Grand Unification with Anomalous U(1) Symmetry

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Abstract

In this talk, we introduce a new scenario of grand unified theory (GUT) with anomalous $U(1)_A$ gauge symmetry. Since generic interactions (including non-renormalizable interactions) are introduced, once we fix the symmetry of the theory, we can define the theory without $O(1)$ coefficients. The number of parameters to determine the symmetry is just 11 or 12 integer parameters which are anomalous $U(1)_A$ charges(integer) for introduced multiplets. It is surprising that the GUT scenario can explain doublet-triplet splitting, quark and lepton masses and mixing angles. In neutrino sector, the scenario realizes LMA solution for solar neutrino problem and large $U_{e3} = O(0.1)$. Moreover, the scenario predicts that the unification scale becomes smaller than the usual GUT scale, namely proton decay via dimension 6 operators may be seen in future experiments. This talk is based on the papers [1]-[5].

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1 Introduction

Grand unified theory (GUT) [6] realizes two kinds of unification. It unifies 3 gauge interactions $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ in the standard model into 1 gauge interaction like $SU(5)$, $SO(10)$ or E_6. It is regarded as the signal for the unification of gauge interaction that three gauge couplings meet at a scale $\Lambda_G \sim 2 \times 10^{16}$ GeV in the minimal supersymmetric (SUSY) standard model (MSSM). Moreover, GUT gives a unification of quark and leptons in a fewer multiplets. In the standard model, 6 multiplets (Q, U^c, D^c, L, E^c and N^c) are required for one family quark and lepton, while in the $SU(5)$ GUT, 3 multiplets ($\mathbf{10} = Q + U^c + E^c$, $\bar{\mathbf{5}} = D^c + L$, $\mathbf{1} = N^c$) for one family. If we adopt $SO(10)$ GUT, just one multiplet ($\mathbf{16} = \mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}$) is needed for one family. However, introducing $\mathbf{10}$ of $SO(10)$ naturally avoids unrealistic $SO(10)$ GUT relations between quark and lepton Yukawa matrices and can realize large mixing angles observed in neutrino sector[7]. Thus, E_6 GUT is more interesting because a multiplet includes $\mathbf{16}$ and $\mathbf{10}$ as $\mathbf{27} = \mathbf{16}+\mathbf{10}+\mathbf{1}$[8]. Therefore, $SO(10)$ or E_6 GUT is more attractive than $SU(5)$ GUT in the sense of unification of quark and leptons. However, in the sense of unification of gauge interactions, $SO(10)$ or E_6 GUT is less natural than $SU(5)$ GUT, because $SO(10)$ or E_6 GUT has additional freedoms which always make the gauge coupling unification possible in principle. This is because the rank of $SO(10)$ or E_6 is larger than that of $SU(5)$ and G_{SM}. But anyway, GUT scenario realizes above two kinds of unification simultaneously.

Unfortunately, it is not so easy to obtain the realistic SUSY GUT. One of the main difficulties is to realize doublet-triplet splitting in a natural way [9, 10, 11] with stable proton. The other difficulty is to avoid unrealistic GUT relations between Yukawa matrices of quark and leptons, which are related with the unification of quark and lepton, and to obtain realistic quark and lepton mass matrices.

It is not so easy to solve these problems in a natural way, keeping t he 2 kinds of unifications in GUT scenario. For example, in the scenario of orbifold GUT which many people have discussed in the literature[11], doublet-triplet splitting is beautifully realized, but the unification of quark and leptons must be given up even in the sense of $SU(5)$ to avoid unrealistic GUT relations.

