Grand Unification and Time Variation of the Gauge Couplings

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Astrophysical indications that the fine structure constant is time dependent are discussed in the framework of grand unification models. A variation of the electromagnetic coupling constant could either be generated by a corresponding time variation of the unified coupling constant or by a time variation of the unification scale, or by both. The case in which the time variation of the electromagnetic coupling constant is caused by a time variation of the unification scale is of special interest. It is supported in addition by recent hints towards a time change of the proton-electron mass ratio. Possible implications for baryogenesis are discussed.

The study of a possible time variation of the fundamental parameters like for example the fine structure constant, has a long history that can be traced back to Dirac [1]. Some extensions of the standard model of particle physics and in particular models that couple gravity to the standard model, e.g. quintessence [2], require or at least allow a time dependence of the parameters of the model. Our motivation to study the implications of a time dependence of the fundamental parameters for particle physics is not that much of a theoretical origin but rather because there are indications [3–8] coming from different astrophysical measurements that the parameters of the standard model could be time dependent. We shall consider the implications of Webb *et al.*'s result which indicate a possible time dependence of the fine structure constant α and make predictions that could be tested in other measurements. If interpreted in the simplest way, the data suggest that α was lower in the past:

$$\Delta\alpha/\alpha = (-0.72 \pm 0.18) \times 10^{-5} \tag{1}$$

for a redshift $z \approx 0.5 \ldots 3.5$ [3]. We note that since then the significance has increased $\Delta\alpha/\alpha = (-0.57 \pm 0.10) \times 10^{-5}$ [4–6]. An interpretation as a spatial dependence of fundamental parameters is also conceivable see e.g. [9].

It should be clear that Webb *et al.*'s result can only be tested in a given theoretical framework. Therefore a negative result in another sector cannot rule out that measurement, but can basically only rule out a set of assumptions made about the model.

It would be surprising if only α was time dependent and we thus expect that other parameters of the standard model should be time dependent too. Unfortunately the standard model has too many parameters which are uncorrelated. This implies a large number of time dependent functions and makes the discussion of time dependence of the fundamental constants in that framework not very efficient. We thus consider grand unified models where one has relations between the different parameters. We shall concentrate on SU(5) with $\mathcal{N} = 1$ supersymmetry and S0(10) broken directly to the standard

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model where unification is still possible due to thresholds effects [10]. The results are quite different which is of great theoretical interest since a possible time variation of the parameters could allow to test the ideas of grand unification.

We make the following assumptions:

- The standard model is embedded into a grand unified theory.
- Unification takes place at all time.
- The physics responsible for the time evolution of the parameters only affects the unified coupling constant α_u and the unification scale Λ_G.
- The Yukawa couplings are time-independent at Λ_G.
- The Higgs bosons vacuum expectation values are time-independent at Λ_G.

Our predictions only test the data coming from astrophysics together with this set of assumptions.

Assuming $\alpha_u = \alpha_u(t)$ and $\Lambda_G = \Lambda_G(t)$ and using the one loop renormalization group equations one finds:

$$\frac{1}{\alpha_i}\frac{\dot{\alpha}_i}{\alpha_i} = \left[\frac{1}{\alpha_u}\frac{\dot{\alpha}_u}{\alpha_u} - \frac{b_i}{2\pi}\frac{\dot{\Lambda}_G}{\Lambda_G}\right] \tag{2}$$

where $b_i^{SM} = (b_1^{SM}, b_2^{SM}, b_3^{SM}) = (41/10, -19/6, -7)$ are the coefficients of the renormalization group equations for the standard model and $b_i^S = (b_1^S, b_2^S, b_3^S) = (33/5, 1, -3)$ are the coefficients of the renormalization group equations in the $\mathcal{N} = 1$ supersymmetric case. This leads to

$$\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} = \frac{8}{3}\frac{1}{\alpha_s}\frac{\dot{\alpha}_s}{\alpha_s} - \frac{1}{2\pi}\left(b_2 + \frac{5}{3}b_1 - \frac{8}{3}b_3\right)\frac{\dot{\Lambda}_G}{\Lambda_G}. \tag{3}$$

