Cosmological Change of Coupling Constants Driven by Dark Matter

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Abstract

In this talk, I review recent works on the Bekenstein-like models of changing alpha. The original model that admits a scalar field coupled to electromagnetic $F^2_{\mu\nu}$ predicts this change at the level of $O(10^{-10} - 10^{-9})$ for $z \sim 1$, far below recent experimental hints at the non-zero change at $O(10^{-5})$ level. I argue that in supersymmetric extensions of the same model, one should expect four-five orders of magnitude enhancement, that make them marginally consistent with the observational claims.

1 Introduction and summary

This talk is based on the recent work [1]. Since then there has been an important development on experimental side [2], which strengthened the statistical evidence for the change of alpha at redshifts $z \sim 1$. On the theoretical side, there appeared a new stringent limit on the change of alpha implied by meteoritic data [3] that goes back to the formation of the Solar system.

Speculations that fundamental constants may vary in time and/or space goes back to the original idea of Dirac [4]. Despite such a reputable origin, this idea has not received much attention during the last fifty years for the two following reasons. First, there exist various sensitive experimental checks that coupling constants do not change (See, e.g. [5]). Second, for a long time there has not been any credible theoretical framework which would predict such changes.

Twenty years ago Bekenstein [6] formulated a dynamical model of "changing α". The model consists of a massless scalar field which has a linear coupling to the F^2 term of the $U(1)$ gauge field, $M_*^{-1}\phi F_{\mu\nu}F^{\mu\nu}$, where M_* is an associated mass scale and thought to be of order the Planck scale. A change in the background value of ϕ, can be interpreted as the change of the effective coupling constant. Bekenstein noticed that F^2 has a non-vanishing matrix element over protons and neutrons, of order $(10^{-3} - 10^{-2})m_N$. This matrix element acts as a source in the ϕ equation of motion and naturally leads to the cosmological evolution of the ϕ field driven by the baryon energy density. Thus, the change in ϕ translates into a change in α on a characteristic time scale comparable to the lifetime of the Universe or larger. However, the presence of the massless scalar field ϕ in the theory leads to the existence of an additional attractive force which does not respect Einstein's weak universality principle. The extremely accurate checks of the latter [7] lead to a firm lower limit on M_*, $M_*/M_{\rm Pl} > 10^3$ that confines possible changes of α to the range $\Delta\alpha < 10^{-10} - 10^{-9}$ for $0 < z < 5$ [6, 8].

This range is five orders of magnitude tighter than the change $\Delta\alpha/\alpha \simeq 10^{-5}$ indicated in the observations of quasar absorption spectra at $z = 0.5 - 3.5$ and recently reported by Webb et al. [9].Therefore, it is interesting to explore the possibility of constructing a dynamical model, including modifications of Bekenstein's model, which could produce a large change in α in the redshift range $z = 0.5 - 3.5$ and still be consistent with the constraints on $\Delta\alpha/\alpha$ from the results of high-precision limits on the violation of equivalence principle by a fifth force. It is also interesting to study whether the range $\Delta\alpha/\alpha \simeq 10^{-5}$ could be made consistent with the limits on $\Delta\alpha/\alpha$ [10]-[13], extracted from the analysis of element abundances from the Oklo phenomenon, a natural nuclear fission reactor that occurred about 1.8 billion years ago.

The gap of five orders of magnitude between the desirable range of 10^{-5} and bounds of order 10^{-10} appear to be insurmountable for any sensible modification of Bekenstein's theory[1]. In this paper, we propose a modification of Bekenstein's idea consistent with experimental constraints, but relies on a large coupling between the non-baryonic dark matter energy density and the ϕ field.

