A modified Starobinsky's model of inflation: Anomaly-induced inflation, SUSY, and graceful exit

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ABSTRACT

We present a model of inflation based on the anomaly-induced effective action of gravity in the presence of a conformally invariant Hilbert-Einstein term. Our approach is based on the conformal representation of the fields action and on the integration of the corresponding conformal anomaly. In contradistinction to the original Starobinsky's model, inflation can be stable at the beginning and unstable at the end. The instability is caused by a slowing down of inflation due to quantum effects associated to the massive fermion fields. In supersymmetric theories this mechanism can be linked to the breaking of SUSY and suggests a natural way to achieve graceful exit from the inflationary to the FLRW phase.

Introduction

Inflation [1] automatically solves five of the six basic cosmological problems [2]: 1) the monopole problem, 2) the horizon problem, 3) the flatness-curvature-entropy problem, 4) the rotation problem, and 5) the large-scale homogeneity versus small-scale inhomogeneity problem. The minimum number of e-folds of inflation required can be (roughly) estimated in many different ways. As an example, take a flat Friedmann-Lemaître-Robertson-Walker (FLRW) model, then the scale factor $a = a(t)$ in the matter-dominated (MD) and radiation-dominated (RD) eras evolve as $a \sim t^{2/3}$ and $a \sim t^{1/2}$ respectively. Since at present $t_0 \sim 10^{18}$ sec (15 Gy) and $a(t_0) \sim 1.5 \times 10^{10} lyr \sim 10^{28} cm$, it follows that the scale factor at the end of the RD epoch ($t \sim 10^{12}$ sec $\sim 10^5$ yr) was $a_R = a(t_0)/z_R = 10^{24} cm$, where $z_R \sim a(t_0)/a_R = (10^{18}/10^{12})^{2/3} = 10^4$ is the redshift at that time. From this the scale factor at the Planck time ($\sim 10^{-44}$ sec) should be $a_P^* = (10^{-44}/10^{12})^{1/2}\ a_R \sim 10^{-4} cm$, which is of course untenable! Therefore, to make this number to match up the correct Planck length, $a_P \sim 10^{-33}$ cm, we need to supplement the standard FLRW evolution with an early inflation period in which the number of e-folds of inflation should be around

$\xi \sim 65$:

$$e^{\xi} = \frac{a_P^*}{a_P} = 10^{29} \Rightarrow \xi \sim 67\,. \tag{1}$$

At present, however, the classical cosmological problems that motivated inflation are no longer regarded as the strongest motivation for inflationary cosmology. For example, the relation "homogeneity → flatness" is not true. Nowadays we have homogeneous open models of inflation. Besides providing the solution to some cosmological problems, the important thing at present is that inflationary models can be (and will be) more and more accurately tested (something that one could not suspect 10 years ago) through their specific predictions on the metric and density perturbations, which should be consistent with structure formation and the anisotropies of the CMB [3]. These are nowadays the real facts behind $\xi > 65$ in Eq.(1), rather than the previous and similar heuristic argumentations.

Inflation, however, does *not* solve the sixth cosmological problem, the cosmological constant (CC) problem [4]. The fact that the CC has been measured non-zero and positive [5] poses a new challenge and may require novel approaches. One possibility is to think of the renormalization group (RG) evolution of the cosmological parameters [6]. We will see that this same approach can be applied to the study of cosmological inflation, if we identify the RG scale with the expansion rate $\mu = H(t)$. This identification can be understood as follows. At low energy the dynamics of gravity is defined by Einstein's equations

$$R_{\mu\nu} - \frac{1}{2}\, R g_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + g_{\mu\nu}\, \Lambda \, \right), \tag{2}$$

where $G_N = 1/M_P^2$ is Newton's constant. Let us use the value of the curvature scalar (R) to construct an order parameter for the gravitational energy. By dimensional analysis the RG scale μ for gravity is naturally associated with $R^{1/2}$. From Eq.(2) we see that this is equivalent to take $\mu \sim \sqrt{T_\mu^\mu / M_P^2}$. But in the cosmological setting the basic dynamical equations refer to the scale factor $a(t)$ of the FLRW metric, and so we must re-express the graviton energy in terms of it. The 00 component of (2) yields the well-known Friedmann-Lemaître equation

