Anomaly Mediation, Yukawa Textures, and Neutrino Masses

Ian Jack and D.R. Tim Jones [1]

Dept. of Mathematical Sciences
University of Liverpool, Liverpool L69 3BX, UK

Abstract

We describe some recent developments in Anomaly Mediated Supersymmetry Breaking. We focus on resolutions of the tachyonic slepton puzzle based on extending the MSSM by an anomaly free U_1 symmetry, so that exact RG invariance is preserved.

1 Introduction

The usual assumption of the CMSSM is that at gauge unification:

$$\begin{aligned} \text{GAUGINO MASSES} \quad (M_i) &\rightarrow M_0 \\ \text{SOFT} \quad \phi^3 \quad \text{TERMS} \quad (h) &\rightarrow AY^{ijk} \\ \phi^*\phi \quad \text{MASSES} \quad (m^2) &\rightarrow m_0^2 \end{aligned}$$

(here Y^{ijk} are the Yukawa couplings). There is, however, no compelling theoretical basis for this. A persuasive alternative is provided by[1] Anomaly Mediated Supersymmetry Breaking (AMSB):

$$\begin{aligned} M_i &= m_{\frac{3}{2}}\beta_{g_i}/g_i \\ h &= -m_{\frac{3}{2}}\beta_Y \\ m^2 &= \frac{1}{2}m_{\frac{3}{2}}m^*_{\frac{3}{2}}\mu\frac{d}{d\mu}\gamma \end{aligned} \tag{1}$$

Here γ is the matter multiplet anomalous dimension and $m_{\frac{3}{2}}$ is the gaugino mass. The above relations are precisely RG invariant, and so can be evaluated at any scale; running from the unification scale is not necessary.

[1]Talk given by DRTJ

1.1 The Gaugino sector

An elementary consequence of Eq. (1) is that at low energies we have (approximately)

$$M_1 : M_2 : M_3 = 0.3 : 0.1 : 1, \tag{2}$$

to be compared with the usual assumption that $M_1 = M_2 = M_3$ at gauge unification, which gives

$$M_1 : M_2 : M_3 = 0.1 : 0.3 : 1. \tag{3}$$

Thus in the AMSB scenario, there is likely to be an approximately degenerate triplet of light winos (a chargino and a neutralino). The characteristic phenomenology (involving the characteristic decay of the chargino to the neutralino and a charged pion) has been explored in a number of papers.

Another interesting consequence[2] is that in both AMSB and the CMSSM there is a sum rule relating the chargino and neutralino masses:

$$\begin{aligned} \Delta M^2 &= 2\sum_{i=1,2}(M_{\chi_i^{\pm}})^2 - \sum_{j=1\cdots 4}(M_{\chi_j^0})^2 \\ &= f(g_i)M_3^2 + 4M_W^2 - 2M_Z^2 \end{aligned} \tag{4}$$

In AMSB type models it is negative while in the CMSSM it is positive.

1.2 The slepton mass problem

Direct application of the AMSB solution to the MSSM leads, unfortunately, to negative $(\text{mass})^2$ sleptons. The solution to this which has been most popular is the replacement $m^2 \to \hat{m}^2$ where

$$(\hat{m}^2)^i{}_j = (m^2)^i{}_j + m_0^2\delta^i{}_j. \tag{5}$$

Here m^2 is the basic AMSB solution from Eq. (1) and m_0^2 is constant. For example, in a recent paper[3], de Campos et al applied this solution to an extension of the MSSM to incorporate bilinear R-parity violation, i.e.

$$W \to W + \sum_i \lambda_i L_i H_2, \tag{6}$$

where W is the MSSM superpotential,

$$W = H_2 Q Y_t t^c + H_1(QY_b b^c + LY_\tau \tau^c) + \mu H_1 H_2. \tag{7}$$

This results in interesting phenomenology; for example the mixing between neutral gauginos and neutrinos induces neutrino masses. For a different approach where R-parity violation is used to resolve the slepton mass problem, see Ref. [4].