Recently, in a series of papers[1, 2, 3, 4, 5, 12], the interesting GUT scenario with anomalous $U(1)_A$ gauge symmetry [13], whose anomaly is canceled by the Green-Schwarz mechanism[14], has been proposed with $SO(10)$ unified group[1] and with E_6 unified group[4, 5]. One of the biggest difference from the previous works[15] is that in our scenario "generic" interactions (including non-renormalizable interactions), namely all the interactions which are allowed by the symmetry of the theory, are introduced. Therefore, once we fix the symmetry of the theory ($SO(10)$ or E_6) $\times\, U(1)_A$, we can define the model except $O(1)$ coefficients. The number of input parameters is essentially the number of the anomalous $U(1)_A$ charges (integer), namely, the number of multiplets we introduce. In our scenario, 4 multiplets ($3 \times \mathbf{16} + \mathbf{10}$) for matter sector and 8 multiplets ($2 \times \mathbf{45} + 2 \times (\mathbf{16} + \overline{\mathbf{16}} + \mathbf{10})$) for Higgs sector are introduced in $SO(10)$ unification, and 3 multiplets ($3 \times \mathbf{27}$) for matter sector and 8 multiplets ($2 \times \mathbf{78} + 3 \times (\mathbf{27} + \overline{\mathbf{27}})$) for Higgs sector are introduced in E_6 unification. Here we neglect the number of singlet fields. Therefore only 11 or 12 integers are input parameters in our models.

It is amazing that under such a small amount of inputs, following various things can be explained in our scenario.

1. GUT scales and other symmetry breaking scales are determined by anomalous $U(1)_A$ charges.

2. Doublet-triplet splitting is realized in a natural way.

3. Proton decay via dimension 5 operators is suppressed.

4. MSSM is obtained at a low energy scale.

5. Mass spectrum of superheavy fields are fixed by anomalous $U(1)_A$ charges.

6. Natural gauge coupling unification in a sense of the minimal $SU(5)$ GUT is realized even if we adopt $SO(10)$ or E_6, whose rank is larger than that of $SU(5)$ or G_{SM}, as the unified group.

7. Realistic quark and lepton masses and mixings are obtained.

8. μ problem is solved.

9. In E_6 unification, a condition for suppression of the flavor changing neutral current is satisfied.

Predictions of our scenario are

1. Bi-large neutrino mixing angles and LMA solution for the solar neutrino problem;

2. $U_{e3} \sim O(0.1)$;

3. CP violation phase in lepton sector $\delta_{CP} \sim O(1)$;

4. small $\tan\beta \sim 5$;

5. Smaller unification scale $\Lambda_u < \Lambda_G \sim 2\times 10^{16}$ GeV (Thus proton decay via dimension 6 operators may be seen in future experiments)

$$\tau(P \to e\pi) \sim \left\{ \begin{array}{ll} 5\times 10^{33}\text{years} & (a=-1) \\ 8\times 10^{34}\text{years} & (a=-1/2) \end{array} \right. ; \qquad (1.1)$$

6. Smaller Cutoff scale $\Lambda \sim \Lambda_G < M_{Pl}$ (It may imply the existence of extra dimension in which only gravity modes can propagate as discussed by Horava-Witten);

7. Low energy SUSY.

2 Doublet-triplet splitting

One of the most interesting feature of anomalous $U(1)_A$ gauge theory is that the vacuum expectation values (VEV) are determined by anomalous $U(1)_A$ charges as

$$\left\langle Z^+ \right\rangle = 0, \tag{2.1}$$

$$\left\langle Z^- \right\rangle \sim \lambda^{-z^-}, \tag{2.2}$$

where $Z^\pm$ are singlet operators with the charges $z^+ > 0$ and $z^- < 0$, and $\lambda = \langle \Theta \rangle / \Lambda$. Here Θ is a Froggatt-Nielsen field [16]. Through this paper, we use unit in which the cutoff $\Lambda = 1$ and denote all the superfields by uppercase letters and their anomalous $U(1)_A$ charges by the corresponding lowercase letters. Such VEVs do not change the order of the coefficients obtained by the Froggatt-Nielsen mechanism:

$$W = \left(\frac{\Theta}{\Lambda}\right)^{x+y+z} XYZ \to \lambda^{x+y+z} XYZ, \tag{2.3}$$

if the total charge $x+y+z$ of the operator XYZ is positive. Using this mechanism, the hierarchy of Yukawa couplings can be realized [17] Note that even if the operator $\frac{Z^-}{\Lambda}$ is used instead of $\left(\frac{\Theta}{\Lambda}\right)^{-z^-}$ in the interactions, the order of the coefficients does not change. This feature is critically different from the naive expectation that the contribution from the higher dimensional operators is more suppressed. If the total charge $x+y+z$ is negative, such interaction is not allowed by the anomalous $U(1)_A$ gauge symmetry because only negatively charged fields have non-vanishing VEVs. This is called SUSY zero mechanism. Note that this mechanism leads to the finite number of non-renormalizable interactions, and therefore we can control the generic superpotential.