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3\, M_P^2}\, (\rho + \Lambda) - \frac{k}{a^2}\,. \tag{3}$$

The space curvature term can be safely set to zero ($k = 0$). The spatial components of (2), combined with the 00 component (3), yields the following dynamical equation for $a(t)$:

$$\ddot{a} = -\frac{4\pi}{3\, M_P^2}\, (\rho + 3\, p - 2\, \Lambda)\; a\,. \tag{4}$$

In these equations $\rho = \rho_M + \rho_R$ is the total energy density of matter and radiation, and p is the pressure. In the modern Universe $p \simeq 0$ and $\rho \simeq \rho_M^0$. Moreover, from the recent supernovae data [5], we know that Λ and ρ_M^0 have the same order of magnitude as the critical density ρ_c^0. Therefore, the source term on the *r.h.s.* of (4) is characterized by a single dimensional parameter $\sqrt{\rho_c^0/M_P^2}$, which according to Eq. (3) is nothing but the experimentally measurable Hubble's constant H_0. This is obviously consistent with the expected result $\sqrt{T_\mu^\mu/M_P^2}$ in the general case because $T_\mu^\mu \sim \rho_M^0 \sim \rho_c^0$ for the present-day universe. Therefore, we conclude that the Ansatz

$$\mu \sim R^{1/2} \sim H(t) \tag{5}$$

is reasonable and we assume that this identification takes place at each stage of the cosmological evolution. With this guiding principle, a semi-classical description of gravitational phenomena of quantum matter in a curved classical background should be possible. In particular, in the early universe the value of $H(t)$ decides which matter particles are active degrees of freedom for the RG evolution of the parameters. Therefore, particles whose masses satisfy $M > H(t)$ will be decoupled from the relevant quantum effects at time t. Clearly, if one would be able to concoct a natural mechanism by which $H(t)$ progressively slows down after the universe has achieved a sufficient number of e-folds of inflation (namely $\xi > 65$), then inflation should eventually stop and the FLRW regime could perhaps start. While many authors have looked for a suitable scalar field (so-called "inflaton") capable of realizing this scenario[1], in the original Starobinsky's model [7] inflation was attempted by looking for a self-consistent solution of Einstein's equations when they are modified to include the vacuum quantum effects. Unfortunately, in Starobinsky's model inflation is unstable from the very beginning (with the flat space stable), so that one has to fine-tune the initial conditions to insure $\xi > 65$ before inflation stops [8]. It would be much desirable to find an improved framework where the self-consistent solution appears first as a stable inflationary solution (hence independent of the initial conditions, even though space-time is unstable) and such that subsequently (after $\xi > 65$ is fulfilled) inflation becomes unstable and the universe transits into the stable and flat FLRW space-time. Indeed, a modified Starobinsky's model like that is possible, provided that we can arrange the condition $H(t) \to 0$ and at the same time distort the stability regime thanks to a change in the number of active degrees of freedom, e.g. due to a phase transition from a supersymmetric Grand Unified Theory (GUT) into the Standard Model (SM) of the strong and electroweak interactions. Such a scenario has been first discussed in Ref.[9]. It is based on a modification of the anomaly-induced effective action [7, 8, 10] resulting from a prior full conformization of the classical action for gravitational fields [11], including the Hilbert-Einstein term, and matter fields [12], and on the decoupling of the supersymmetric particles at low energy [13]. Furthermore, there are strong indications that the spectrum and the amplitude of the gravitational waves in this model [14, 15] are in agreement with the existing CMBR data [3]. Recently, also the stability with respect to small perturbations of the conformal factor of the metric has been studied in the presence of a cosmological constant [6, 15].