Eq. (5) has a major defect which is that it is not RG invariant. If instead we have[5]

$$(\hat{m}^2)^i{}_j = (m^2)^i{}_j + m_0^2\sum_{a=1}^{\mathcal{N}} k_a(Y_a)^i{}_j \tag{8}$$

Table 1: Table of U_1 and U_1' hypercharges.

	Q	L	t^c	b^c	τ^c	H_1	H_2
Y	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{2}{3}$	$\frac{1}{3}$	1	$-\frac{1}{2}$	$\frac{1}{2}$
Y'	$\frac{7}{3}$	-7	$\frac{5}{3}$	$-\frac{19}{3}$	3	4	-4

then $\hat{m}^2$ *is* RG invariant as long as each Y_a corresponds to a U_1 invariance of the superpotential W and also has vanishing mixed anomaly with each MSSM gauge group factor. (One may also employ a set of Y_a corresponding to a U_1 R-symmetry, in which case Eq. (8) is modified[6].) In the MSSM there are two possible flavour-blind[5] linearly independent sets of such Y_a; the hypercharge gauge group U_1^Y and another, which could be chosen[7] to be U_1^{B-L}, or a linear combination of U_1^Y and U_1^{B-L}, which we call U_1'.
A set of possible U_1' charges (chosen so as to satisfy $\mathrm{Tr}(YY') = 0$) and the corresponding U_1 ones are shown in Table 1. It is easy to show that for $k' < 0$ and $-3 > k/k' > -14$, the contributions to both slepton mass terms from the k_a terms in Eq. (8) are positive. With, for example, $\zeta_1 = km_0^2 = 0.2\mathrm{TeV}^2$ and $\zeta_2 = k'm_0^2 = -0.02\mathrm{TeV}^2$, electroweak symmetry breaking occurs and a distinctive sparticle spectrum is obtained[5]. An interesting feature is the existence of sum rules for combinations of masses in which the dependence on $\zeta_{1,2}$ cancels. For example:

$$\begin{aligned}
m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2 + m_{\tilde{b}_1}^2 + m_{\tilde{b}_2}^2 - 2(m_t^2 + m_b^2) &= 2.79\left(\frac{m_{\frac{3}{2}}}{40}\right)^2 \mathrm{TeV}^2, \\
m_{\tilde{\tau}_1}^2 + m_{\tilde{\tau}_2}^2 + m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2 - 2(m_t^2 + m_\tau^2) &= 1.15\left(\frac{m_{\frac{3}{2}}}{40}\right)^2 \mathrm{TeV}^2, \\
m_{\tilde{e}_L}^2 + 2m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2 &= 0.90\left(\frac{m_{\frac{3}{2}}}{40}\right)^2 \mathrm{TeV}^2, \\
m_{\tilde{u}_R}^2 + m_{\tilde{d}_R}^2 + m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2 &= 3.56\left(\frac{m_{\frac{3}{2}}}{40}\right)^2 \mathrm{TeV}^2, \\
m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2 - m_{\tilde{u}_R}^2 - m_{\tilde{e}_R}^2 &= 0.90\left(\frac{m_{\frac{3}{2}}}{40}\right)^2 \mathrm{TeV}^2, \\
m_A^2 - 2\sec 2\beta\left(m_{\tilde{\tau}_1}^2 + m_{\tilde{\tau}_2}^2 - 2m_\tau^2\right) &= 0.49\left(\frac{m_{\frac{3}{2}}}{40}\right)^2 \mathrm{TeV}^2. \qquad (9)
\end{aligned}$$

(The numerical results above apply for $\tan\beta = 5$.)

