Grand Unification
and
Time Variation of the Gauge Couplings

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Plan

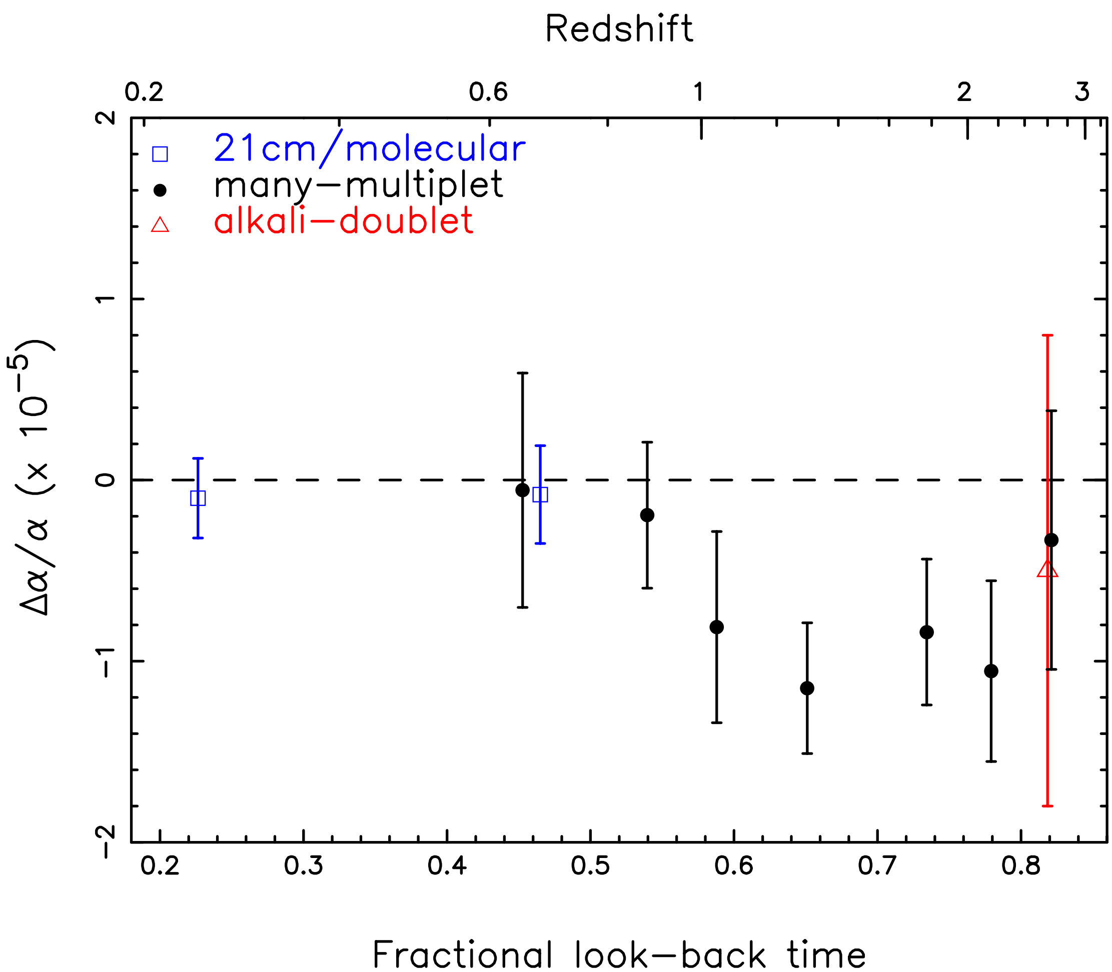
- Introduction / Experimental data
- Time Variation of GUT's parameters
- Tests in Quantum Optics
- Conclusions

Experimental data

- We have an indication that α might be time dependent. redshift $z \approx 0.5 \ldots 3.5$:

$$\Delta\alpha/\alpha = (-0.72 \pm 0.18) \times 10^{-5},$$

J. K. Webb *et al.*, Phys. Rev. Lett. **87** 091301 (2001).



– many multiplet and alkali-doublet (40 objects): measurement of the fine structure constant $\propto \alpha^2$
– 21 cm /molecular: measurement of the hyperfine structure constant $\propto g_p\alpha^2$
be careful QCD enters the game!!!

- That's the main message of our work: be careful with the interpretation of the data.