We first consider the SU(5) supersymmetric case. One may consider different scenarios. We first keep Λ_G invariant and consider the case where $\alpha_u = \alpha_u(t)$ is time dependent. One gets [11]

$$\frac{\dot{\Lambda}}{\Lambda} = -\frac{3}{8}\frac{2\pi}{b_3^{SM}}\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} = R\frac{\dot{\alpha}}{\alpha}, \tag{4}$$

where Λ is the QCD scale. If we calculate $\dot{\Lambda}/\Lambda$ using the relation above in the case of 6 quark flavors, neglecting the masses of the quarks, we find $R \approx 46$. There are large theoretical uncertainties in R. Taking thresholds into account one gets $R = 37.7 \pm 2.3$ [11]. The uncertainty in R is given, according to $\Lambda = 213^{+38}_{-35}$MeV, by the uncertainty in the ratio α/α_s, which is dominated by the uncertainty in α_s.

We now consider the case where α_u is invariant, but $\Lambda_G = \Lambda_G(t)$ is time dependent. One gets [12]

$$\frac{\dot{\Lambda}}{\Lambda} = \frac{b_3^S}{b_3^{SM}}\left[\frac{-2\pi}{b_2^S + \frac{5}{3}b_1^S}\right]\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} \approx -30.8\frac{\dot{\alpha}}{\alpha}. \tag{5}$$

It is interesting to notice that the effects of a time variation of the unified coupling constant or of a time variation of the grand unified scale are going in opposite directions. Clearly those are two extreme cases and a time variation of both parameters is conceivable. Another possibility is dynamics between the grand unification scale and low energy physics [13]. In that case it is conceivable to have a time variation of α but no time variation in the QCD sector. Such an effect could also be achieved in our approach by a fine tuning of the parameters α_u and Λ_G.

In a grand unified theory, the grand unified scale and the unified coupling constant may be related to each other via the Planck scale e.g.

$$\frac{1}{\alpha_u} = \frac{1}{\alpha_{Pl}} + \frac{b_G}{2\pi} \ln\left(\frac{\Lambda_{Pl}}{\Lambda_G}\right) \tag{6}$$

where Λ_{Pl} is the Planck scale, α_{Pl} the value of the grand unified coupling constant at the Planck scale and b_G depends on the grand unified group under consideration. This leads to

$$\frac{\dot{\alpha}}{\alpha} = \frac{-b_3^{SM}}{2\pi} \left(\frac{\frac{8}{3}b_G - b_2^S - \frac{5}{3}b_1^S}{b_G - b_3^S}\right) \alpha \frac{\dot{\Lambda}}{\Lambda}, \tag{7}$$

in which case a test of the nature of the grand unified group is in principle possible. It should be mentioned that the scale of supersymmetry could also vary with time. One obtains:

$$\frac{1}{\alpha_i}\frac{\dot{\alpha}_i}{\alpha_i} = \left[\frac{1}{\alpha_u}\frac{\dot{\alpha}_u}{\alpha_u} - \frac{b_i^S}{2\pi}\frac{\dot{\Lambda}_G}{\Lambda_G}\right] + \frac{1}{2\pi}(b_i^S - b_i^{SM})\frac{\dot{\Lambda}_S}{\Lambda_S}\theta(\Lambda_S - \mu). \tag{8}$$

However without a specific model for supersymmetry breaking relating the supersymmetry breaking scale to e.g. the grand unified scale, this expression is not very useful since it only introduces a new unknown function in the discussion.

The case in which the time variation of α is related to a time variation of the unification scale is of particular interest. Λ_G could be related in specific models to vacuum expectation values of scalar fields. Since the universe expands, one might expect a decrease of the unification scale due to a dilution of the scalar field. A lowering of Λ_G implies according to (3):

$$\frac{\dot{\alpha}}{\alpha} = -\frac{1}{2\pi}\alpha\left(b_2^S + \frac{5}{3}b_1^S\right)\frac{\dot{\Lambda}_G}{\Lambda_G} = -0.014\frac{\dot{\Lambda}_G}{\Lambda_G}. \tag{9}$$

If $\dot{\Lambda}_G/\Lambda_G$ is negative, $\dot{\alpha}/\alpha$ increases in time. That is consistent with the experimental observation. Taking $\Delta\alpha/\alpha = -0.72 \times 10^{-5}$, we would conclude $\Delta\Lambda_G/\Lambda_G = 5.1 \times 10^{-4}$, i.e. the scale of grand unification about 8 billion years ago was about 8.3×10^{12} GeV higher than today.