2 AMSB and the FN mechanism

Although the flavour-blind scenario described in section 1 has the advantage that flavour changing neutral currents (FCNCs) are naturally suppressed, it is interesting to explore

Table 2: Table of U_1 and U_1' hypercharges:FN case

Q_i	b_1^c	b_2^c	b_3^c	t_2^c	t_3^c
$8 - t_1^c - h_2$	$2h_2 + t_1^c - 16$	$2h_2 + t_1^c - 12$	$2h_2 + t_1^c - 8$	$t_1^c - 4$	$t_1^c - 8$
L_i	τ_1^c	τ_2^c	τ_3^c	h_1	
$3t_1^c + 3h_2 - 24$	$16 - 2h_2 - 3t_1^c$	$20 - 2h_2 - 3t_1^c$	$24 - 2h_2 - 3t_1^c$	$-h_2$	

a marriage between the U_1-based solution to the AMSB slepton mass problem and the Froggatt-Nielsen (FN) mechanism by replacing U_1^{B-L} with a generation-dependent U_1 symmetry: in the hope of providing an explanation for the mass hierarchies and also a more natural framework for the introduction of neutrino masses.

At first sight this is unattractive because evidently generation-dependent contributions proportional to Y_a in Eq. (8) will give rise to off-diagonal squark and slepton masses when we rotate to the quark and lepton mass-diagonal bases. However, the following set of textures provide a solution[8] to this conundrum:

$$
\begin{aligned}
Y_t &\sim \begin{pmatrix} \lambda^8 & \lambda^4 & 1+\mathcal{O}(\lambda^2) \\ \lambda^8 & \lambda^4 & 1+\mathcal{O}(\lambda^2) \\ \lambda^8 & \lambda^4 & 1+\mathcal{O}(\lambda^2) \end{pmatrix}, \quad Y_b \sim \begin{pmatrix} \lambda^4 & \lambda^2 & 1+\mathcal{O}(\lambda^2) \\ \lambda^4 & \lambda^2 & 1+\mathcal{O}(\lambda^2) \\ \lambda^4 & \lambda^2 & 1+\mathcal{O}(\lambda^2) \end{pmatrix}, \\
Y_\tau &\sim \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}
\end{aligned} \tag{10}
$$

In Y_τ the third column entries are $O(1)$; likewise in $Y_{t,b}$ except that in these cases we require them to differ from each other at $O(\lambda^2)$ only. Imposing this leads to a CKM matrix of the form

$$
CKM \sim \begin{pmatrix} 1 & 1 & \lambda^2 \\ 1 & 1 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix} \tag{11}
$$

which is not of the form of the standard Wolfenstein parametrisation, but does reproduce its most significant feature, which is the smallness of the couplings to the third generation. Textures of this "democratic" form manifestly correspond to generation-independent charge assignments for the left-handed fields; however the Yukawa matrices are diagonalised by rotations (to a good approximation) of *the left handed fields only*. Thus looking again at Eq. (8) we see that FCNC problems are avoided: for LH fields (which are rotated) because their charges are flavour-independent, and for RH fields (whose charges differ) because they are not rotated.

We assume the texture pattern is produced via higher order terms such as $H_2 Q_i t_j^c (\frac{\theta_t}{M_U})^{a_{ij}}$ where M_U represents the scale of new physics. We assume that each Yukawa matrix $Y_{t,b,\tau}$ gains its texture from the vev of a *particular* θ-charge and that the vevs of the θ-charges are approximately the same. A set of charges leading to automatic cancellation of mixed anomalies (for arbitrary charges h_2 and t_1^c) is shown in Table 2. It is easy to verify that with these charges we obtain the democratic texture of Eq. (10) if we use θ-charges $\theta_t = -1$, $\theta_b = \theta_\tau = 2$.