2. Local conformal invariance and effective action

The expansion of an homogeneous, isotropic universe means a conformal transformation of the metric $g_{\mu\nu}(t) \to a^2(\eta)\, g_{\mu\nu}$, where $a(\eta) = \exp\, \sigma(\eta)$ and η is the conformal time ($d\eta = dt/a(\eta)$). Suppose that one starts from the conformal invariant formulation of the Standard Model of the strong and electroweak interactions [12, 16] and of gravity[11], and then one uses the well-known methods to derive the anomaly-induced action [17, 18]. It is natural to think that the latter can be applied at high energies, where the masses of the matter fields are negligible. At the classical level, the theory which results from this procedure is always equivalent to the original theory. Nevertheless, in the quantum theory the equivalence is destroyed by the anomaly, which can be calculated explicitly. In particular, the massive fields may also contribute to it. Besides the anomalous terms, there are the conformal invariant quantum corrections to the classical vacuum action. Our first purpose is to construct such a formulation of the SM in curved space-time which possesses local conformal invariance in $d = 4$. Actually, the procedure can be applied to any gauge theory, e.g. the SM and extensions thereof, including GUT's and supersymmetric generalizations like the Minimal Supersymmetric Standard Model (MSSM) [19].

The original action of the theory includes kinetic terms for spinor and gauge boson fields, as well as interaction terms, all of them already conformal invariant. As for scalars (e.g. Higgs bosons) we suppose that their kinetic terms appear in the combination $g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + 1/6 \cdot R\varphi^2$ providing the local conformal invariance. The non-invariant terms are the massive ones for the scalar and spinor fields, but also the Hilbert-Einstein term giving General Relativity at low energies. In all these cases the conformal non-invariance is caused by the presence of dimensional parameters m_H^2, m, $M_P^2 = 1/G$. The central idea is to replace these parameters by functions of some new auxiliary scalar field χ. For instance, we replace [12]

$$m_H^2 \to \frac{m_H^2}{M^2}\,\chi^2\,, \qquad m \to \frac{m}{M}\,\chi\,, \qquad M_P^2 \to \frac{M_P^2}{M^2}\,\chi^2\,, \tag{6}$$

where M is some dimensional parameter, e.g. related to a high scale of spontaneous breaking of dilatation symmetry [12]. Then the scalar and fermion mass terms become quartic interactions and Yukawa couplings respectively,

$$\frac{1}{2}\int d^4x\sqrt{-g}\;\frac{m_H^2}{M^2}\,\varphi^2\,\chi^2\,, \qquad\qquad \int d^4x\sqrt{-g}\;\frac{m}{M}\;\bar\psi\psi\,\chi\,, \tag{7}$$

which are of course (local and global) conformal invariant. Furthermore, the Hilbert-Einstein term gets conformized too:

$$S^*_{EH} = -\frac{1}{16\pi G\, M^2}\int d^4x\sqrt{-g}\;\left[\, R\chi^2 + 6\,(\partial\chi)^2\,\right]\,. \tag{8}$$

After setting $\chi \to M$ this expression becomes identical to the ordinary gravitational term, and from (7) the ordinary mass terms for scalars and fermions are recovered at the same time . This fixing can be called "conformal unitary gauge" in analogy with the unitary gauge of ordinary gauge theories, and the scale M can be associated to the vacuum expectation value of the spontaneously broken dilatation symmetry at high energies [12]

It is supposed that the new scalar field χ takes the values close to M, especially at low energies. But, there is a great difference between χ and M with respect to the conformal transformation. The mass does not transform, while χ does. Then, the action of the new model becomes invariant under the conformal transformation

$$\chi \rightarrow \chi\, e^{-\sigma}\,, \tag{9}$$

which is performed together with the usual transformations for the other fields

$$g_{\mu\nu} \rightarrow g_{\mu\nu}\, e^{2\sigma}\,,\;\; \varphi \rightarrow \varphi\, e^{-\sigma}\,,\;\; \psi \rightarrow \psi\, e^{-3/2\,\sigma}\,. \tag{10}$$

Thus, in the matter sector our program of "conformization" is complete. When we quantize the theory, it is important to separate the quantum fields from the ones which represent a classical background. In order to maintain the correspondence with the usual formulation of the SM, we avoid the quantization of the field χ which will be considered, along with the metric, as an external classical background for the quantum matter fields. It is well known (see, e.g. [18]) that the renormalizability of the quantum field theory in external fields requires some extra terms in the classical action of the theory. The list of such terms includes the nonminimal term of the $\int R\varphi^2$-type in the Higgs sector, and the action of external fields with the proper dimension and symmetries. The higher derivative part of the vacuum action has the form