- Constraints:
  a) Oklo in Gabon (2 billion years ago i.e. $z \sim 0.1$), limit $\dot{\alpha}/\alpha = (-0.2 \pm 0.8) \times 10^{-17}$/yr, T. Damour and F. Dyson, Nucl. Phys. B **480**, 37 (1996). BUT WATCH OUT: QCD enters the game!!
  b) Direct laboratory measurements provide the constraint:
  $$\left|\frac{\dot{\alpha}}{\alpha}\right| \leq 3.7 \times 10^{-14} \text{ yr}^{-1}$$
  J. D. Prestage *et al.* Phys. Rev. Lett. **74** 3511 (1995).
  c) Cosmic Microwave Background/Large Scale Structure: at 1 σ compatibility with a variation of α but no signal at 2 σ $\rightarrow$ too early to call!!! Martins *et al.* astro-ph/0203149
- There is another indication that fundamental parameters are time dependent.
  Define $\mu = M_p/m_e$. Measuring the vibrational lines of H_2, a small effect was seen. The data allows two different interpretations:
  a) $\Delta\mu/\mu = (5.7 \pm 3.8) \times 10^{-5}$
  b) $\Delta\mu/\mu = (12.5 \pm 4.5) \times 10^{-5}$.
  A. V. Ivanchik *et al.* astro-ph/0112323, depending on which of the 2 available tables of standard laboratory wavelengths is used.

- If such an effect is confirmed, that would be the sign that something weird/unpredictable is going on.
- We must make a minimal number of assumptions about the physics responsible for such a time variation, test these assumptions and be ready to drop them.
- If α is time-dependent, other parameters will probably be time-dependent too.
- We can try to make predictions to test Webb $et\ al.$´ s result in other sectors.
- The Standard Model (SM) has too many parameters to allow an useful description of the variation of fundamental parameters: 18 parameters for massless neutrinos $\rightarrow$ more natural framework are GUTs.

List of assumptions

- SM is embedded into a GUT (reduce the number of scales/parameters).
- Unification is taking place at all time.
- Physics responsible for the time evolution of the parameters is acting above the GUT scale.
  - reasonable if the effect is connected to gravity.
- We have 2 time-dependent parameters α_u and Λ_G
- Yukawa couplings are time-independent (at Λ_G).
- low energy Higgs Bosons vacuum expectation values are time-independent (at Λ_G).
- Webb *et al.* measurement is correct!!!!

Consequences

- renormalization group equations (RGEs) can be used. New physics only affects the initial conditions of the RGEs. We are sure that QFT works below the GUT scale.
- Time translation invariance is lost, but it is anyway the case because of the Big Bang $\rightarrow$ time dependence of "constants" more probable than space dependence.

Time Variations in SUSY SU(5) GUT

- 1 loop RGEs

$$\begin{aligned}\alpha_i(\mu)^{-1} &= \left(\frac{1}{\alpha_i^0(\Lambda_G)} + \frac{1}{2\pi} b_i^S \ln\left(\frac{\Lambda_G}{\mu}\right)\right)\theta(\mu - \Lambda_S) \\ &\quad + \left(\frac{1}{\alpha_i^0(\Lambda_S)} + \frac{1}{2\pi} b_i^{SM} \ln\left(\frac{\Lambda_S}{\mu}\right)\right)\theta(\Lambda_S - \mu),\end{aligned}$$

with $b_i^{SM} = (b_1^{SM}, b_2^{SM}, b_3^{SM}) = (41/10, -19/6, -7)$ below the SUSY scale Λ_S and $b_i^S = (b_1^S, b_2^S, b_3^S) = (33/5, 1, -3)$ when $\mathcal{N} = 1$ supersymmetry is restored, and with

$$\frac{1}{\alpha_i^0(\Lambda_S)} = \frac{1}{\alpha_i^0(M_Z)} + \frac{1}{2\pi} b_i^{SM} \ln\left(\frac{M_Z}{\Lambda_S}\right).$$

- We adopt the usual definition

$$\begin{aligned}\alpha_1 &= 5/3 g_1^2/(4\pi) = 5\alpha/(3\cos^2(\theta)_{\overline{\mathsf{MS}}}) \\ \alpha_2 &= g_2^2/(4\pi) = \alpha/\sin^2(\theta)_{\overline{\mathsf{MS}}} \\ \alpha_s &= g_3^2/(4\pi).\end{aligned}$$

- Assuming $\alpha_u = \alpha_u(t)$ and $\Lambda_G = \Lambda_G(t)$, one finds:

$$\frac{1}{\alpha_i}\frac{\dot{\alpha}_i}{\alpha_i} = \left[\frac{1}{\alpha_u}\frac{\dot{\alpha}_u}{\alpha_u} - \frac{b_i^S}{2\pi}\frac{\dot{\Lambda}_G}{\Lambda_G}\right]$$

which leads to

$$\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} = \frac{8}{3}\frac{1}{\alpha_s}\frac{\dot{\alpha}_s}{\alpha_s} - \frac{1}{2\pi}\left(b_2^S + \frac{5}{3}b_1^S - \frac{8}{3}b_3^S\right)\frac{\dot{\Lambda}_G}{\Lambda_G}.$$

One may consider different scenarios:

- Λ_G invariant, $\alpha_u = \alpha_u(t)$

$$\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} = \frac{8}{3}\frac{1}{\alpha_s}\frac{\dot{\alpha}_s}{\alpha_s}$$

and

$$\frac{\dot{\Lambda}}{\Lambda} = -\frac{3}{8}\frac{2\pi}{b_3^{SM}}\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} = R\frac{\dot{\alpha}}{\alpha}$$

i.e. there is no dependence on Λ_S. If we calculate $\dot{\Lambda}/\Lambda$ using the relation above in the case of 6 quark flavors, neglecting the masses of the quarks, we find $R \approx 46$. A more precise calculation yields $R = 37.7 \pm 2.3$. This case is not very interesting for quantum optics because of Oklo constraint.