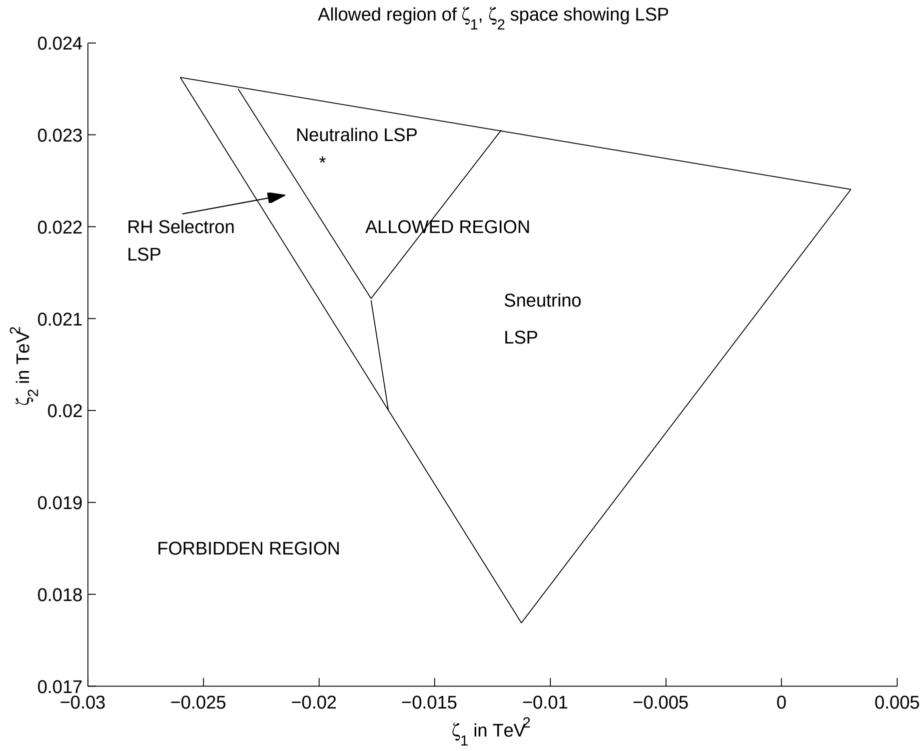

Figure 1: Allowed values of $\zeta_{1,2}$ for $\tan\beta = 5$, $m_{\frac{3}{2}} = 40\text{TeV}$ and $\text{sign}\,\mu = -1$.

It is straightforward to show that in order to resolve the slepton mass problem we then require

$$3t_1^c + 4h_2 < 24 \quad \text{or} \quad 3t_1^c + 4h_2 > 32, \tag{12}$$

For the particular choices $h_2 = 12$, $t_1^c = -7/2$, $m_{\frac{3}{2}} = 40\text{TeV}$, $\tan\beta = 5$ and $\text{sign}\,\mu = -1$, we show in Fig. 1 the triangular region in the $\zeta_{1,2}$ plane which leads to an acceptable electroweak vacuum. For a typical point in the neutralino LSP region ($\zeta_1 = -0.02\text{TeV}^2$, $\zeta_2 = 0.0227\text{TeV}^2$), we obtain the following spectrum:

$$\begin{aligned}
m_{\tilde{t}_{1,2}} &= 869, 484,\ m_{\tilde{b}_{1,2}} = 825, 1082, \\
m_{\tilde{\tau}_{1,2}} &= 148, 442,\ m_{\tilde{u}_L,\tilde{c}_L} = 931,\ m_{\tilde{u}_R} = 908, \\
m_{\tilde{c}_R} &= 856,\ m_{\tilde{d}_L,\tilde{s}_L} = 934,\ m_{\tilde{d}_R} = 998, \\
m_{\tilde{s}_R} &= 1042,\ m_{\tilde{e}_L,\tilde{\mu}_L} = 149,\ m_{\tilde{e}_R} = 117, \\
m_{\tilde{\mu}_R} &= 323,\ m_{\tilde{\nu}_e,\tilde{\nu}_\mu} = 126,\ m_{\tilde{\nu}_\tau} = 125 \\
m_{h,H} &= 122, 166, m_A = 161,\ m_{H^\pm} = 181, \\
m_{\tilde{\chi}^\pm_{1,2}} &= 112, 575,\ m_{\tilde{\chi}_{1\cdots4}} = 111, 369, 579, 579 \\
m_{\tilde{g}} &= 1007
\end{aligned} \tag{13}$$

where all masses are given in GeV. The squarks $\tilde{t}_1, \tilde{b}_1$ and $\tilde{\tau}_1$ couple more strongly to t_L, b_L and τ_L respectively, though (for our chosen $\tan\beta$) the $\tilde{t}_{1,2}$ mixing is of course substantial.