Monitoring the ratio $\mu = M_p/m_e$ could allow to see an effect. Measuring the vibrational lines of H_2, a small effect was seen recently: $\Delta\mu/\mu = (5.7 \pm 3.8) \times 10^{-5}$ [7]. Supersymmetric SU(5) predicts $\frac{\Delta\mu}{\mu} = 22 \times 10^{-5}$ with a rather large theoretical uncertainty. It is interesting that the data suggests that μ is indeed decreasing, while α seems

to increase. If confirmed, this would be a strong indication that the time variation of α at low energies is caused by a time variation of the unification scale. We would like to emphasize that our calculation is based on the assumption that the proton mass is mainly determined by Λ. In particular, we neglect the possible time changes of the electron mass or of the quarks masses.

Under a further assumption, namely that $\dot{\alpha}/\alpha$ is constant, tests could be performed in quantum optics. We consider the case in which $\Lambda(t)$ is time dependent. If the rate of change is extrapolated linearly, Λ_G is decreasing at a rate $\frac{\dot{\Lambda}_G}{\Lambda_G} = -7 \times 10^{-14}/\text{yr}$. The magnetic moments of the proton μ_p as well as that of nuclei would increase according to $\frac{\dot{\mu_p}}{\mu_p} = 30.8\frac{\dot{\alpha}}{\alpha} \approx 3.1 \times 10^{-14}/\text{yr}$. The wavelength of the light emitted in hyperfine transitions, e.g. the ones used in the cesium clocks being proportional to $\alpha^4 m_e/\Lambda$ will vary in time like $\frac{\dot{\lambda}_{hf}}{\lambda_{hf}} = 4\frac{\dot{\alpha}}{\alpha} - \frac{\dot{\Lambda}}{\Lambda} \approx 3.5 \times 10^{-14}/\text{yr}$ taking $\dot{\alpha}/\alpha \approx 1 \times 10^{-15}/\text{yr}$. The wavelength of the light emitted in atomic transitions varies like α^{-2}: $\frac{\dot{\lambda}_{at}}{\lambda_{at}} = -2\frac{\dot{\alpha}}{\alpha}$. One has $\dot{\lambda}_{at}/\lambda_{at} \approx -2.0 \times 10^{-15}/\text{yr}$. A comparison gives:

$$\frac{\dot{\lambda}_{hf}/\lambda_{hf}}{\dot{\lambda}_{at}/\lambda_{at}} = -\frac{4\dot{\alpha}/\alpha - \dot{\Lambda}/\Lambda}{2\dot{\alpha}/\alpha} \approx -17.4. \tag{10}$$

It should be clear that our results are strongly model dependent. For example in SO(10) without supersymmetry, varying the grand unification scale, one finds:

$$\frac{\dot{\Lambda}}{\Lambda} = \left[\frac{-2\pi}{b_2^{SM} + \frac{5}{3}b_1^{SM}}\right]\frac{1}{\alpha}\frac{\dot{\alpha}}{\alpha} = -234.8\frac{\dot{\alpha}}{\alpha}, \tag{11}$$

neglecting the threshold corrections. But, this model dependence is what makes a possible time variation of the fundamental parameters so interesting. In principle, we could test grand unified theories without seeing any particle from a grand unified model. A time variation of the gauge couplings could thus provide a new condition for a grand unified theory besides reasonable proton decay and the unification of the coupling constants. But, it would require a more careful approach: calculations should take thresholds effects into account and would become quite complicated. Furthermore it would require measuring different time dependent quantities.

Clearly there are many constraints coming from different sectors and different redshifts. See [14] for a review. One important constraint is the Oklo phenomenon which allows to derive a severe constraint for the time variation of α during the last two billion years ago [15]. But, this analysis is performed under the assumption that only α is time dependent. As we have shown the effects could be much larger in QCD but go in the opposite direction. Some partial cancellation could possibly take place. But, it has been shown that extracting a limit for a time variation of the strong coupling constant is not an easy task [16]. Clearly it would be difficult to rule out the results coming from astrophysics using data coming from a later time. But, it would be very surprising if no effect was observed at a previous time. Such effects could show up in the cosmic microwave background [17]. Nucleosynthesis also allows to constrain severely time variations of fundamental parameters (see e.g. [18]).

