Reconstruction of Supersymmetric High Scale Theories

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Abstract

We demonstrate how the fundamental supersymmetric theory at high energy scales can be reconstructed using precision data expected at future high energy collider experiments. We have studied a set of representative examples in this context: minimal supergravity; gauge mediated supersymmetry breaking; and superstring effective field theories.

1 Introduction

Supersymmetry is one of the most attractive extensions of the Standard Model. Therefore the discovery of supersymmetric particles as well as the accurate measurement of their properties are among the main topics in the experimental programs for future high–energy colliders, such as LHC [1] and prospective e^+e^- linear colliders [2]. In this report we summarize how high–precision measurements of supersymmetric particles can be used to extract information on the underlying high scale theory at the GUT / Planck and to eventually reconstruct this theory [3].

The general strategy is based on the bottom–up approach. It can be summarized shortly as follows (see [3] for more details). From a specific theory at the high scale one calculates the observables at the electroweak scale, e.g. masses and cross sections. These observables are endowed with errors as expected at future high–energy experiments. This set of data is adopted as input and the origin of data in terms of the high scale theory is "forgotten". Next one extracts the parameters plus the corresponding errors at the electroweak scale from the experimental observables. These parameters are extrapolated to the high scale by means of renormalization group techniques. In this way one gains insight to which extent and with which accuracy the original theory can be reconstructed. In the following we exemplify this procedure by considering three examples: supergravity [4], gauge mediated supersymmetry breaking [5], and superstring effective field theories motivated by orbifold compactified heterotic string theories [6].

2 Gravity Mediated SUSY Breaking

Supersymmetry is broken in a hidden sector in supergravity models (SUGRA) and the breaking is transmitted to our eigenworld by gravitational interactions [4]. In this scheme it is suggestive although not compulsory that the soft SUSY breaking parameters are universal at the high scale, e.g. the GUT scale M_U.

Particle	M(GeV)	Δ M(GeV)	Particle	M(GeV)	Δ M(GeV)
h^0	113.33	0.05	$\tilde{e}_L$	269.1	0.3
A^0	435.5	1.5	$\tilde{e}_R$	224.82	0.15
$\tilde{\chi}_1^+$	183.05	0.15	$\tilde{\tau}_1$	217.7	1.00
$\tilde{\chi}_2^+$	383.3	0.3	$\tilde{\tau}_2$	271.5	0.9
$\tilde{\chi}_1^0$	97.86	0.2	$\tilde{u}_L$	589	10
$\tilde{\chi}_2^0$	184.6	0.3	$\tilde{u}_R$	572	10
$\tilde{g}$	598	10	$\tilde{t}_1$	412	10

Table 1: *Representative experimental mass errors used in the fits to the mass spectra; with the exception of the gluino mass, all the other parameters are based on LC measurements.*

The SUGRA point we have analyzed in detail, was chosen close to the Snowmass Point SPS#1 [7], except for the scalar mass parameter M_0 which was taken slightly larger for merely illustrative purpose: $M_{1/2} = 250$ GeV, $M_0 = 200$ GeV, $A_0 = -100$ GeV, $\tan \beta = 10$ and $sign(\mu) = +$. This set of parameters is compatible with the present results of low–energy experiments. The initial "experimental" values, have been generated by evolving the universal parameters down to the electroweak scale according to standard procedures [8, 9]. These parameters define the experimental observables, including supersymmetric particle masses and production cross sections. They are endowed with errors as expected for threshold scans as well as for measurements in the continuum at e^+e^- linear colliders (LC). The errors given in Ref.[10] are scaled in proportion to the masses of the spectrum. Typical examples are shown in Table 1. The LC errors on the squark masses, see *e.g.* Ref.[11], are set to an average value of 10 GeV. For the cross-sections we use only statistical errors, while assuming a (conservative) reconstruction efficiency of 20%. These observables are interpreted as the experimental input values for the evolution of the mass parameters in the bottom-up approach to the Grand Unification scale.

