SO(10) GUTs in Higher Dimensions*

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Abstract

We discuss supersymmetic SO(10) unified theories constructed in 6 dimensions. The breaking of SO(10) group is achieved by the orbifold compactification. In 4 dimensions we obtain a $\mathcal{N} = 1$ supersymmetric standard model which gauge group is enlarged by an additional U(1) symmetry. This unbroken gauge group is obtained as intersection of the Pati-Salam and the Georgi-Glashow subgroups of SO(10). The doublet-triplet splitting in Higgs multiplet arises as in SU(5) models in 5 dimensions.

*This talk is based on the works with W. Buchmüller and L. Covi [1].

The supersymmetric grand unified theory (GUT) is an attractive physics beyond the standard model. The simplest GUT to unify electroweak and strong interactions is based on the SU(5) gauge group [2]. With the present experimental evidence for neutrino masses and mixings the larger gauge group SO(10) [3] appears particularly attractive, since it unifies one family of quarks and leptons, including the right-handed neutrino, in a single irreducible representation.

However, we have to overcome several issues in the construction of a successful GUT model. The breaking of GUT gauge groups is in general rather involved and requires often large Higgs representations. In particular, the serious problem is how to achieve the mass splitting between the weak doublet and the color triplet Higgs fields. An attractive solution to these issues has been suggested by Kawamura [4] in the case of SU(5) GUT in 5 dimensions (5d). Compactification on the orbifold $S^1/(\mathcal{Z}_2 \times \mathcal{Z}_2')$ offers a simple and elegant explanation of the SU(5) breaking into the standard model gauge group G_{SM}= SU(3)×SU(2)×U(1) as well as the wanted doublet-triplet splitting. Various aspects of such SU(5) GUTs in 5d have been extensively studied [5].

In this talk we would like to discuss the breaking of the SO(10) GUT group by the orbifold compactification. In contrast to the SU(5) case the SO(10) breaking is not straightforward. In fact, an ideal breaking pattern of SO(10) into the standard gauge group G_{SM} cannot be obtained by the orbifold boundary conditions (of inner automorphism). This is because the orbifold breaking does not reduce the rank of the gauge group and also because G_{SM} is not a symmetric subgroup of SO(10).[1] However, it is remarkable that the extended standard model group, $G_{SM'}$=G_{SM}×U(1), is the maximal common subgroup of two symmetric subgroups of SO(10), say G_{PS} = SU(4) × SU(2) × SU(2) [7] and G_{GG} = SU(5) × U(1) (cf. Figure 1). This suggests that the SO(10) breaking into $G_{SM'}$ (rather than G_{SM}) can be realized by starting from the six dimensional theory and by orbifolding to the two different subgroups in the two orthogonal compact dimensions.[2] This is the reason why SO(10) GUTs are constructed in 6d [1, 9].

Let us consider a SO(10) Yang-Mills theory in 6d with $\mathcal{N} = 1$ supersymmetry (SUSY). The gauge multiplet consists of gauge fields A_M $(M = 0, 1, 2, 3, 5, 6)$ and two gauginos λ_1 and λ_2. In the usual 4d superspace, they correspond to one vector superfield $V = (A_\mu, \lambda_1)$ and one chiral superfield $\Sigma = (A_{5,6}, \lambda_2)$. Here $\mu = 0, 1, 2, 3$, $V = V^a T^a$ and $\Sigma = \Sigma^a T^a$ with SO(10) generators T^a. The action for the gauge multiplet is given by [10]

$$S_{6d} \;=\; \int d^6x \; \Big\{ \mathrm{Tr} \Big[\int d^2\theta \; \frac{1}{4k} W^\alpha W_\alpha + h.c. \Big]$$

[1]For a general discussion of the orbifold breaking, see, *e.g.,* Ref. [6].

[2]SO(10) breakings into G_{PS} and G_{GG} can be done similar to the SU(5) model in 5d. See, *e.g.*, Ref. [8].