At first, such a coupling may appear strange. Indeed, why should dark matter interact with the ϕ field when it is known that dark matter particles are not charged [15] and their electromagnetic form-factors are also tightly constrained [16]? It turns out that in certain classes of models for dark matter, and in supersymmetric models in particular, it is natural to expect that ϕ would couple more strongly to dark matter particles than to baryons. It is easy to demonstrate this idea by a simple supersymmetrization of Bekenstein's interaction. In addition to the coupling of ϕ to the kinetic term, F^2, of the gauge boson, ϕ will acquire an additional coupling to the kinetic term of the gaugino, $M_*^{-1}\phi\bar{\chi}\not{\partial}\chi$. If this gaugino constitutes a significant fraction of the stable LSP neutralino, as is often the case, the source of ϕ due to the energy density of dark matter turns out to be dramatically enhanced compared to the baryonic source,

$$\frac{\text{Dark matter source}}{\text{baryonic source}} \sim (10^2 - 10^3)\frac{\Omega_{\rm matter}}{\Omega_{\rm baryon}} \sim 10^3 - 10^4. \tag{1.1}$$

Such an enhancement factor compensates, although not entirely, for the tremendous suppression of $\Delta\alpha$ once the Eötvös-Dicke-Braginsky (EDB) limits on M_* are imposed. It is then reasonable to study this class of models in further detail as they are numerically much more promising than the original Bekenstein framework.

[1] A recent publication claiming that the 10^{-5} change in α is realistic in this framework [14] does not impose the limits from Eötvös-Dicke-Braginsky experiments.

2 Generalization of Bekenstein's model

We start our analysis by formulating a generic action that includes spin-2 gravity, kinetic and potential terms of the modulus ϕ, kinetic terms for the electromagnetic field and baryons as well as the dark matter action,

$$\begin{aligned} S = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}M_{\rm Pl}^2 R + \frac{1}{2}M_*^2\partial_\mu\phi\partial^\mu\phi - M_{\rm Pl}^2\Lambda_0 B_\Lambda(\phi) \\ -\frac{1}{4}B_F(\phi)F_{\mu\nu}F^{\mu\nu} + \sum_{i=p,n}\bar{N}_i(i\not{D} - m_i B_{Ni}(\phi))N_i \\ +\frac{1}{2}\bar{\chi}\not{\partial}\chi - \frac{1}{2}M_\chi B_\chi(\phi)\chi^T\chi\Big] \end{aligned} \tag{2.2}$$

Throughout this paper we assume a $+---$ signature for the metric tensor. In (2.3), $M_{\rm Pl} = (8\pi G_N)^{-1/2} = 2.4\times10^{18}$GeV is the Planck mass and M_* is its analogue in the ϕ sector. Defined this way, ϕ is dimensionless. N_i stands for neutrons and protons, and $\not{D} = \gamma^\mu(\partial_\mu - ie_0A_\mu)$ for protons and $\not{D} = \gamma^\mu\partial_\mu$ for neutrons. Here e_0 is the *bare* charge which remains constant throughout the cosmological evolution (modulo standard RG evolution of e_0 which can be neglected in our analysis). For definiteness, we assume that the dark matter is predominantly the non-relativistic Majorana fermion χ. While it is clear that one can associate χ with a neutralino, our approach can be easily generalized to other forms of cold dark matter. Ellipses stand for the omitted electron and neutrino terms, as well as for a number of possible interaction terms (i.e. baryon anomalous magnetic moments, nucleon-nucleon interactions etc.). All mass and kinetic terms are supplied with ϕ-dependent factors denoted $B_i(\phi)$. In this sense, the cosmological constant term acts as a the potential for ϕ.

We shall further assume that the change of ϕ over cosmological scales is small, $\Delta\phi \equiv \phi(t=t_0) - \phi(t) \ll 1$, where t_0 is the present age of the universe. As such, we can expand all couplings around the current value of ϕ, which we choose to be zero, $\phi(t=t_0) = 0$,

$$\begin{aligned} B_\Lambda(\phi) &= 1 + \zeta_\Lambda\phi \\ B_F(\phi) &= 1 + \zeta_F\phi \\ B_{Ni}(\phi) &= 1 + \zeta_i\phi \\ B_\chi(\phi) &= 1 + \zeta_\chi\phi. \end{aligned} \tag{2.3}$$

The effective fine structure constant depends on the value of ϕ. As such, $\phi(t)$ and $\Delta\alpha/\alpha$ are directly related,

$$\begin{aligned} \alpha(\phi) &= \frac{e_0^2}{4\pi B_F(\phi)} \\ \frac{\Delta\alpha}{\alpha} &= \zeta_F\phi, \end{aligned} \tag{2.4}$$

and we have defined $\Delta\alpha/\alpha$ as $(\alpha_0 - \alpha(t))/\alpha_0$.