The presumably strongest support, though indirect, for supersymmetry is related to the tremendous success of this theory in predicting the unification of the gauge couplings [12]. The precision, being at the per–cent level, is surprisingly high after extrapolations over fourteen orders of magnitude in the energy from the electroweak scale to the unification scale M_U. The expected accuracies in M_U and α_U, based on the GigaZ option, are: $M_U = (2.000 \pm 0.016) \cdot 10^{16} \, \text{GeV}$ and $\alpha_U^{-1} = 24.361 \pm 0.007$.

For the evolution of the gaugino and scalar mass parameters two–loop RGEs [13] have been used. One–loop threshold effects are incorporated using the formulas given in [9] and in case of Higgs bosons we have included two–loop effects as given in [14]. The results for the evolution of the mass parameters to the GUT scale M_U are shown in Fig. 1. Fig. 1(a) presents the evolution of the gaugino parameters M_i^{-1} which clearly is under excellent control, as are the extrapolations of the slepton mass parameters squared of the first (and second) and the third generation in Fig. 1(c) and (d), respectively. The accuracy deteriorates for the squark mass parameters and for the Higgs mass parameter $M_{H_2}^2$. The origin of the differences between the errors for slepton, squark and Higgs mass parameters can be traced back to the structure of the RGEs. This can easily be
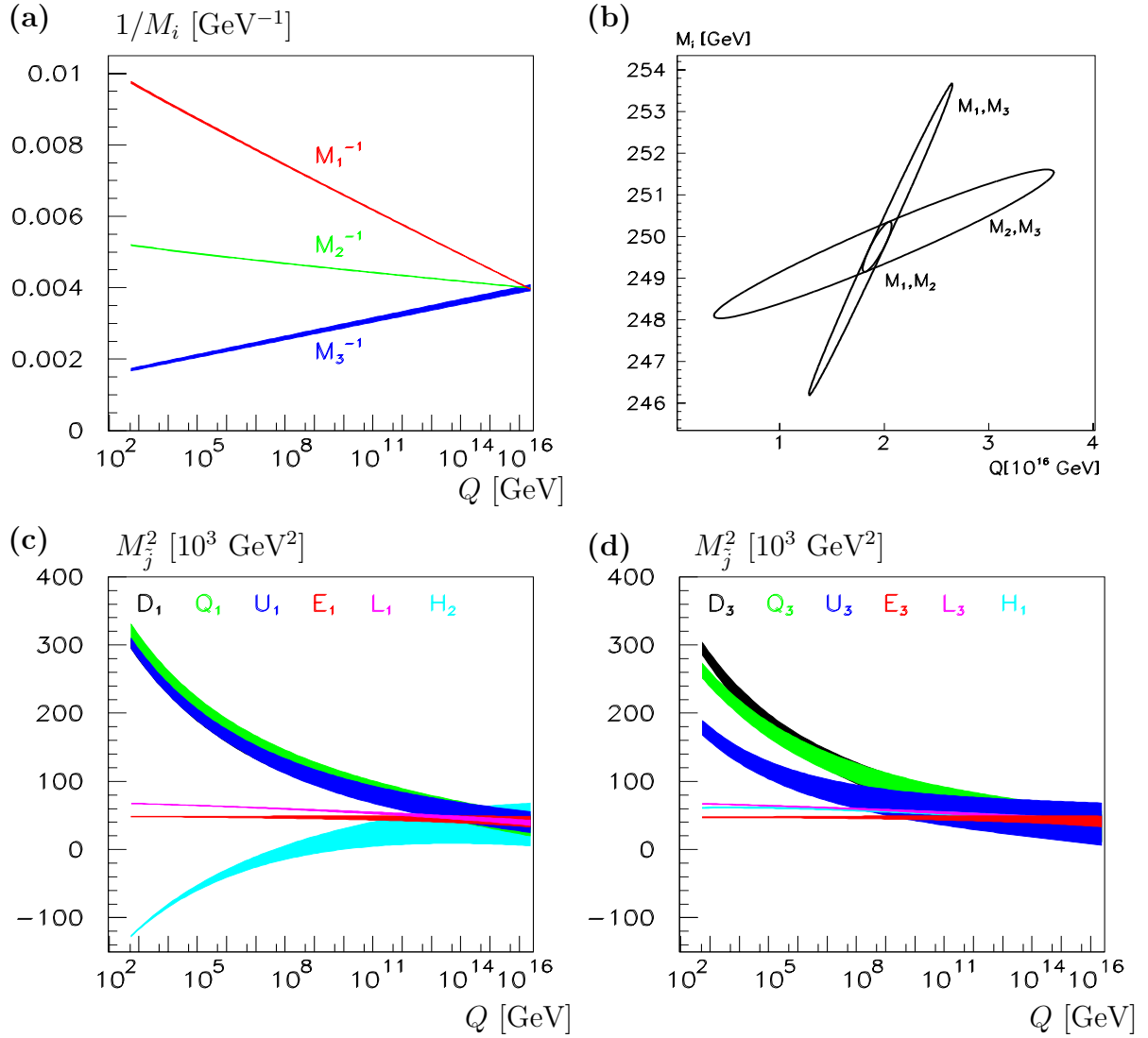
(a) $1/M_i$ [GeV^{-1}]