$$S_{vac} = \int d^4x\sqrt{-g}\,\left\{l_1 C^2 + l_2 E + l_3 \nabla^2 R\,\right\}\,, \tag{11}$$

where, $l_{1,2,3}$ are some parameters, C^2 is the square of the Weyl tensor and E is the integrand of the Gauss-Bonnet topological invariant. Now, since there is an extra field χ, the vacuum action should be supplemented by the χ-dependent term. The only possible, conformal and diffeomorphism invariant, terms with dimension 4 are (8) and the $\int \chi^4$-term. The last contributes to the cosmological constant, which we suppose to cancel and do not consider here. Its effect is reported elsewhere [6, 15].

The next step is to derive the conformal anomaly in the theory with two background fields $g_{\mu\nu}$ and χ. The anomaly results from the renormalization of the vacuum action including the terms (8) and (11). For the sake of generality, let us suppose that there is also some background gauge field with strength tensor $F_{\mu\nu}$. Then the conformal anomaly has the form

$$< T_\mu^\mu > = \,-\,\Big\{\,wC^2 + bE + c\nabla^2 R + dF^2 + f\,[\,R\chi^2 + 6\,(\partial\chi)^2\,]\Big\}\,, \tag{12}$$

where w, b, c are β-functions for the parameters l_1, l_2, l_3; f is the β-function for the dimensionless parameter $1/(16\pi G\, M^2)$ of the action (8), and d is the β-function for the gauge coupling constant. The values of w, b and c depend on the matter content (N_i being the number of particles with spin i):

$$w = \frac{N_0 + 6N_{1/2} + 12N_1}{120 \cdot (4\pi)^2}\,, \quad b = -\,\frac{N_0 + 11N_{1/2} + 62N_1}{360 \cdot (4\pi)^2}\,, \quad c = \frac{N_0 + 6N_{1/2} - 18N_1}{180 \cdot (4\pi)^2}\,. \tag{13}$$

Recall that the condition for stable inflation is $c > 0$ [7]. Then one can play with various models. For instance, from the previous equation it follows that the particle content of the SM ($N_0 = 4, N_{1/2} = 24, N_1 = 12$) leads to $c < 0$ (unstable inflation), which suggests that with only SM matter fields the inflation period cannot be easily sustained, and it might be insufficient [7]. On the other hand, for the MSSM [19] ($N_0 = 104, N_{1/2} = 32, N_1 = 12$) one has $c > 0$ (stable inflation) etc. Clearly, we need physics beyond the SM in order to possibly arrange a graceful transit from a regime of stability (insuring the condition $\xi > 65$) into another of instability that might hopefully end into the FLRW phase.

We will argue that the presence of the non-zero β-function f could be the necessary quantum dynamical mechanism for the graceful exit. From direct calculation using the Schwinger-DeWitt method (see e.g. [18]) we get

$$f \;=\; \sum_i \frac{N_i}{3\,(4\pi)^2}\,\frac{m_i^2}{M^2}\,, \tag{14}$$

where N_i are the number of Dirac spinors with masses m_i. We note that bosons do not contribute to f.

In order to obtain the anomaly-induced effective action, we put $g_{\mu\nu} = \bar{g}_{\mu\nu} \cdot e^{2\sigma}$ and $\chi = \bar{\chi} \cdot e^{-\sigma}$, where the metric $\bar{g}_{\mu\nu}$ has fixed determinant and the field $\bar{\chi}$ does not change under the conformal transformation. Then, the solution of the equation for the effective action $\bar{\Gamma}$ proceeds in the usual way [17]. Disregarding the conformal invariant term we arrive at the following expression [9]:

$$\bar{\Gamma} = \int d^4x\sqrt{-\bar{g}}\,\{w\bar{C}^2 + b(\bar{E} - \frac{2}{3}\bar{\nabla}^2\bar{R}) + 2b\,\sigma\bar{\Delta} + d\bar{F}^2 +$$

$$+ f\,[\,\bar{R}\bar{\chi}^2 + 6\,(\partial\bar{\chi})^2\,]\,\}\sigma - \frac{3c + 2b}{36}\int d^4x\sqrt{-g}\,R^2\,. \tag{15}$$