The cosmological evolution of ϕ follows from the scalar field equation

$$M_*^2 \Box\phi = -M_{\rm Pl}^2 \Lambda_0 B'_\Lambda - B'_F \frac{1}{4}\langle F_{\mu\nu}F^{\mu\nu}\rangle \\ -\langle B'_n m_n \bar{n}n + B'_p m_p \bar{p}p\rangle - \frac{1}{2} B'_\chi M_\chi \langle \chi^T \chi\rangle. \tag{2.5}$$

In this formula, primes denote $d/d\phi$, and the average $\langle ...\rangle$ denotes a statistical average over a current state of the Universe. The term with $F_{\mu\nu}F^{\mu\nu}$ can be neglected to a good approximation as its average is zero for photons, and its contribution mediated by the baryon density, $\sum_{n,p} n_i \langle i|F_{\mu\nu}F^{\mu\nu}|i\rangle$ is already included in the terms proportional to $B'_{n,p}$. We further note that for a Dirac fermion ψ, the mass term $m_\psi \bar{\psi}\psi$ (and the analogous combination for a Majorana fermion) coincides with the trace of the ψ-contribution to stress-energy tensor, or $\rho_\psi - 3p_\psi$. Thus, the only term, that drives ϕ in the radiation domination epoch when $\rho = 3p$ is $\Lambda_0 B'_\Lambda$ (see e.g. [17, 18]). One can easily check that the change of ϕ induced by this term during radiation domination will be small compared to the $\Delta\phi$ developed in the subsequent matter domination epoch. Restricting Eq. (2.6) to matter domination, and assuming a linearized regime (2.3), we derive the following equation

$$M_*^2(\ddot{\phi} + 3H\dot{\phi}) = -\rho_m \zeta_m - M_{\rm Pl}^2 \Lambda \zeta_\Lambda, \tag{2.6}$$

where $H = \dot{a}/a$ and ζ_m is defined as

$$\rho_m \zeta_m \equiv \rho_\chi \zeta_\chi + \rho_b (Y_p \zeta_p + Y_n \zeta_n). \tag{2.7}$$

Here, Y_p and Y_n are the abundances of neutrons and protons in the Universe, including those bound in nuclei. We also assume that $\rho_m = \rho_\chi + \rho_b$. In a more sophisticated treatment, one may include the contributions of electrons, the Coulomb energy stored in nuclei and other minor effects. As discussed in Refs. [6, 8], to good accuracy, ζ_m remains constant during the matter dominated epoch.

If the ϕ-dependent energy density becomes comparable to ρ_m or $\rho_\Lambda \equiv M_{\rm Pl}^2\Lambda$, Eq. (2.6) must be solved along with Einstein's equations and energy conservation as a coupled set of equations. However, the small ϕ solutions that we are interested in imply that the ρ_ϕ is small and (2.6) can be treated separately, with $a(t)$ as an input function.

3 Cosmological evolution of the fine structure constant

In this section, we study the cosmological evolution of ϕ determined by the ζ_i terms in Eq. (2.6).

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{1}{M_*^2}\left[\zeta_m \rho_m + \zeta_\Lambda \rho_\Lambda\right] = -\frac{\rho_c}{M_*^2}\left[\zeta_m \Omega_m \left(\frac{a_0}{a}\right)^3 + \zeta_\Lambda \Omega_\Lambda\right], \tag{3.8}$$