Actually, under the vacua (2.1), the generic superpotential to determine the VEVs of Z^- can be written as

$$W = \sum_i^{n_+} W_{Z_i^+}, \tag{2.4}$$

where W_X denotes the terms linear in the X field. This is because the F-flatness conditions of negatively charged fields are automatically satisfied and the terms with more than two positively charged fields do not contribute in the F-flatness condition of positively charged fields.

Let us discuss an $SO(10)$ GUT model with anomalous $U(1)_A$ gauge symmetry in which doublet-triplet splitting is naturally realized. The Higgs content is listed in Table I. Here the symbols $\pm$ denote the Z_2 parity. The VEVs of the negatively charged Higgs fields are determined by the superpotential

$$W = W_{A'} + W_{C'} + W_{\bar{C}'} + W_{H'} + W_S. \tag{2.5}$$

We do not have spaces enough to explain the vacuum structure in detail, so we here point out only one good feature in realizing the doublet-triplet splitting. Actually this observation is important in solving doublet-triplet splitting with generic interactions.

If $-3a \le a' < -5a$, the superpotential $W_{A'}$ is in general written

$$W_{A'} = \lambda^{a'+a} \alpha A'A + \lambda^{a'+3a} (\beta (A'A)_{\mathbf{1}} (A^2)_{\mathbf{1}} + \gamma (A'A)_{\mathbf{54}} (A^2)_{\mathbf{54}}), \tag{2.6}$$

Table I. Typical values of anomalous $U(1)_A$ charges.

	non-vanishing VEV	vanishing VEV
45	$A(a=-1,-)$	$A'(a'=3,-)$
16	$C(c=-4,+)$	$C'(c'=3,-)$
$\overline{\mathbf{16}}$	$\bar{C}(\bar{c}=-1,+)$	$\bar{C}'(\bar{c}'=6,-)$
10	$H(h=-3,+)$	$H'(h'=4,-)$
1	$\Theta(\theta=-1,+)$,$Z(z=-2,-)$, $\bar{Z}(\bar{z}=-2,-)$	$S(s=5,+)$

where the suffices **1** and **54** indicate the representation of the composite operators under the $SO(10)$ gauge symmetry, and α, β and γ are parameters of order 1. Here we assume $a+a'+c+\bar{c}<0$ to forbid the term $\bar{C}A'AC$, which destabilizes the DW form of the VEV $\langle A\rangle$. The D-flatness condition requires the VEV $\langle A\rangle = i\tau_2\times\mathrm{diag}(x_1,x_2,x_3,x_4,x_5)$, and the F-flatness conditions of the A' field requires $x_i(-\alpha\lambda^{-2a}+(2\beta-\frac{\gamma}{5})(\sum_j x_j^2)+\gamma x_i^2)=0$. This allows only two solutions, $x_i^2=0$ and $x_i^2=v^2\sim\lambda^{-2a}$. Here $N_0=0-5$ is the number of $x_i=0$ solutions. When $N_0=2$, the vacuum becomes $\langle A(\mathbf{45})\rangle_{B-L}=\tau_2\times\mathrm{diag}(v,v,v,0,0)$, which breaks $SO(10)$ into $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ at the scale $\Lambda_A\equiv\langle A\rangle\sim\lambda^{-a}$. This Dimopoulos-Wilczek form of the VEV plays an important role in solving the DT splitting problem. Actually through the interaction $W=H'AH$, the DW type of the VEV gives superheavy masses only to the triplet Higgs, and therefore the doublet Higgs remains massless. Taking account of the mass term H'^2, only one pair of Higgs doublets becomes massless.