3. The role of masses in slowing down inflation

In order to understand the role of the particle masses in the anomaly-induced inflation, let us consider the total action with quantum corrections

$$S_t = S_{matter} + S_{EH} + S_{vac} + \bar{\Gamma}\,. \tag{16}$$

One of the approximations we made was to disregard higher loop and non-perturbative effects in the vacuum sector. Another approximation is that we take only the leading-log corrections. Usually, this is justified if the process goes at high energy scale. If the quantum theory has UV asymptotic freedom, the higher loops effects are suppressed, and our approximation is reliable. At the low-energy limit, we suppose that the massive fields decouple and their contributions are not important. Then Eq. (16) can be presented in the form

$$S_t = \int d^4x\sqrt{-\bar{g}}\,\Big\{ \Big(-\frac{M_P^2}{16\pi M^2} + f\sigma \Big)\,[\,\bar{R}\bar{\chi}^2 + 6\,(\partial\bar{\chi})^2\,]$$
$$-\Big(\frac{1}{4} - d\sigma\Big)\,\bar{F}^2\Big\} + S_{matter} + high.\,deriv.\,terms\,. \qquad (17)$$

One can see that the modifications with respect to the case of free massless fields [14] are an additional f-term and the contribution to anomaly due to the background gauge fields. In order to restore the Hilbert-Einstein term and get the inflationary solution, we fix the conformal unitary gauge and put $\chi = \bar{\chi}\,e^{\sigma} = M$. Furthermore, we can choose the conformally flat metric $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$. Then the gravitational part of the action (17) becomes

$$S_{grav} = \int d^4x\,\Big\{ 2b\,(\partial^2\sigma)^2 - (3c+2b)\,[(\partial\sigma)^2 + \partial^2\sigma)]^2 -$$

$$-\,6M_P^2\,e^{2\sigma}\,(\partial\sigma)^2\,\Big[1 - \frac{16\pi M^2}{M_P^2}\,f\,\Big] - \Big(\frac{1}{4} - d\sigma\Big)\,\bar{F}^2\Big\}\,. \qquad (18)$$

Computing the equation of motion of $a = \ln\sigma$ in terms of the physical time t (where $dt = a(\eta)d\eta$) we find

$$a^2\ddddot{a} + 3a\dot{a}\dddot{a} - \Big(5 + \frac{4b}{c}\Big)\,\dot{a}^2\ddot{a} + a\ddot{a}^2 - \frac{M_P^2}{8\pi c}\,\Big(a^2\ddot{a} + a\dot{a}^2\Big) +$$

$$+\frac{2fM^2}{c}\,\ln a\,\Big(a^2\ddot{a} + a\dot{a}^2\Big) + \frac{2fM^2}{c}\,\frac{\dot{a}^2}{a} - \frac{d\bar{F}^2}{6ca} = 0. \qquad (19)$$

An exact solution of (19) does not look possible, but it can be easily analyzed within the approximation that f is not too large. Then the new terms (collected in the second line of Eq. (19)) can be considered as perturbations. Moreover, the last two of them are irrelevant, because during inflation they decrease exponentially with respect to the other terms. Thus, in this approximation, the only one relevant change is the replacement $M_P^2 \longrightarrow M_P^2\,\Big[1 - \tilde{f}\,\ln a(t)\Big]$ where for future convenience we have introduced the dimensionless parameter

$$\tilde{f} \equiv \frac{16\pi f\,M^2}{M_P^2} = \sum_i \frac{N_i}{3\pi}\,\frac{m_i^2}{M_P^2}\,. \qquad (20)$$

Notice that f is given by Eq. (14) and so $\tilde{f}$ does not depend on the scale M. Since f is small, the effect of the masses may be approximated through the modification of the