Here $\rho_c = 3H_0^2 M_{\rm Pl}^2$ is the critical density of the Universe at $t = t_0$ and $\Omega_i = \rho_i/\rho_c$. The solution to this equation can be easily found [8, 13, 14]. Throughout this paper we shall assume that the Universe is flat and is presently dominated by non-relativistic matter and a cosmological constant, $\Omega_m + \Omega_\Lambda = 1$. In this case, the time dependence of the scale factor is given by

$$a(t)^3 = a_0^3 \frac{\Omega_m}{\Omega_\Lambda} \left[\sinh(\frac{3}{2}\Omega_\Lambda^{1/2} H_0 t) \right]^2 \tag{3.9}$$

and Eq. (3.8) can be integrated in the analytical form. The first integral is given by

$$\dot{\phi} = -3\Omega_m H_0^2 \frac{M_{\rm Pl}^2}{M_*^2} \frac{a_0^3}{a^3} \left[\zeta_m t + \frac{\zeta_\Lambda}{4b}(\sinh(2bt) - 2bt) - t_c \right], \tag{3.10}$$

where $b = \frac{3}{2}\Omega_\Lambda^{1/2} H_0$. In principle, the constant of integration t_c could be kept arbitrary. There is, however, only one natural way of fixing it by imposing initial conditions for $\dot{\phi}$ deep inside the radiation domination epoch, i.e. at t close to 0. As discussed in the previous section, during radiation domination the r.h.s of (3.8) is effectively zero. This leads to a $\dot{\phi} \sim a^{-3}$ scaling behavior and means that any initial value of $\dot{\phi}$ will be efficiently damped by the Hubble expansion over a few Hubble times. Thus, for the solution in the matter dominated epoch we can safely take $\dot{\phi}(t=0) = 0$ or equivalently $t_c = 0$.

Integrating of (3.10) gives ϕ as a function of time,

$$\phi(t) = \frac{4}{3}\frac{M_{\rm Pl}^2}{M_*^2} \left[(\frac{\zeta_\Lambda}{2} - \zeta_m)(bt_0 \coth(bt_0) - bt\coth(bt)) - \zeta_m \ln \frac{\sinh(bt)}{\sinh(bt_0)} \right]. \tag{3.11}$$

Figure 1 shows three different types of solutions for $\Delta\alpha/\alpha$ as a function of the red-shift z, $1+z = a_0/a$. In this plot, we have chosen $\zeta_F = 10^{-5}$, $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$. Comparing the three curves, one can see that the variation of α at high red-shifts is mostly determined by ζ_m. If ζ_F is negative, one would need to choose negative ζ_m in order to get smaller values of α in the past. Opposite signs of ζ_F and ζ_m lead to the larger values of α in the past.

Given the large parameter space, $(M_*,\ \zeta_F,\ \zeta_m,\ \zeta_\Lambda)$, one could expect that it is easy to get $\Delta\alpha(z = 0.5 - 3.5)/\alpha \sim 10^{-5}$ as suggested by the analysis of the quasar absorption spectra by Webb et al. [9]. On the other hand, it is clear that the EDB constraints should severely restrict the parameter space of our model. The best constraints on the long-range forces are extracted from $\Delta g/\bar{g}$ measured in experiments that compare the the acceleration of light and heavy elements. The differential acceleration of platinum and aluminium is $\leq 2 \times 10^{-12}$ at the 2σ level (last reference in [7] as quoted in [6]), and the differential acceleration of the Moon (silica-dominated) and the Earth (iron-dominated) towards the Sun is $\leq 0.92 \times 10^{-12}$ [19]. Choosing the appropriate values of Z and A and retaining only the hydrogen contribution to the mass of the Sun, we get

$$\begin{aligned} &\frac{1}{\omega}\left|\zeta_p(\zeta_n - \zeta_p + 2.9\times 10^{-2}\zeta_F)\right| < 2.5\times 10^{-11} \qquad \text{Al/Pt system} \\ &\frac{1}{\omega}\left|\zeta_p(\zeta_n - \zeta_p + 1.8\times 10^{-2}\zeta_F)\right| < 2.5\times 10^{-11} \qquad \text{Si/Fe system} \end{aligned} \tag{3.12}$$

$\zeta_n - \zeta_p$ and ζ_F enter in Eqs. (3.12) in different linear combinations. Thus, it is possible to extract *separate* limits on $\omega^{-1}\zeta_p\zeta_F$ and $\omega^{-1}\zeta_p(\zeta_n - \zeta_p)$. Models that have non-zero ζ_F