Note that the higher terms $A'A^{2L+1}$ $(L>1)$ are forbidden by the SUSY zero mechanism. If they were allowed, the number of possible VEVs other than the DW form would become larger, and thus it would become less natural to obtain the DW form. This is a critical point of this mechanism, and the anomalous $U(1)_A$ gauge symmetry plays an essential role in forbidding the undesired terms.

The spinor Higgs fields C and $\bar{C}$ break $SU(2)_R\times U(1)_{B-L}$ into $U(1)_Y$ by developing $\langle C\rangle$ $(=\langle\bar{C}\rangle\equiv\Lambda_C\sim\lambda^{-(c+\bar{c})/2})$. Then this model becomes MSSM at a low energy scale.

3 Gauge coupling unification

Unfortunately, the mass spectrum of superheavy Higgs fields does not respect $SU(5)$ gauge symmetry. Naively thinking, it spoils the success of gauge coupling unification in SUSY GUT scenario. In the early stage of these works, we expected that there must be tuning parameters to realize the gauge coupling unification because the rank of $SO(10)$ or E_6 is larger than that of $SU(5)$. However, the fact is more exciting than what we expected.

In our scenario, the mass spectrum of superheavy fields and GUT breaking scales are determined by anomalous $U(1)_A$ charges, so the conditions for gauge coupling unification

$$\alpha_1(\Lambda_u)=\alpha_2(\lambda_u)=\alpha_3(\Lambda_u) \tag{3.1}$$

are rewritten by using anomalous $U(1)_A$ charges, cutoff scale Λ and usual GUT scale Λ_G as

$$\Lambda\sim\Lambda_G \tag{3.2}$$

$$h \sim 0. \tag{3.3}$$

It is miraculous that all the charges except doublet Higgs's are canceled out. And the first condition is nothing but for determining the scale of the theory, and the second condition is essentially the same freedom as that of colored Higgs mass in the minimal $SU(5)$ GUT because h is anomalous $U(1)_A$ charge not only of doublet Higgs but also of colored Higgs. Therefore, we have no other parameters to be adjusted than in the minimal $SU(5)$ GUT. Let us explain the situation. In our scenario, since mass spectrum of superheavy fields and GUT breaking scales are determined by charges, we can estimate the gauge coupling constants $\alpha_i(\Lambda_W)$ at a low energy scale from the cutoff scale for any charges and any cutoff. By using the estimated value $\alpha_i(\Lambda_W)$, we can calculate the running gauge couplings in MSSM. It is surprising that for any charges and for any cutoff, three gauge couplings meet at a scale. The scale is the same as the cutoff scale in our scenario. If our scenario is real, since it is known that three gauge couplings meet at the usual GUT scale $\Lambda_G \sim 2 \times 10^{16}$ GeV in MSSM, the cutoff scale in our scenario must be taken as Λ_G. On the other hand, the real unification scale is usually smaller than the cutoff $\Lambda_u \sim \lambda^{-a} < \Lambda = \Lambda_G$. Therefore proton decay via dimension 6 operators is interesting in our scenario, and actually, rough estimation leads to

$$\tau(P \to e\pi) \sim \begin{cases} 5 \times 10^{33}\text{years} & (a=-1) \\ 8 \times 10^{34}\text{years} & (a=-1/2) \end{cases} . \tag{3.4}$$

We would like to emphasize that the above picture is independent of the details of Higgs sector, namely independent of their charges and of how to realize doublet-triplet splitting. Actually sufficient conditions for the miracle cancellation[12] are

1. The unification group is simple.
2. $$\langle O_i \rangle \sim \begin{cases} \lambda^{-o_i} & o_i \le 0 \\ 0 & o_i > 0 \end{cases} . \tag{3.5}$$
3. At a low energy scale, MSSM(+singlets) is realized.

The vacuum structure is naturally obtained if generic interactions are introduced and F-flatness conditions determine the scale of the VEVs. Thus the above picture is fairly general one in GUT with anomalous $U(1)_A$ gauge symmetry.