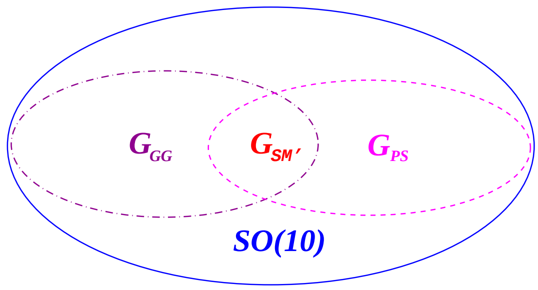

Figure 1: The extended standard model gauge group $G_{SM'}$ =SU(3)× SU(2)×U(1)^2 as intersection of the two symmetric subgroups of SO(10), G_{PS} =SU(4)×SU(2)×SU(2) and G_{GG} =SU(5)×U(1).

$$+ \ \mathrm{Tr}\left[\int d^4\theta \ \frac{1}{2kg^2}\left((-\overline{\partial}+\sqrt{2}g\overline{\Sigma})e^{2gV}(\partial+\sqrt{2}g\Sigma)e^{-2gV}+\frac{1}{2}\overline{\partial}e^{2gV}\partial e^{-2gV}\right)\right]\Bigg\} \, , \quad (1)$$

where tr $\left(T^aT^b\right) = k\delta^{ab}$, $\partial = \partial_5 - i\partial_6$ and $\overline{\partial} = \partial_5 + i\partial_6$.

The breaking of SO(10) gauge symmetry is achieved by compactifying the extra two dimensions on the orbifold $T^2/(\mathcal{Z}_2^O \times \mathcal{Z}_2^{PS} \times \mathcal{Z}_2^{GG})$. These discrete symmetries are reflection symmetries of the extra coordinates $y = (x_5, x_6)$ with respect to the points $y_O = (0,0)$, $y_{PS} = (\pi R_5/2, 0)$ and $y_{GG} = (0, \pi R_6/2)$, respectively. Here R_5 and R_6 denote the compactification radii. They are singular fixpoints on T^2. Further, there appears the fourth fixpoint at $y_{fl} = (\pi R_5/2, \pi R_6/2)$ [9]. The boundary conditions associated with the $\mathcal{Z}_2$ symmetries break SO(10) into its subgroups, *i.e.*, SO(10), G_{PS}, G_{GG} and $G_{fl} = SU(5)' \times U(1)'$, at the four fixpoints y_O, y_{PS}, y_{GG} and y_{fl}, respectively. In practice, the party transformations of the gauge multiplet induce these gauge symmetry breakings. For example, the transformations of the vector superfield V are

$$P_O \, V(x, \ y_O - y) \, P_O^{-1} = \eta_O \, V(x, y_O + y) \, , \quad (2)$$
$$P_{PS} \, V(x, \ y_{PS} - y) \, P_{PS}^{-1} = \eta_{PS} \, V(x, \ y_{PS} + y) \, , \quad (3)$$
$$P_{GG} \, V(x, \ y_{GG} - y) \, P_{GG}^{-1} = \eta_{GG} \, V(x, \ y_{GG} + y) \, , \quad (4)$$
$$P_{fl} \, V(x, \ y_{fl} - y) \, P_{fl}^{-1} = \eta_{fl} \, V(x, \ y_{fl} + y) \, . \quad (5)$$

Those of the chiral superfield Σ are obtained by replacing V by Σ. Here the parity matrices P_α ($\alpha = O, PS, GG$ and fl) in the vector representation are taken as $P_O = I$ (I

is a 10×10 unit matrix),

$$P_{PS} = \begin{pmatrix} -\sigma_0 & 0 & 0 & 0 & 0 \\ 0 & -\sigma_0 & 0 & 0 & 0 \\ 0 & 0 & -\sigma_0 & 0 & 0 \\ 0 & 0 & 0 & +\sigma_0 & 0 \\ 0 & 0 & 0 & 0 & +\sigma_0 \end{pmatrix}, \quad P_{GG} = \begin{pmatrix} \sigma_2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_2 \end{pmatrix}, \quad (6)$$

and $P_{fl} = P_{PS}P_{GG}$, where σ_0 is a 2×2 unit matrix and σ_i $(i = 1, 2, 3)$ denote the Pauli matrices.