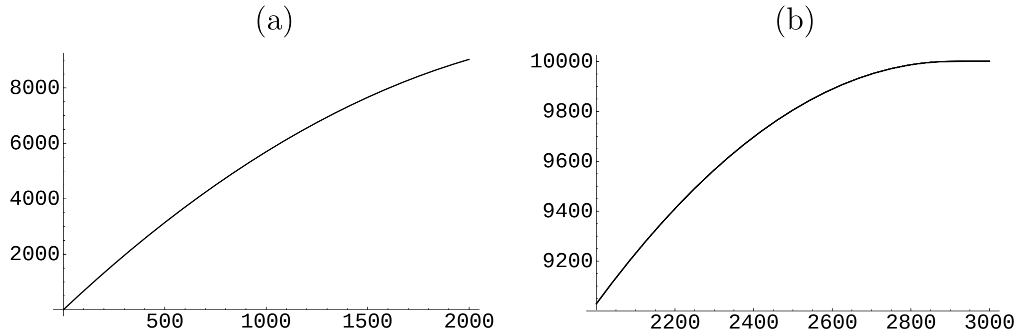

Figure 1. ***(a)*** *Plot of $\sigma = \ln(a)$ versus the physical time t as a result of the numerical analysis of Eq.(19); t is given in units of $16\,\pi/M_P$ and we fixed the parameter (20) as $\tilde{f} = 10^{-4}$. Initial data: $a(0) = 1\,, \dot{a}(0) = H_1\,, \ddot{a}(0) = H_1^2\,, \dddot{a}(0) = H_1^3$. In this time interval, inflation does not stop, yet;* ***(b)*** *As in (a), but extending the numerical analysis until reaching an approximate plateau marking the end of stable inflation.*

inflation law $a(t) = a_0\, e^{H_1 t}$ according to [1]

$$H_1 = \frac{M_P}{\sqrt{-16\pi b}} \;\longrightarrow\; \frac{M_P}{\sqrt{-16\pi b}} \left[1 - \tilde{f}\, \ln a(t)\right]^{1/2} = H(t)\,, \tag{21}$$

To substantiate our claim, we have solved Eq. (19) directly using the numerical methods. The plots corresponding to the numerical solution of the Eq. (19) are shown in Fig. 1. Since in the first period of inflation masses do not play much role and the stabilization of the exponential inflation proceeds very fast, the initial data (in both Eq. (21) and the plots of Fig. 1) were chosen according to the exponential inflation law. According to the numerical analysis, the total number of e-folds in the "fast phase" of inflation (until the Hubble constant becomes comparable to the transition scale M^* where instability develops) is about 10^4 for our particular values of the parameters, and at the last stage the expansion essentially slows down.

The chosen value of the parameter (20) $\tilde{f} = 10^{-4}$ in the plot is, as we warned before, independent of the scale M, and it determines where the process of stable inflation finishes as well as the number of e-folds. On the other hand, if the scale M^* is chosen near the typical SUSY GUT value $M_{SUSY} \sim M_X \approx 10^{16}\,GeV$ for supersymmetry breaking at high energies, some of the spinor masses m_i will be of order $M_{SUSY} \approx 10^{16}\,GeV$. The latter assumption is indeed sound because the m_i will include the supersymmetric fermions associated to the super-heavy gauge and Higgs bosons at the GUT scale, and so $m_i \sim M_X \sim M_{SUSY}$. From Eq. (20) it follows that the parameter $\tilde{f}$ will be numerically smaller than the one we have assumed in Fig. 1, and consequently the amount of inflation will

[1]We remind the reader that the coefficient b is negative for any particle content, see (13).

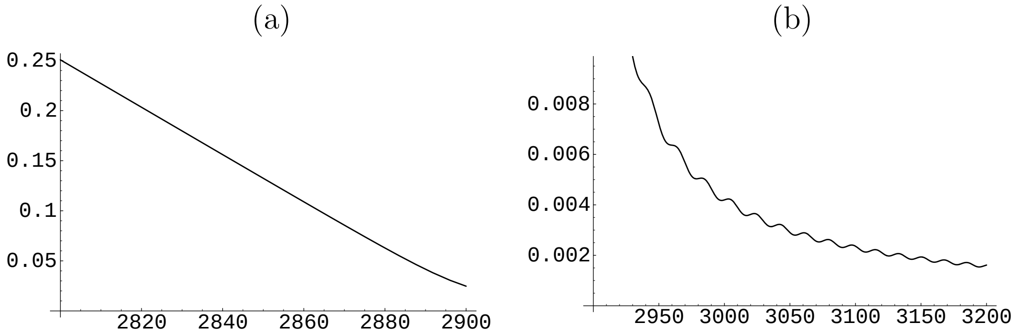

Figure 2. *Plot of $H(t) = \dot{a}(t)/a(t)$ versus t as a result of the numerical analysis of Eq.(19) and parameter values as in Fig.1:* ***(a)*** *$H(t)$ near the onset of the plateau;* ***(b)*** *$H(t)$ well over the plateau.*

be larger. But the important qualitative point is that for any value of $\tilde{f}$ the approximate plateau eventually appears and signals the end of stable inflation. Also notice from Fig. 1 that the initial evolution is close to the exponential inflation, but after that the expansion slows down due to the quantum effects of massive fermions.