(b) M_i [GeV]

(c) $M_{\tilde{j}}^2$ [10^3 GeV^2]

(d) $M_{\tilde{j}}^2$ [10^3 GeV^2]

Figure 1: **mSUGRA:** *Evolution, from low to high scales, of (a) gaugino mass parameters, and (b) unification of gaugino mass parameter pairs; (c) evolution of first–generation sfermion mass parameters and the Higgs mass parameter $M_{H_2}^2$; (d) evolution of third–generation sfermion mass parameters and the Higgs mass parameter $M_{H_1}^2$. The initial parameters are given by: $M_0 = 200$ GeV, $M_{1/2} = 250$ GeV, $A_0 = -100$ GeV, $\tan\beta = 10$, and $\mathrm{sign}(\mu) = (+)$. [The widths of the bands indicate the 1σ CL.]*

	Exp. Input	GUT Value
M_1 [GeV]	102.31 ± 0.25	250.00 ± 0.33
M_2 [GeV]	192.24 ± 0.48	250.00 ± 0.52
M_3 [GeV]	586 ± 12	250.0 ± 5.3
μ	358.23 ± 0.28	355.6 ± 1.2
$M_{L_1}^2$ [GeV^2]	$(6.768 \pm 0.005) \cdot 10^4$	$(3.99 \pm 0.41) \cdot 10^4$
$M_{E_1}^2$ [GeV^2]	$(4.835 \pm 0.007) \cdot 10^4$	$(4.02 \pm 0.82) \cdot 10^4$
$M_{Q_1}^2$ [GeV^2]	$(3.27 \pm 0.08) \cdot 10^5$	$(3.9 \pm 1.5) \cdot 10^4$
$M_{L_3}^2$ [GeV^2]	$(6.711 \pm 0.050) \cdot 10^4$	$(4.00 \pm 0.41) \cdot 10^4$
$M_{E_3}^2$ [GeV^2]	$(4.700 \pm 0.087) \cdot 10^4$	$(4.03 \pm 0.83) \cdot 10^4$
$M_{Q_3}^2$ [GeV^2]	$(2.65 \pm 0.10) \cdot 10^5$	$(4.1 \pm 3.0) \cdot 10^4$
$M_{H_1}^2$ [GeV^2]	$(6.21 \pm 0.08) \cdot 10^4$	$(4.01 \pm 0.54) \cdot 10^4$
$M_{H_2}^2$ [GeV^2]	$(-1.298 \pm 0.004) \cdot 10^5$	$(4.1 \pm 3.2) \cdot 10^4$