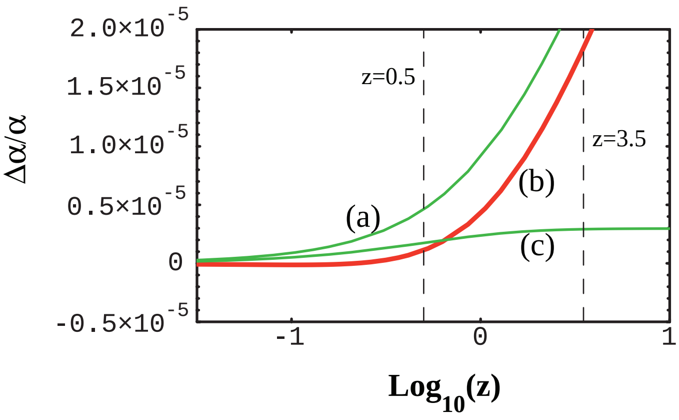

Figure 1: Three qualitatively different types of solutions for $\Delta\alpha(z)/\alpha_0$ that give smaller values of α in the past for positive ζ_F. They correspond to the choice of $\zeta_F = 10^{-5}$ and (a) $\zeta_m = 1$, $\zeta_\Lambda = 0$ (b) $\zeta_m = 1$, $\zeta_\Lambda = -2$ and (c) $\zeta_m = 0$, $\zeta_\Lambda = 1$. The interval of z, considered by Webb et al., $0.5 \le z \le 3.5$ is shown by two vertical dashed lines.

also have non-vanishing $\zeta_{p,n}$ unless some intricate conspiracy of quark, gluon and photon contributions occur. Barring possible cancellations, one obtains $|\zeta_{n,p}| \gtrsim |\zeta_n - \zeta_p| \gtrsim 10^{-3}|\zeta_F|$. Using these relations, we can combine the preferred range of Ref. [9] with the constraints, imposed by Eqs. (3.12).

The region excluded by the EDB constraints in the $(\zeta_m/\sqrt{\omega},\ \zeta_F/\sqrt{\omega})$ parameter space is shown by the light shaded (blue) region in Figure 2. Here we have set $\zeta_\Lambda = 0$. The long negative-slope band that connects the upper-left and lower-right hand corners is the range that reproduces $\Delta\alpha/\alpha = 10^{-5}$ in the interval $0.5 \le z \le 3.5$. In the original Bekenstein model, $\zeta_m = (10^{-4}$ to $10^{-3})\zeta_F$ and corresponds to the positive slope band close to the upper-left corner [2]. As one can see, the diamond-shaped intersection is deep inside the range *excluded* by the EDB experiments. Of course, this is in agreement with conclusions of [6, 8]. Finally, the dark-shaded (green) area represents the choice of parameters that can reproduce [9] and still be in agreement with the EDB constraints. For this region, $\zeta_m/\sqrt{\omega} \gtrsim 3 \times 10^{-3}$ and $\zeta_F/\sqrt{\omega} < 10^{-3}$, which points towards models in which ϕ couples to dark matter and the couplings to baryons and ζ_F are suppressed.

4 Conclusions

In the framework of a very generic model, we can show that the result of Webb et al. cannot be explained by the simplest Bekenstein model. The preferred source for the evolution of the scalar field responsible for the change of the coupling constant is its coupling to dark matter or self potential. We have shown that the supersymmetrization of the Bekenstein model leads to a large coupling between the scalar field and the susy partner of photon and to four-five orders of magnitude enhancement in the strength of

[2] $\zeta_m = 10^{-3}\zeta_F$ would require rather “generous” assumptions about nucleon matrix elements and Ω_b