4 Quark and lepton mass matrices

One of the most attractive features of grand unified theory is to unify the quark and lepton into fewer multiplets. For example, in $SO(10)$ GUT scenario, a **16** representation field contains one family quark and lepton fields including right-handed neutrino field. However, this attractive feature directly leads to unrealistic Yukawa relations. For example, if we introduce $3 \times \mathbf{16}$ $\Psi_i (i = 1, 2, 3)$ for 3 family quark and leptons, the Yukawa couplings are obtained from the interaction

$$W = Y_{ij} \Psi_i \Psi_j H \tag{4.1}$$

as $Y_u = Y_d$ and $Y_d = Y_e$, which lead to unrealistic mass relation. We have to pick up the VEV $\langle C \rangle$ in the Yukawa matrices to avoid the former unrealistic relation $Y_u = Y_d$, and the VEV $\langle A \rangle$ to avoid the latter unrealistic relation $Y_d = Y_e$. In our scenario, we introduce an additional matter field $T(\mathbf{10})$. Then after breaking the GUT gauge group into the standard model gauge group, one pair of vector-like fields $\mathbf{5}$ and $\bar{\mathbf{5}}$ of $SU(5)$ becomes massive. The mass matrix is obtained from the interaction

$$W = \lambda^{\psi_i+t+c}\Psi_i TC + \lambda^{2t}T^2 \tag{4.2}$$

as

$$\mathbf{5}_T(\lambda^{t+\psi_1+(c-\bar{c})/2}, \lambda^{t+\psi_2+(c-\bar{c})/2}, \lambda^{t+\psi_3+(c-\bar{c})/2}, \lambda^{2t}) \begin{pmatrix} \bar{\mathbf{5}}_{\Psi 1} \\ \bar{\mathbf{5}}_{\Psi 2} \\ \bar{\mathbf{5}}_{\Psi 3} \\ \bar{\mathbf{5}}_{\mathbf{T}} \end{pmatrix}, \tag{4.3}$$

where actually the VEV $\left\langle \bar{C} \right\rangle = \langle C \rangle \sim \lambda^{-(c+\bar{c})/2}$ appear in the mass matrix. Since $\psi_3 < \psi_2 < \psi_1$, the massive mode $\bar{\mathbf{5}}_M$, the partner of $\mathbf{5}_T$, must be either $\bar{\mathbf{5}}_{\Psi 3}(\Delta \equiv 2t - (t+\psi_3+(c-\bar{c})/2) > 0)$ or $\bar{\mathbf{5}}_T(\Delta < 0)$. The former case is interesting, and in this case, the three massless modes $(\bar{\mathbf{5}}_1, \bar{\mathbf{5}}_2, \bar{\mathbf{5}}_3)$ can be written $(\bar{\mathbf{5}}_{\Psi 1} + \lambda^{\psi_1-\psi_3}\bar{\mathbf{5}}_{\Psi 3}, \bar{\mathbf{5}}_T + \lambda^{\Delta}\bar{\mathbf{5}}_{\Psi 3}, \bar{\mathbf{5}}_{\Psi 2} + \lambda^{\psi_2-\psi_3}\bar{\mathbf{5}}_{\Psi 3})$. If we adopt their charges $(\psi_1, \psi_2, \psi_3, t) = (9/2, 7/2, 3/2, 5/2)$ in addition to the charges of Higgs fields, then we can estimate quark and lepton mass matrices as

$$M_u = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle, \quad M_d = M_e = \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^{3.5} & \lambda^3 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \\ \lambda^1 & \lambda^{0.5} & 1 \end{pmatrix} \langle H_d \rangle. \tag{4.4}$$

And for the neutrino sector, we take into account the interaction

$$\lambda^{\psi_i+\psi_j+2\bar{c}}\Psi_i\Psi_j\bar{C}\bar{C}, \tag{4.5}$$

which lead to the right-handed neutrino masses

$$M_R = \lambda^{\psi_i+\psi_j+2\bar{c}} \left\langle \bar{C} \right\rangle^2 = \lambda^{2n+\bar{c}-c} \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \tag{4.6}$$