In Eqs. (2)–(5) the eigenvalues η_α must be ± 1 and $\eta_{fl} = \eta_{PS}\eta_{GG}$. The action in Eq. (1) requires eigenvalues of V^a and Σ^a to be opposite. We choose $\eta_\alpha = +1$ for V and $\eta_\alpha = -1$ for Σ for all the parity transformations. The naive dimensional reduction of the 6d theory with $\mathcal{N} = 1$ SUSY results in the 4d theory with $\mathcal{N} = 2$ SUSY. Our choice of eigenvalues breaks this unwanted extended SUSY in 4d, while keeps the minimal SUSY in 4d being unbroken for the stabilization of the gauge hierarchy between the GUT scale and the electroweak scale.

From the viewpoint of the effective 4d theory, each field appears together with the infinite tower of the Kaluza-Klein states. Their masses are determined by the parities and only fields for which all parities are positive $+1$ have massless zero modes. We can see from Table 1 that zero modes form a vector superfield of the gauge group $G_{SM'} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)_X$. Therefore, as a low energy theory, we obtain the minimal SUSY standard model which gauge group is enlarged by an additional $\mathrm{U}(1)_X$.

In Figure 2 we show the fixpoints on T^2. The physical region is obtained by the folding the shaded region along the dotted line and gluing the edges. The result is a "pillow" with the four fixpoints, O, O_{PS}, O_{GG} and O_{fl}, as corners, at which the gauge groups SO(10), G_{PS}, G_{GG} and G_{fl} are manifest, respectively. The unbroken gauge group $G_{SM'}$ of the effective 4d theory is obtained as intersection of these SO(10) subgroups, SO(10), G_{PS}, G_{GG} and G_{fl}.

We should stress that the orbifold $T^2/(\mathcal{Z}_2^O \times \mathcal{Z}_2^{PS} \times \mathcal{Z}_2^{GG})$ has the periodicity of πR_5 and πR_6 in the extra coordinates, although we start from the T^2 with the radii R_5 and R_6. It is found that the successive operations $\mathcal{Z}_2^O \times \mathcal{Z}_2^{PS}$ and $\mathcal{Z}_2^O \times \mathcal{Z}_2^{GG}$ generate the translations as $\mathcal{Z}_2^O \times \mathcal{Z}_2^{PS} : (x_5, x_6) \to (x_5 + \pi R_5, x_6)$ and $\mathcal{Z}_2^O \times \mathcal{Z}_2^{GG} : (x_5, x_6) \to (x_5, x_6 + \pi R_6)$. This shows that the considering model can be constructed also on the orbifold $\widetilde{T}^2/\mathcal{Z}_2^O$ where $\widetilde{T}^2$ denotes the two torus with the radii $R_5/2$ and $R_6/2$. (See Ref. [9].)

One of the great achievements in GUT constructed on the orbifold is the elegant

			$V^a(A_\mu,\lambda_1)$			$\Sigma^a(A_{5,6},\lambda_2)$		
$G_{SM'}$	G_{PS}	G_{GG}	$\mathcal{Z}_2^O$	$\mathcal{Z}_2^{PS}$	$\mathcal{Z}_2^{GG}$	$\mathcal{Z}_2^O$	$\mathcal{Z}_2^{PS}$	$\mathcal{Z}_2^{GG}$
$(\mathbf{8},\mathbf{1})_{0,0}$	$(\mathbf{15},\mathbf{1},\mathbf{1})$	$\mathbf{24}_0$	+	+	+	−	−	−
$(\mathbf{1},\mathbf{3})_{0,0}$	$(\mathbf{1},\mathbf{3},\mathbf{1})$	$\mathbf{24}_0$	+	+	+	−	−	−
$(\mathbf{1},\mathbf{1})_{0,0}$	$(\mathbf{1},\mathbf{1},\mathbf{3})$	$\mathbf{24}_0$	+	+	+	−	−	−
$(\mathbf{1},\mathbf{1})_{0,0}$	$(\mathbf{15},\mathbf{1},\mathbf{1})$	$\mathbf{1}_0$	+	+	+	−	−	−
$(\mathbf{3}^*,\mathbf{1})_{-\frac{2}{3},4}$	$(\mathbf{15},\mathbf{1},\mathbf{1})$	$\mathbf{10}_4$	+	+	−	−	−	+
$(\mathbf{3},\mathbf{1})_{\frac{2}{3},-4}$	$(\mathbf{15},\mathbf{1},\mathbf{1})$	$\mathbf{10}^*_{-4}$	+	+	−	−	−	+
$(\mathbf{1},\mathbf{1})_{1,4}$	$(\mathbf{1},\mathbf{1},\mathbf{3})$	$\mathbf{10}_4$	+	+	−	−	−	+
$(\mathbf{1},\mathbf{1})_{-1,-4}$	$(\mathbf{1},\mathbf{1},\mathbf{3})$	$\mathbf{10}^*_{-4}$	+	+	−	−	−	+
$(\mathbf{3},\mathbf{2})_{-\frac{5}{6},0}$	$(\mathbf{6},\mathbf{2},\mathbf{2})$	$\mathbf{24}_0$	+	−	+	−	+	−
$(\mathbf{3}^*,\mathbf{2})_{\frac{5}{6},0}$	$(\mathbf{6},\mathbf{2},\mathbf{2})$	$\mathbf{24}_0$	+	−	+	−	+	−
$(\mathbf{3},\mathbf{2})_{\frac{1}{6},4}$	$(\mathbf{6},\mathbf{2},\mathbf{2})$	$\mathbf{10}_4$	+	−	−	−	+	+
$(\mathbf{3}^*,\mathbf{2})_{-\frac{1}{6},-4}$	$(\mathbf{6},\mathbf{2},\mathbf{2})$	$\mathbf{10}^*_{-4}$	+	−	−	−	+	+