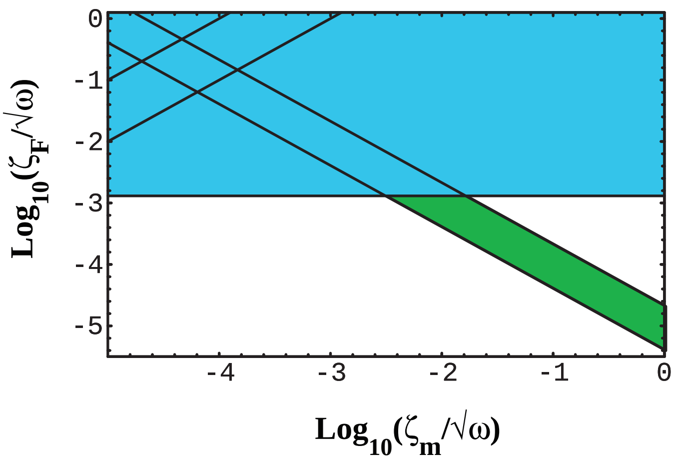

Figure 2: The $(\zeta_m/\sqrt{\omega},\ \zeta_F/\sqrt{\omega})$ parameter space. The dark-shaded (green) region is consistent with both the EDB constraints and with a possible relative change of α at the 10^{-5} level, as suggested by Webb et al. The light shaded (blue) region is excluded by EDB constraints. ζ_Λ is set to zero in this plot.

the cosmological source for the scalar field compared to the baryonic source. Even in this case, however, there is a significant difficulty in constructing a model that would give a non-zero result at quasar times and be consistent with constraints on the change of alpha imposed by Oklo and meteorite data.

References

[1] K. A. Olive and M. Pospelov, Phys. Rev. D **65** (2002) 085044.

[2] M. T. Murphy, J. K. Webb, V. V. Flambaum, and S. J. Curran, arXiv:astro-ph/0210532.

[3] K. A. Olive, M. Pospelov, Y. Z. Qian, A. Coc, M. Casse, and E. Vangioni-Flam, Phys. Rev. D **66** (2002) 045022.

[4] P.A.M. Dirac, Nature, **139** (1937) 323.

[5] P. Sisterna and H. Vucetich, Phys. Rev. D **41** (1990) 1034.

[6] J. D. Bekenstein, Phys. Rev. D **25** (1982) 1527.

[7] R.V. Eötvös, V. Pekar, and E. Fekete, Ann. Phys. (Leipzig) **68** (1922) 11;
P. G. Roll, R. Krotkov, and R. H. Dicke, Annals Phys. **26** (1964) 442;
V.B. Braginsky and V.I. Panov, Zh. Eksp. Teor. Fiz. **61** (1972) 873 [Sov. Phys. JETP **34** (1972) 463.

[8] M. Livio and M. Stiavelli, Ap. J. Lett. **507** (1998) L13.

[9] J.K. Webb *et al.*, Phys. Rev. Lett. **87** (2001) 091301.

[10] A. I. Shlyakhter, Nature **264** (1976) 340.

[11] T. Damour and F. Dyson, Nucl. Phys. B **480** (1996) 37.

[12] Y. Fujii *et al.*, Nucl. Phys. **B573** (2000) 377.

[13] S. J. Landau and H. Vucetich, astro-ph/0005316; N. Chamoun, S. J. Landau, and H. Vucetich, Phys. Lett. B **504** (2001) 1.

[14] H. B. Sandvik, J. D. Barrow, and J. Magueijo, Phys. Rev. Lett. **88**, 031302 (2002).

[15] S. Davidson, B. Campbell and D. Bailey, Phys. Rev.**D43** (1991) 2314.

[16] M. Pospelov, and T. ter Veldhuis, Phys. Lett. B **480** (2000) 181 [hep-ph/0003010].

[17] N. Kaloper and K. A. Olive, Astropart. Phys. **1** (1993) 185.

[18] A.A. Tseytlin, Mod. Phys. Lett. **A6** (1991) 1721; Class. Quant. Grav. **9** (1992) 979; Int. J. Mod. Phys. **D1** (1992) 223.

[19] J.O. Dickey *et al.*, Science **265** (1994) 482;
J.G. Williams *et al.*, Phys. Rev. **D53** (1996) 6730;
T. Damour, arXiv:gr-qc/0109063.