Table 2: *Representative gaugino/scalar mass parameters and couplings as determined at the electroweak scale and evolved to the GUT scale in the mSUGRA scenario; based on LHC and LC simulations. [The errors quoted correspond to 1σ.]*

understood by inspecting the approximate solutions of the RGEs. Typical examples evaluated at the scale $Q = 500$ GeV read as follows:

$$M_{\tilde{L}_1}^2 \simeq M_0^2 + 0.47 M_{1/2}^2 \tag{1}$$

$$M_{\tilde{Q}_1}^2 \simeq M_0^2 + 5.0 M_{1/2}^2 \tag{2}$$

$$M_{\tilde{H}_2}^2 \simeq -0.03 M_0^2 - 1.34 M_{1/2}^2 + 1.5 A_0 M_{1/2} + 0.6 A_0^2 \tag{3}$$

While the coefficients for sleptons are of order unity, the coefficient for squarks in front of $M_{1/2}^2$ is 5, so that small errors in $M_{1/2}^2$ are magnified by nearly an order of magnitude in the solution for M_0. This feature becomes even more enhanced for the Higgs mass parameter, giving rise to large errors in this case. A representative set of mass values and the associated errors, after evolution from the electroweak scale to M_U, is presented in Table 2. The corresponding error ellipses for the unification of the gaugino masses are shown in Fig. 1(b).

Inspecting Figs. 1(c) and (d) leads us to the conclusion that a blind top-down approach eventually may generate an incomplete picture. Global fits based on mSUGRA without allowing for deviations from universality, are dominated by $M_{1,2}$ and the slepton mass parameters due to the pseudo-fixed point behaviour of the squark mass parameters. Therefore, the structure of the theory in the squark sector is not scrutinized stringently at the unification scale in the top-down approach let alone the Higgs sector. By contrast, the bottom-up approach demonstrates very clearly the extent to which the theory can be tested at the high scale quantitatively. The quality of the global test is apparent from Table 2, in which

the evolved gaugino values should reproduce the universal mass $M_{1/2} = 250$ GeV and all the scalars the mass $M_0 = 200$ GeV. They are compared with the global mSUGRA fit of the universal parameters where we find $M_{1/2} = 250 \pm 0.08$ GeV and $M_0 = 200 \pm$
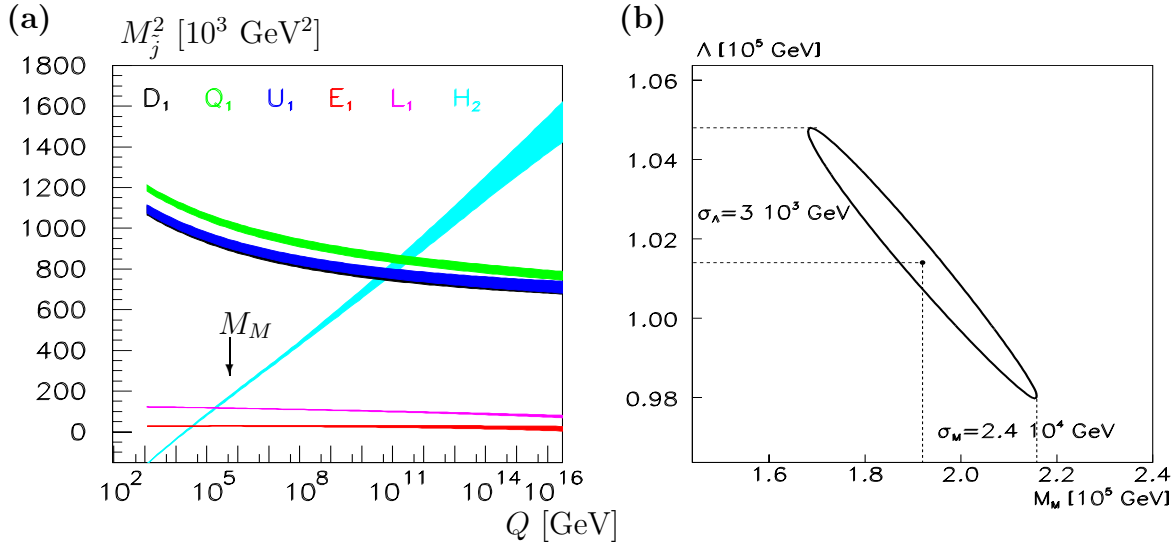
Figure 2: **GMSB:** *Evolution of (a) first–generation sfermion mass parameters and Higgs mass parameter $M_{H_2}^2$ and (b) Λ–M_M determination in the bottom–up approach. The point probed, SPS#8, is characterized by the parameters $M_M = 200$ TeV, $\Lambda = 100$ TeV, $N_5 = 1$, $\tan\beta = 15$, and $\text{sign}(\mu) = (+)$. [The widths of the bands indicate the 1σ CL.]*