Table 1: Parity assignment for the bulk gauge multiplet.

realization of the mass splitting between the weak doublet and the color triplet Higgs fields, as illustrated by Kawamura using the SU(5) model [4]. This attractive mechanism can work well in the considering model. We introduce two Higgs hypermultiplets in the 6d bulk, which are **10**-plets of SO(10). A hypermultiplet $\mathcal{H}^{6d}$ in 6d can be grouped into two chiral superfields $(\mathcal{H},\mathcal{H}')$ in the 4d superspace. Note that $\mathcal{H}$ and $\mathcal{H}'$ transform as complex conjugates of each other under the gauge group. Hereafter we refer the representation of $\mathcal{H}^{6d}$ to that of $\mathcal{H}$. The bulk action for $\mathcal{H}$ and $\mathcal{H}'$ is given by

$$\begin{aligned} S &= \int d^6x \left\{ \int d^4\theta \left(\overline{\mathcal{H}} e^{2gV}\mathcal{H} + \mathcal{H}' e^{-2gV}\overline{\mathcal{H}}' \right) \right. \\ &\qquad \left. + \left[\int d^2\theta\ \mathcal{H}'(\ \partial + \sqrt{2} g\Sigma\ )\mathcal{H} + h.c. \right] \right\} . \end{aligned} \tag{7}$$

The doublet-triplet splitting is realized by the orbifold boundary conditions since it can project out some components as zero modes from the original SO(10) multiplet. Similar to the gauge multiplet, the parity transformations of $\mathcal{H}$ under the discrete symmetries are given by

$$P_O\ \mathcal{H}(x,\ y_O - y) = \eta_O\ \mathcal{H}(x, y_O + y)\ , \tag{8}$$
$$P_{PS}\ \mathcal{H}(x,\ y_{PS} - y) = \eta_{PS}\ \mathcal{H}(x,\ y_{PS} + y)\ , \tag{9}$$
$$P_{GG}\ \mathcal{H}(x,\ y_{GG} - y) = \eta_{GG}\ \mathcal{H}(x,\ y_{GG} + y)\ , \tag{10}$$
$$P_{fl}\ \mathcal{H}(x,\ y_{fl} - y) = \eta_{fl}\ \mathcal{H}(x,\ y_{fl} + y)\ , \tag{11}$$