- α_u invariant, $\Lambda_G = \Lambda_G(t)$. One finds

$$\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} = -\frac{1}{2\pi}\left(b_2^S + \frac{5}{3}b_1^S\right)\frac{\dot{\Lambda}_G}{\Lambda_G},$$

with

$$\Lambda_G = \Lambda_S\left[\frac{\Lambda}{\Lambda_S}\exp\left(-\frac{2\pi}{b_3^{SM}}\frac{1}{\alpha_u}\right)\right]^{\left(\frac{b_3^{SM}}{b_3^S}\right)}$$

which follows from the extraction of the Landau pole. One obtains

$$\frac{\dot{\Lambda}}{\Lambda} = \frac{b_3^S}{b_3^{SM}}\left[\frac{-2\pi}{b_2^S + \frac{5}{3}b_1^S}\right]\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} \approx -30.8\frac{\dot{\alpha}}{\alpha}$$

- $\alpha_u = \alpha_u(t)$ and $\Lambda_G = \Lambda_G(t)$. One has

$$\begin{aligned}
\frac{\dot{\Lambda}}{\Lambda} &= -\frac{2\pi}{b_3^S}\frac{1}{\alpha_u}\frac{\dot{\alpha_u}}{\alpha_u} + \frac{b_3^S}{b_3^{SM}}\frac{\dot{\Lambda}_G}{\Lambda_G} \\
&= -\frac{3}{8}\frac{2\pi}{b_3^{SM}}\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} - \frac{3}{8}\frac{1}{b_3^{SM}}\left(b_2^S + \frac{5}{3}b_1^S - \frac{8}{3}b_3^S\right)\frac{\dot{\Lambda}_G}{\Lambda_G} \\
&= 46\frac{\dot{\alpha}}{\alpha} + 1.07\frac{\dot{\Lambda}_G}{\Lambda_G}
\end{aligned}$$

- In a grand unified theory, the GUT scale and the unified coupling constant may be related to each other via the Planck scale e.g.

$$\frac{1}{\alpha_u} = \frac{1}{\alpha_{Pl}} + \frac{b_G}{2\pi}\ln\left(\frac{\Lambda_{Pl}}{\Lambda_G}\right)$$

  where Λ_{Pl} is the Planck scale, α_{Pl} the value of the GUT group coupling constant at the Planck scale and b_G depends on the GUT group under consideration. This leads to

$$\frac{\dot{\Lambda}_G}{\Lambda_G} = \frac{2\pi}{b_G}\frac{1}{\alpha_u}\frac{\dot{\alpha}_u}{\alpha_u}$$

  and thus to

$$\frac{\dot{\Lambda}}{\Lambda} = \frac{2\pi}{-b_3^{SM}}\frac{b_G - b_3^S}{\frac{8}{3}b_G - b_2^S - \frac{5}{3}b_1^S}\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha}$$

  or

$$\frac{\dot{\alpha}}{\alpha} = \frac{-b_3^{SM}}{2\pi}\left(\frac{\frac{8}{3}b_G - b_2^S - \frac{5}{3}b_1^S}{b_G - b_3^S}\right)\alpha\frac{\dot{\Lambda}}{\Lambda}.$$

- Test of GUT theory/SUSY (in principle) possible!!!

- Finally, it should be mentioned that the scale of supersymmetry could also vary with time. One obtains:

$$\frac{1}{\alpha_i}\frac{\dot{\alpha}_i}{\alpha_i} = \left[\frac{1}{\alpha_u}\frac{\dot{\alpha}_u}{\alpha_u} - \frac{b_i^S}{2\pi}\frac{\dot{\Lambda}_G}{\Lambda_G}\right] + \frac{1}{2\pi}(b_i^S - b_i^{SM})\frac{\dot{\Lambda}_S}{\Lambda_S}\theta(\Lambda_S - \mu).$$

  However without a specific model for supersymmetry breaking relating the supersymmetry breaking scale to e.g. the GUT scale, this expression is not very useful.

- In particular if the physics generating a time variation of α was taking place between the GUT scale and the scale for supersymmetry breaking, our analysis might not be very reliable as there would then be no reason to assume that quantum field theory remains valid between these two scales.