0.09 GeV.

3   Gauge Mediated Supersymmetry Breaking

In gauge mediated supersymmetry breaking (GMSB) [5] the scalar and the F components of a Standard–Model singlet superfield S acquire vacuum expectation values $\langle S \rangle$ and $\langle F_S \rangle$ through interactions with other fields in the secluded sector, thus breaking supersymmetry. The messenger fields mediating the breaking to our eigen-world and the two vacuum expectation values characterize the system. The general scale is given by the messenger mass $M_M \sim \langle S \rangle$ whereas the size of gaugino and scalar masses is set by $\Lambda = \langle F_S \rangle / \langle S \rangle$. The gaugino masses are generated at 1–loop level by loops of scalar and fermionic messenger component fields. Masses of the scalar fields in the visible sector are generated by 2-loop effects of gauge/gaugino and messenger fields. The masses are equal at the messenger scale M_M for scalar particles with identical Standard–Model charges squared. In the minimal version of GMSB, the A parameters are generated at 3-loop level and they are practically zero at M_M.

We have investigated this scheme for the point $\Lambda = 100$ TeV, $M_M = 200$ TeV, $N_5 = 1$, $N_{10} = 0$, $\tan\beta = 15$ and $\mu > 0$ corresponding to the Snowmass Point SPS#8 [7]. The evolution of the sfermion mass parameters of the first generation as well as the Higgs mass parameter $m_{H_2}^2$ is shown in Fig. 2a. It is obvious from the figure that the GMSB scenario cannot be confused with the universal supergravity scenario. [Specific experimental signatures generated in the decays of the next to lightest supersymmetric particle to gravitinos provide a complementary experimental discriminant, see e.g. [15]].

The bands of the slepton L–doublet mass parameter $M_{\tilde{L}}^2$ and the Higgs parameter $M_{H_2}^2$, which carry the same moduli of standard–model charges, cross at the scale M_M. The crossing, which is indicated by an arrow in Fig. 2(a), is a necessary condition for the GMSB scenario to be realized. The determination of scalar and gaugino mass parameters at this "meeting point" can be used to extract Λ, M_M and the multiplicity coefficient $(N_5 + 3N_{10})$. For the point analyzed one finds:

$$\Lambda = (1.01 \pm 0.03) \cdot 10^2 \text{ TeV} \tag{4}$$

$$M_M = (1.92 \pm 0.24) \cdot 10^2 \text{ TeV} \tag{5}$$

$$N_5 + 3N_{10} = 0.978 \pm 0.056 \tag{6}$$

in good agreement with the theoretical ideal input values. The correlation between Λ and M_M is shown in Fig. 2(b).