3 Massive Neutrinos

We could just introduce Dirac masses for the neutrinos, preserving U_1^{B-L}, and hence maintain the flavour-blind scenario described in Section 1. This is obviously unappealing however, as it provides no explanation for the smallness of neutrino masses. So we adopt the seesaw mechanism:

$$W \to W + \tfrac{1}{2}\nu^c M_{\nu^c}\nu^c + H_2 L Y_\nu \nu^c \tag{14}$$

where Y_ν is $3 \times n_\nu$, n_ν being the number of RH neutrinos. In fact we can still preserve the flavour-blind solution if we assign zero U_1' charge to all the ν^c, and generate the Y_ν matrix via the FN mechanism. We eschew this for two reasons; firstly because it seems unnatural to thus generate *only* Y_ν (and not $Y_{t,b,\tau}$); and secondly because we would then have to consider (or somehow exclude) R-parity violation since clearly, for example the LH_2 bilinear is allowed if $H_2L\nu^c$ is (with zero charge for ν^c). We prefer therefore to use a more exotic assignment of charges such as described in Section 2 with a view to simultaneously producing an acceptable neutrino spectrum with a U_1' which (without further assumption) naturally suppresses the R-parity violation sector. The simplest such possibility we have found[8] is to begin with the charge assignments given in Table 2. Then with two RH neutrinos with U_1' charges $\pm q_\nu$ and a FN field θ_ν such that

$$\begin{aligned} L_i + h_2 + q_\nu + nq_{\theta_\nu} &= 0 \\ L_i + h_2 - q_\nu + mq_{\theta_\nu} &= 0, \end{aligned} \tag{15}$$

it is easy to show that if we choose, for example, $n = 2$ and $m = 1$ and $q_{\theta_\nu} = -9$ then no renormalisable R-parity violating interactions can be generated using the available θ-charges.

The resulting pattern of neutrino masses and mixings can accommodate the currently favoured pattern, i.e. $m_1 \ll m_2 \ll m_3$ (with $m_1 = 0$) and a mixing matrix of the form, for example,

$$U_{MNS} \approx \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \tag{16}$$

While these patterns can indeed be obtained they are not specifically predicted by the model in its present form. It would be interesting if a union of the AMSB and texture paradigms that naturally led to an appropriate hierarchy and form of U_{MNS} could be found. There is already a considerable literature devoted to building neutrino mass models; for recent reviews of the experimental and theoretical situations see for example Ref. [8].

Acknowledgements

DRTJ is grateful for the support of a PPARC Senior Fellowship.

References

[1] L. Randall and R. Sundrum, *Nucl. Phys.* **B557** 79 (1999);
G.F. Giudice et al, *JHEP* **9812** 27 (1998);
I. Jack and D.R.T. Jones, *Phys. Lett.* **B465** 148 (1999).

[2] K. Huitu, J. Laamanen, and P.N. Pandita, Phys. Rev. **D65** 115003 (2002).

[3] F. de Campos et al, *Nucl. Phys.* **B623** 47 (2002).

[4] B.C. Allanach and A. Dedes, JHEP **0006** 017 (2000).

[5] I. Jack and D.R.T. Jones, *Phys. Lett.* **B482** 167 (2000).

[6] A. Pomarol and R. Rattazzi, *JHEP* **9905** 013 (1999);
I. Jack and D.R.T. Jones, *Phys. Lett.* **B491** 151 (2000).

[7] N. Arkani-Hamed et al, *JHEP* **0102** 041 (2001).

[8] I. Jack, D.R.T. Jones, and R. Wild, *Phys. Lett.* **B535** 193 (2002).

[9] C. Giunti, hep-ph/0209103;
S.F. King, hep-ph/0208266