- One case is of particular interest: the time variation of α is related to a time variation of the unification scale.

- the GUT scale could be related in specific models to vacuum expectation values of scalar fields. Since the universe expands, one might expect a decrease of the unification scale due to a dilution of the scalar field. A lowering of Λ_G implies according to

$$\frac{\dot{\alpha}}{\alpha} = -\frac{1}{2\pi}\alpha\left(b_2^S + \frac{5}{3}b_1^S\right)\frac{\dot{\Lambda}_G}{\Lambda_G} = -0.014\frac{\dot{\Lambda}_G}{\Lambda_G}.$$

  If $\dot{\Lambda}_G/\Lambda_G$ is negative, $\dot{\alpha}/\alpha$ increases in time, consistent with the experimental observation. Taking $\Delta\alpha/\alpha = -0.72 \times 10^{-5}$, we would conclude $\Delta\Lambda_G/\Lambda_G = 5.1 \times 10^{-4}$, i.e. the scale of grand unification about 8 billion years ago was about 8.3×10^{12} GeV higher than today.

- Monitoring the ratio $\mu = M_p/m_e$ could allow to see an effect. Measuring the vibrational lines of H_2, a small effect was seen recently. The data allow two different interpretations:

  a) $\Delta\mu/\mu = (5.7 \pm 3.8) \times 10^{-5}$

  b) $\Delta\mu/\mu = (12.5 \pm 4.5) \times 10^{-5}$.

  The interpretation b) agrees with the expectation based on the assumption that Λ_G is time dependent.

$$\frac{\Delta\mu}{\mu} = 22 \times 10^{-5}.$$

  It is interesting that the data suggest that μ is indeed decreasing, while α seems to increase. If confirmed, this would be a strong indication that the time variation of α at low energies is caused by a time variation of the unification scale.

- Unification based on $SO(10)$ without supersymmetry (Lavoura and Wolfenstein Phys.Rev.**D48**:264-269,1993). Varying the GUT scale, one finds:

$$\frac{\dot{\Lambda}}{\Lambda} = \left[\frac{-2\pi}{b_2^{SM} + \frac{5}{3}b_1^{SM}}\right] \frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} = -234.8\frac{\dot{\alpha}}{\alpha}.$$

  Taking tresholds into account, we get:

$$\frac{\dot{\Lambda}}{\Lambda} = \left[\frac{-2\pi}{b_2^{SM} + \frac{5}{3}b_1^{SM}}\right] \frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} = -290.8\frac{\dot{\alpha}}{\alpha}.$$

Tests in Quantum Optics

- New assumption $\dot{\alpha}/\alpha$ is constant. Linear extrapolation of Webb´ s results.
- Consider case with $\Lambda(t)$.
- If the rate of change is extrapolated linearly, Λ_G is decreasing at a rate $\frac{\dot{\Lambda}_G}{\Lambda_G} = -7 \times 10^{-14}$/yr.
- The magnetic moments of the proton μ_p as well of nuclei would increase according to

$$\frac{\dot{\mu}_p}{\mu_p} = 30.8 \frac{\dot{\alpha}}{\alpha} \approx 3.1 \times 10^{-14}/yr.$$

- The wavelength of the light emitted in hyperfine transitions, e.g. the ones used in the cesium clocks being proportional to $\alpha^4 m_e/\Lambda$ will vary in time like

$$\frac{\dot{\lambda}_{hf}}{\lambda_{hf}} = 4\frac{\dot{\alpha}}{\alpha} - \frac{\dot{\Lambda}}{\Lambda} \approx 3.5 \times 10^{-14}/\text{yr}$$

taking $\dot{\alpha}/\alpha \approx 1.0 \times 10^{-15}$/yr.

- The wavelength of the light emitted in atomic transitions varies like α^{-2}:

$$\frac{\dot{\lambda}_{at}}{\lambda_{at}} = -2\frac{\dot{\alpha}}{\alpha}.$$

- One has $\dot{\lambda}_{at}/\lambda_{at} \approx -2.0 \times 10^{-15}$/yr. A comparison gives:

$$\frac{\dot{\lambda}_{hf}/\lambda_{hf}}{\dot{\lambda}_{at}/\lambda_{at}} = -\frac{4\dot{\alpha}/\alpha - \dot{\Lambda}/\Lambda}{2\dot{\alpha}/\alpha} \approx -17.4.$$

- Study underway at MPQ (Haensch, Walther)

Conclusions

- It is important to test Webb´ s results in other sectors.
- Ideas of GUT can be tested without seeing any new particle.
- Depending on $\alpha(t)$, there could be an effect observable in quantum optics.
- Any prediction is strongly model dependent. It is crucial to test assumptions.
- Negative results in Quantum Optics would not rule out Webb´ s results.