4   String Induced Supersymmetry Breaking

Superstring theories are among the most exciting candidates for a comprehensive theory of matter and interactions. Here we summarize results obtained for a string effective theory in four dimensions based on orbifold compactification of the heterotic superstring [6]. SUSY breaking is generated by non–pertubative effects, mediated by a Goldstino field which is a superposition of the dilaton field S and the moduli field T [all moduli fields are assumed to be identical]: $\tilde{G} = \sin\theta\,\tilde{S} + \cos\theta\,\tilde{T}$. Universality is generally broken in such a scenario by a set of non–universal modular weights $\{n_j\}$ that determine the couplings of T to the SUSY matter fields Φ_j.

$$M_i = -g_i^2 m_{3/2} s \sqrt{3} \sin\theta + \ldots \tag{7}$$

$$M_{\tilde{j}}^2 = m_{3/2}^2 \left(1 + n_j \cos^2\theta\right) + \ldots \tag{8}$$

In next–to–leading order, indicated by the ellipses, also the vacuum value $< T >$ and the Green–Schwarz parameter δ_{GS} enter. The one loop effects give rise to non–universal corrections for the gaugino mass parameters as well as for the gauge couplings at the unification scale M_{GUT}. A precise measurement sensitive to these one–loop effects for the gauge couplings and the SUSY parameters can thus be used to get information about the string scenario.

In Fig. 3 we show the evolution of the gaugino mass parameters with the crucial high–scale region expanded in the insert. Here we have taken the parameters presented in Table 3 as input. Relevant parameters derived from an overall–fit to couplings and masses are given in Table 3. One clearly sees that the ideal data, from which the experimental observables were deduced, can indeed by extracted from the data collected at future hadron– and lepton colliders performing high precision measurements.
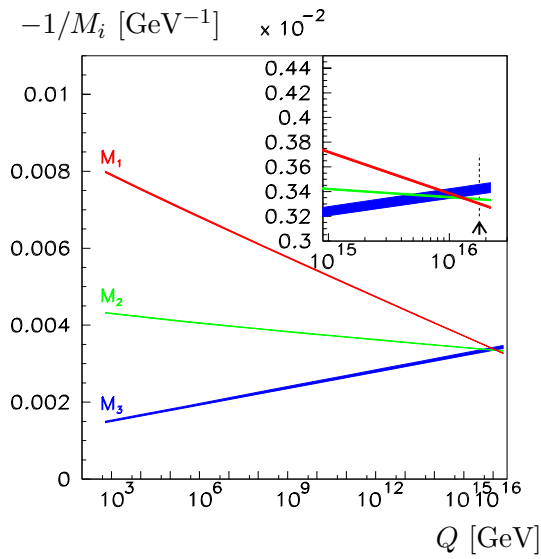
Figure 3: **String Scenario:** *Evolution of gaugino mass parameters [the insert expands on the breaking of universality at the GUT scale].*

Parameter	Ideal	Reconstructed		
$m_{3/2}$	180	179.9	$\pm$	0.4
$\langle S \rangle$	2	1.998	$\pm$	0.006
$\langle T \rangle$	14	14.6	$\pm$	0.2
$\sin^2 \theta$	0.9	0.899	$\pm$	0.002
δ_{GS}	0	0.1	$\pm$	0.4
n_L	-3	-2.94	$\pm$	0.04
n_E	-1	-1.00	$\pm$	0.05
n_Q	0	0.02	$\pm$	0.02
n_U	-2	-2.01	$\pm$	0.02
n_D	+1	0.80	$\pm$	0.04
n_{H_1}	-1	-0.96	$\pm$	0.06
n_{H_2}	-1	-1.00	$\pm$	0.02
$\tan \beta$	10	10.00	$\pm$	0.13

Table 3: *Comparison of the experimentally reconstructed values with the ideal fundamental parameters in a specific example for a string effective field theory.*

5 Conclusions

In this report we have demonstrated that fundamental parameters of the underlying supersymmetric theory at the high scale can be reconstructed in practice. The reconstruction is based on future high–precision data from e^+e^- linear colliders, TESLA in particular, combined with results from LHC and CLIC. The bottom–up approach of evolving the parameters from the electroweak scale to the high scale provides a transparent picture of the underlying theory at the high scale. We have exemplified this conclusions in the cases of minimal supergravity theories, gauge mediated supersymmetry breaking, and for a string effective theory. In the latter example we have demonstrated that one can indeed extract the string parameters – a truly exciting observation – from future high–precision measurements at hadron– and lepton–colliders.

