CMSSM parameters which is consistent with all the above constraints. We found that, within the investigated part of the overall allowed parameter space, the parameter $\tan\beta$ ranges between about 58 and 61 and the 'asymptotic' splitting between the bottom (or tau) and the top Yukawa couplings varies in the range $26 - 35\%$ for central values of $m_b(M_Z)$ and $\alpha_s(M_Z)$.

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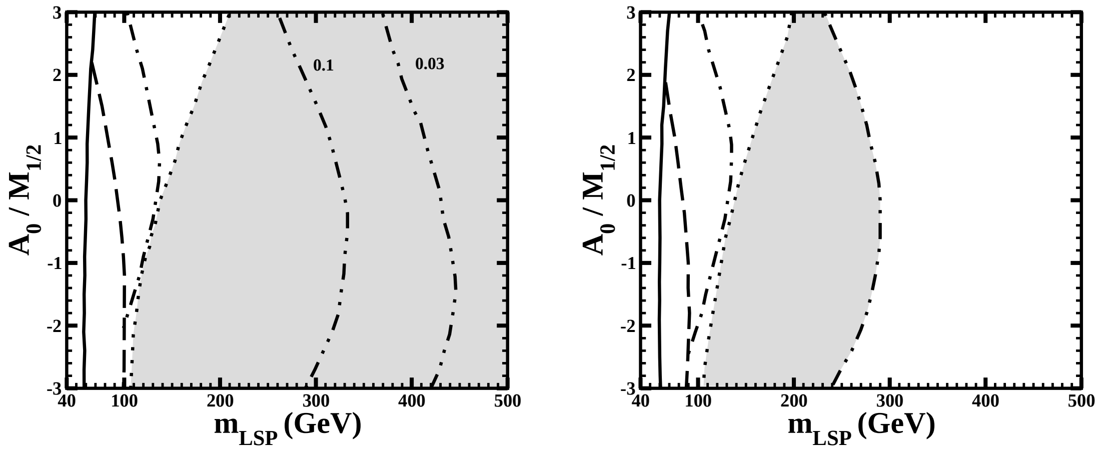

Figure 4: *Restrictions on the* $m_{LSP} - A_0/M_{1/2}$ *plane for* $\Delta_{\tilde{\tau}_2} = 0$ *(left) and 1 (right),* $m_b(M_Z) = 2.888$ GeV*,* $\alpha_s(M_Z) = 0.1185$*. The solid, dashed, dot-dashed and dotted lines correspond to the lower bounds on* m_{LSP} *from* $m_A > 110$ GeV, BR$(b \to s\gamma) > 1.9 \times 10^{-4}$, $\delta a_\mu < 58 \times 10^{-10}$ *and* $m_h > 114.1$ GeV *respectively. The upper bound on* m_{LSP} *from* $\Omega_{LSP}\ h^2 < 0.22$ *does not appear in the left panel since it lies at* $m_{LSP} > 500$ GeV*. The allowed area is shaded. For comparison, we also display on the left panel the bounds from* $\Omega_{LSP}\ h^2 < 0.22$ *(double dot-dashed lines) for* $\Delta_{\tilde{\tau}_2} = 0.1$ and 0.03*, as indicated. The upper bound on* m_{LSP} *from the cosmological constraint* $\Omega_{LSP}\ h^2 < 0.22$ *corresponds to the double dot-dashed line on the left panel.*

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