Since the Dirac neutrino mass is given by

$$M_{\nu_D} = \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda \\ \lambda^{3.5} & \lambda^{2.5} & \lambda^{0.5} \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \langle H_u \rangle \eta, \tag{4.7}$$

the neutrino mass matrix is obtained by the seesaw mechanism as

$$M_\nu = M_{\nu_D} M_R^{-1} M_{\nu_D}^T = \lambda^{4-2n+c-\bar{c}} \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \langle H_u \rangle^2 \eta^2. \tag{4.8}$$

Note that the ratio $\frac{m_{\nu_\mu}}{m_{\nu_\tau}} \sim \lambda$ is realized, that predicts LMA solution for the solar neutrino problem. It is interesting that we obtain the small mixing angles for the Cabibbo-Kobayashi-Maskawa matrix

$$U_{\rm CKM} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \tag{4.9}$$

and the large mixing angles for the Maki-Nakagawa-Sakata matrix

$$U_{\rm MNS} = \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}. \tag{4.10}$$

Since we use a rule that $\lambda^{0.5} + \lambda^{0.5} \sim \lambda^{0.5}$ in calculating the MNS matrix and $\lambda^{0.5} \sim 0.5$, this model gives large mixing angles [18, 19] for the atmospheric neutrino problem and for the solar neutrino problem. And $U_{e3} \sim \lambda$ is predicted, which is around the present upper limit given by CHOOZ[20]. At this stage, the unrealistic GUT relation $Y_d = Y_e$ still remains. However, in our scenario, the same amount of the Yukawa couplings are given by the higher dimensional interactions

$$W = \lambda^{\psi_i+\psi_j+na+h}\Psi_i A^n \Psi_j H \tag{4.11}$$

by developing the VEV $\langle A\rangle \sim \lambda^{-a}$. It is critical that the Yukawa couplings from the higher dimensional interactions have not kept the unrealistic GUT relation. Usually, the corrections from such higher dimensional interactions are suppressed by the factor $\frac{\langle A\rangle}{\Lambda}$. But in our scenario, the suppression factor $\frac{\langle A\rangle}{\Lambda}$ is just canceled by the enhancement factor λ^a in the coefficients, and therefore we can obtain the same order coefficients as from the tree interaction. This is an attractive feature in our scenario, and the realistic mass matrices are naturally obtained.

5 Discussions and summary

It is familiar to introduce generic interactions to explain hierarchical structure of Yukawa couplings, using Froggatt-Nielsen mechanism. However, it has not been done to introduce generic interactions in Higgs sector. This is because if we introduce generic interactions in Higgs sector, the gauge group is expected to be broken maximally into $U(1)^n$. In this talk, we showed that even with generic interactions, non-Abelian structure can be obtained and doublet-triplet splitting is realized. If we adopt $SO(10)$ or E_6 gauge group to unify quark and lepton, usually it becomes less natural to realize coupling unification. However, in our scenario, natural gauge coupling unification is realized in $SO(10)$ or E_6 GUT. Moreover, we obtained realistic quark and lepton mass matrices.

The proton decay via dimension 5 operators[21] is suppressed in our scenario, because effective colored Higgs mass is given by $m_H^c \sim \lambda^{2h}$ and the dimension 5 operators are suppressed due to anomalous $U(1)_A$ gauge symmetry. On the other hand, proton decay via dimension 6 operators is reachable, because the cutoff scale must be around the usual

GUT scale $\Lambda_G \sim 2 \times 10^{16}$ GeV and the unification scale is given by λ^{-a}. If we adopt $a = -1$, then the lifetime of the proton is roughly estimated as

$$\tau_p(p \to e\pi^0) \sim 5 \times 10^{33} \left(\frac{\Lambda_A}{5 \times 10^{15} \text{ GeV}} \right)^4 \left(\frac{0.015(\text{GeV})^3}{\alpha} \right)^2 \text{years}, \qquad (5.1)$$

which is near the present experimental lower bound[22]. Here α is the hadron matrix element parameter. Here we use the formula[23] and the value α given by lattice calculation [24]. Though the prediction is strongly dependent on the actual unification scale which is dependent on the order one coefficients, this rough estimation gives a strong motivation for the future experiments for proton decay search.

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