Acknowledgements

References

[1] I. Hinchliffe et al., Phys. Rev. **D 55**, 5520 (1997); Atlas Collaboration, Technical Design Report 1999, Vol. II, CERN/LHC/99-15, ATLAS TDR 15.

[2] P.M. Zerwas, Proceedings, 1999 Cargèse Institute for High-Energy Physics, hep-ph/0003221; H. Murayama, M.E. Peskin, Ann. Rev. Nucl. Part. Sci. **46**, (1996) 533; E. Accomando *et al.*, ECFA/DESY LC Working Group, Phys. Rep. **299** (1998) 1; J.A. Aguilar-Saavedra *et al.* [ECFA/DESY LC Physics Working Group Collaboration], "TESLA Technical Design Report Part III: Physics at an e+e- Linear Collider," hep-ph/0106315.

[3] G.A. Blair, W. Porod, P.M. Zerwas, Phys. Rev. **D 63** (2001) 017703; and preprint hep-ph/0210058.

[4] A.H. Chamseddine, R. Arnowitt, P. Nath, Phys. Rev. Lett. **49** (1982) 970.

[5] M. Dine, A.E. Nelson, Phys. Rev. **D 48** (1993) 1277.

[6] V.S. Kaplunovsky, Nucl. Phys. **B 307** (1988) 145 [Erratum-ibid. **B 382** (1992) 436]; L.E. Ibáñez, D. Lüst and G.G. Ross, Phys. Lett. **B 272** (1991) 251; M. Cvetič et al., Nucl. Phys. **B 361** (1991) 194; J.P. Derendinger et al., Nucl. Phys. **B 372** (1992) 145; A. Brignole, L.E. Ibáñez, C. Muñoz, Nucl. Phys. **B 422** (1994) 125 [Erratum-ibid. **B 436** (1995) 747]; A. Love, P. Stadler, Nucl. Phys. **B 515** (1998) 34. P. Binetruy, M.K. Gaillard, B.D. Nelson, Nucl. Phys. **B 604** (2001) 32.

[7] B.C. Allanach *et al.*, Eur. Phys. J. **C 25** (2002) 113.

[8] H. Arason et al., Phys. Rev. **D46**, 3945 (1992).

[9] J. Bagger et al., Nucl. Phys. **B 491** (1997) 3.

[10] G.A. Blair, U. Martyn, Proceedings, LC Workshop, Sitges 1999, hep-ph/9910416.

[11] J.L. Feng, D.E. Finnell, Phys. Rev. **D49**, 2369 (1994); A. Bartl et al., EPJdirect **C 6**, 1 (2000).

[12] S. Dimopoulos, S. Raby, F. Wilczek, Phys. Rev. **D 24** (1981) 1681; L.E. Ibáñez, G.G. Ross, Phys. Lett. **B 105** (1981) 439; U. Amaldi, W. de Boer, H. Fürstenau, Phys. Lett. **B 260** (1991) 447; P. Langacker, M. Luo, Phys. Rev. **D 44** (1991) 817; J. Ellis, S. Kelley, D.V. Nanopoulos, Phys. Lett. **B 260** (1991) 161.

[13] S. Martin, M. Vaughn, Phys. Rev. **D50**, 2282 (1994); Y. Yamada, Phys. Rev. **D 50**, 3537 (1994); I. Jack, D.R.T. Jones, Phys. Lett. **B333** (1994) 372.

[14] G. Degrassi, P. Slavich, F. Zwirner, Nucl. Phys. **B 611** (2001) 403; A. Brignole et al., Nucl. Phys. **B 631** (2002) 195.

[15] S. Ambrosanio, G.A. Blair, Eur. Phys. J. **C 12** (2000) 287.