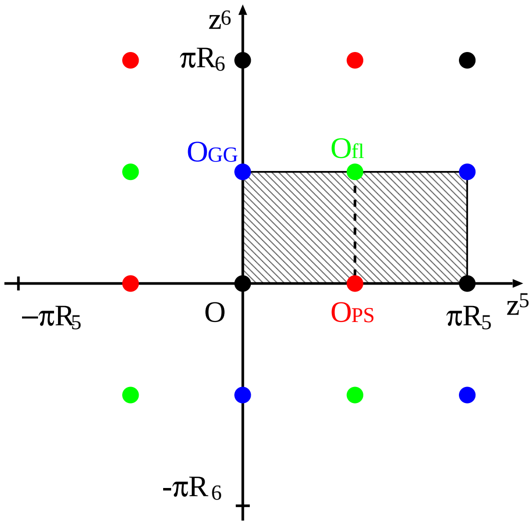

Figure 2: Fixpoints on T^2. Four different fixpoints are denoted by O, O_{PS}, O_{GG} and O_{fl}. Physical region is shown by the shaded region.

and those of $\mathcal{H}$ are obtained by replacing $\mathcal{H}$ by $\mathcal{H}'$. The action in Eq. (7) demands eigenvalues η_α of $\mathcal{H}$ and $\mathcal{H}'$ are opposite. From now on we denote by η_α the parities of $\mathcal{H}$. We choose $\eta_O = +1$, which means that only one chiral superfield $\mathcal{H}$ contains the zero mode and survives in the effective 4d theory. In Table 2 we show the parity assignment of the two Higgs hypermultiplets $H_1^{6d} = (H_1, H_1')$ and $H_2^{6d} = (H_2, H_2')$. It is found that the $\mathcal{Z}_2^{PS}$ symmetry ensures the splitting between the SU(2) weak-doublets and the SU(3) color-triplets since it distinguishes between $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ and $(\mathbf{6}, \mathbf{1}, \mathbf{1})$ components under G_{PS}. The choice of $\eta_{PS} = +1$ leads to massless weak-doublets and massive color-triplets in H_1 and H_2. Further, we take $\eta_{GG} = +1$ for H_1 and $\eta_{GG} = -1$ for H_2, which means the weak doublet comes from the SU(5) $\mathbf{5}^*$-plet for H_1, while from the SU(5) $\mathbf{5}$-plet for H_2. Therefore, the two Higgs doublets H_d and H_u in the minimal SUSY standard model can be obtained as the zero modes in the two hypermultiplets H_1^{6d} and H_2^{6d}, respectively. It should be noted that two $\mathbf{10}$-plets are crucial for the bulk anomaly cancellation. In fact, the irreducible SO(10) anomalies in 6d do cancel between the gaugino and these two Higgs $\mathbf{10}$-plets. The detail discussion of the bulk and brane anomalies in 6 dimension is found in Ref. [11].

In this talk we had discussed the breaking of the SO(10) GUT gauge group in 6d. Unfortunately, the additional $\mathrm{U}(1)_X$ in $G_{SM'}$ still survives after the orbifold compactification. The unbroken electroweak and $\mathrm{U}(1)_{B-L}$ gauge symmetries forbid masses for all fermions

SO(10)	**10**							
G_{SM}	$(\mathbf{1},\mathbf{2})_{-1/2}$		$(\mathbf{1},\mathbf{2})_{+1/2}$		$(\mathbf{3}^*,\mathbf{1})_{+1/3}$		$(\mathbf{3},\mathbf{1})_{-1/3}$	
G_{PS}	$(\mathbf{1},\mathbf{2},\mathbf{2})$		$(\mathbf{1},\mathbf{2},\mathbf{2})$		$(\mathbf{6},\mathbf{1},\mathbf{1})$		$(\mathbf{6},\mathbf{1},\mathbf{1})$	
G_{GG}	$\mathbf{5}^*{}_{-2}$		$\mathbf{5}_{+2}$		$\mathbf{5}^*{}_{-2}$		$\mathbf{5}_{+2}$	
	H^c		H		G^c		G	
	$\mathcal{Z}_2^{PS}$	$\mathcal{Z}_2^{GG}$	$\mathcal{Z}_2^{PS}$	$\mathcal{Z}_2^{GG}$	$\mathcal{Z}_2^{PS}$	$\mathcal{Z}_2^{GG}$	$\mathcal{Z}_2^{PS}$	$\mathcal{Z}_2^{GG}$
H_1	+	+	+	−	−	+	−	−
H_2	+	−	+	+	−	−	−	+

Table 2: Parity assignment for the bulk **10**-plets.

including the right-handed neutrinos, and their breakings are related to the generation of fermion masses. Some models are found in the literature [9, 12, 13, 14].

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