Another question is what is really measured in [3–6]. The authors use a so-called many multiplet method, and fit the relativistic correction $\Delta = (Z\alpha)^2 \left(\frac{1}{j+1/2} - C\right)$ taking $C \sim 0.6$ calculated from QED using a many-body method [4]. But, the question is at what level would QCD affect this measurement? Most probably, the splitting of the observed spectral lines is only due to the QED atomic structure. Only inner shells of heavy atoms are sensitivity to the nuclear size influenced by the QCD parameter. But, a change in nuclear size, and thus of the charge distribution would at some level impact the magnitude of the parameter C. So it is a justified question to ask what is exactly measured if the effect is really much stronger in QCD, as expected from grand unified models.

We shall finally discuss the possible implications for baryogenesis of a time dependence of fundamental parameters. The aim of baryogenesis is to explain the matter/anti-matter asymmetry. Any model must fulfill the Sakharov's conditions:

a) The baryon number must be violated by some process (the net baryon number must change over time).

b) C and CP must be violated (no perfect equality between rates of $\Delta B \neq 0$ processes otherwise no asymmetry could evolve from initially symmetric state).

c) A departure from thermal equilibrium is required, otherwise CPT would assure compensation between processes increasing or decreasing the baryon number.

The standard model has a problem with points b) and c). It has not enough CP violation and the Higgs boson mass is too high to have a first order phase transition. The Higgs boson mass m_H should be smaller than 40 GeV (see e.g. [19]). This is actually a constraint on λ_H, the Higgs self-coupling. The condition for point c) is $E_B/\lambda(T_C) \geq 1$ which is equivalent to (see e.g. [19]):

$$\sqrt{2}\frac{2M_W^3 + M_Z^3}{3\pi v^3 \lambda(T_C)} \geq 1. \tag{12}$$

On the other hand the baryon number is proportional to (see e.g.[20]) $J = \sin(\theta_{12})\sin(\theta_{13})\sin(\theta_{23})\sin(\delta_{CP})$

$$\begin{aligned} n_B \propto\ & (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \\ & (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) J/T^{12} \sim 10^{-21}. \end{aligned} \tag{13}$$

which is much smaller that the measured baryon number $\sim (4-10)\times 10^{-11}$ (see e.g. [20] for a discussion). Thus the standard model is not able to provide the right baryon number and a phase transition. But, things might slightly change if the fundamental parameters of the standard model are time dependent. It seems impossible to explain the baryon number in that framework in view of the large discrepancy. But, if the parameters of the Higgs potential were time dependent there could be a chance to get a phase transition of 1st order. In general a parameter p can be decomposed into one $p = p_{gut} + p_{loop}(t)$, where p_{gut} is the value of the parameter at the grand unified scale and $p_{loop}(t)$ is the time dependence induced by loop corrections. In our case we assumed that the functions p_{gut} for the Higgs and Yukawa sectors are time independent. But, these parameters get a

time dependence through the radiative corrections which involve the coupling constants. An estimate in SO(10) yields $\frac{\Delta v}{v} = 0.4\%$, $\frac{\Delta \lambda_H}{\lambda_H} = -2\%$ and $\frac{\Delta m_t}{m_t} = 20\%$ in 10^{10} years, doing a linear extrapolation of the results of [3]. The effect is stronger for the top mass than for the $SU(2)$ Higgs sector because the top mass has a wild running. Obviously the time variation of the parameters of the Higgs potential obtained from that effect alone cannot explain the phase transition. But, the time variation of the parameter λ_H is rather unconstrained by experiment. We could relax the assumption that we made concerning the time invariance of λ_H at the grand unified scale thereby having a phase transition of 1st order in the early universe. Keeping v roughly constant (notice that a time variation of v is strongly constrained by nucleosynthesis which is very sensitive to G_F), the Higgs boson mass of the $SU(2)$ sector could have been around 40 GeV in the early universe if $(\lambda_H)_{gut}$ is strongly time dependent. Clearly the physics of the early universe could be affected by a time variation of the fundamental parameters. It remains to see if any predictions can be made in that framework. This will be difficult in view of the potentially large number of uncorrelated time dependent functions.

Acknowledgment: We shall like to thank J. Rafelski for enlightening discussions.

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