4. Graceful exit from anomaly-induced inflation

Recall from Eq.(5) that $H(t)$ sets the scale of the RG running for the gravitational part. So if we consider the SUSY breaking and the corresponding change in the number of active degrees of freedom, then the necessary and sufficient condition for the applicability of our approach is that $H(t)$ decreases from the initial value about[2] $M_P/\sqrt{-16\pi b} \sim 10^{18}\,GeV$, down to the lower scale $H = M^* \lesssim M_{SUSY}$. The outcome is that the evolution according to (21) lasts until reaching the scale M^*, and after that most of the SUSY particles are decoupled, the inflationary solution becomes unstable and the FLRW phase can start. In fact, the crucial point is the existence of a nonvanishing f as it eventually tempers stable inflation allowing favorable conditions for the universe to tilt into the FLRW phase.

To justify our claim that $M^* < M_{SUSY}$, recall that for a really successful exit from the inflationary phase we need to make sure that the amplitude of the gravitational waves is consistent with the observable range of anisotropy in the CMBR. This will be the case if during the last 65 e-folds of the inflation, the expansion rate $H(t)$ does not exceed $10^{-5}M_P$. Then the fluctuations in the amplitude h of these waves, $\delta h/h = H/M_P$, will preserve the measured fluctuations in the temperature of the relic radiation according to the relation $\delta h/h = \delta T/T = \mathcal{O}(10^{-5})$. At the lowest end of the inflation interval this condition corresponds, in our framework, to fix the instability value $M_* \approx 10^{-5}M_P = 10^{14}GeV$. It means that, in reality, we expect that after the onset of the approximate plateau in Fig. 1 (b), where SUSY breaking occurs, the universe will take a while before entering the FLRW phase, i.e. the latter will actually initiate at some point well over the plateau

[2]Notice that $|16\pi b| = \mathcal{O}(1)$ in the MSSM, and it is much larger than 1 in any typical SUSY GUT.

where $H = M_* \sim 10^{-5} M_P$. To better assess this issue we have numerically analyzed $H(t)$ over the plateau, see Fig. 2. We see that $H(t)$ decreases very fast on it. For instance, from the comparison of Fig. 2(a) and Fig. 2(b), we find that a 15% increase of the time after the onset of the plateau amounts $H(t)$ to diminish two orders of magnitude. So in general $H(t)$ will decrease further below M_{SUSY}, and the difference between M^* and M_{SUSY} at the moment of the transition can be significant, say one or two orders of magnitude. Hence M_{SUSY} can be $10^{16}\,GeV$ and this does not create problems with the CMBR. Let us also notice the oscillatory behavior of $H(t)$ after reaching the plateau, i.e. when the system is about jumping into the FLRW phase. Interestingly enough, this behavior could perhaps be related to the reheating phenomenon, which is of course indispensable before the universe stabilizes in the FLRW regime.

Overall, we arrive at a consistent picture of the graceful exit in this inflationary scenario. No inflaton field is needed, but only the dynamical work of gravity itself, provided one starts from a renormalizable, fully conformal-invariant classical picture, together with the trace anomaly of the matter fields at the quantum level. Moreover, according to (14), the obtained picture is universal, for it does not depend on the choice of the dilatation symmetry breaking scale M. If interpreted physically, one can put constraints on M using the macroscopic forces mediated by the field σ, demanding that this forces should have the sub-millimeter range, similarly as in [20].

Acknowledgments. The work of J.S. has been supported in part by MECYT and FEDER under project FPA2001-3598. The work of I.Sh. has been supported by the research grant from FAPEMIG and fellowship from CNPq.

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