Product Groups, Discrete Symmetries, and Grand Unification

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Abstract

We present GUT models based on an $SU(5) \times SU(5)$ GUT group. These models maintain the main successes of simple-group GUTs but permit simple solutions to the doublet-triplet splitting problem. Moreover, GUT breaking is triggered by supersymmetry breaking so that the GUT scale is naturally generated as a combination of the Planck scale and the supersymmetry breaking scale.

1 Introduction and summary

The unification of couplings in the MSSM is often viewed as a hint for a grand unified theory (GUT) as well as for supersymmetry. Here we will explore the possibility that the relation between supersymmetry and grand unification is even deeper, and that GUT breaking is a result of supersymmetry breaking [1]. The GUT breaking VEV in our models corresponds to an approximately flat direction, which is only lifted by higher dimension superpotential terms, suppressed by M_{Planck}. Once supersymmetry is broken, if some of the GUT-breaking fields obtain negative soft masses^2 around the TeV, the GUT is broken, and the GUT scale is determined as a combination of the Planck scale and the TeV. How does one generate an almost flat potential for the GUT breaking fields? An obvious way is to charge these fields under some discrete symmetry. Such a symmetry can forbid all superpotential terms up to some desired order.

Discrete symmetries can also lead to doublet-triplet splitting. Witten recently revisited this problem[2], and beautifully explained why it is readily solved in product-group GUTs[3, 4]. The idea is to impose a (global) discrete symmetry which is broken at the GUT scale. Below the GUT scale, a combination of this symmetry and the GUT symmetry remains unbroken and forbids the doublet mass. If the GUT group is simple, a discrete symmetry cannot distinguish between the triplet and doublet mass term. In a semi-simple GUT however, with Higgses transforming under different group factors, the doublet mass term and triplet mass term can have different charges under the unbroken discrete symmetry. We will explain this mechanism in detail in Section 2.1.

As it turns out, the product group structure[5], together with the discrete symmetry, is

crucial both for doublet-triplet splitting, and for ensuring an approximately flat potential for the GUT breaking fields.

Furthermore, in a product group GUT, the standard model matter fields and Higgses may originate from different GUT group factors. This has a number of consequences, which are all related to the fact that some of the Yukawas will appear as non-renormalizable terms, and will therefore be suppressed by some power of $M_{\mathrm{GUT}}/M_{\mathrm{Pl}}$. Clearly one gets some non-trivial hierarchies of Yukawa couplings. Second, the Higgs-triplet couplings to standard model fields can be suppressed relative to the usual Yukawa couplings. Thus the proton decay constraint on the triplet mass can in principle be relaxed. Proton decay is then suppressed by a combination of doublet-triplet "mass splitting" (the usual mechanism) and "Yukawa splitting". Third, matter fields of different generations may transform under different GUT group factors, so that bottom-tau mass unification can still hold, while similar relations for the first generations are avoided.

Product group GUTs often seem problematic, because the standard model gauge couplings arise from different couplings at the high scale. To maintain unification some GUT group factors have to be strongly coupled. In addition, hypercharge quantization is lost. None of this happens in the models we consider. Because the standard model is a subgroup of the diagonal $SU(5)$, hypercharge arises in the usual way, and the three couplings originate from a single coupling—the coupling of the diagonal group.

2   A model

We will now present an explicit model and use it to demonstrate the ideas we discussed. The gauge group is $SU(5) \times SU(5)$, and we impose a discrete symmetry $Z_N \times Z_M^R$, the latter being an R-symmetry. The fields and their charges are summarized in Table 1. The

Field	$SU(5) \times SU(5) \times Z_N \times Z_M^R$
Φ_1	$(5, \bar{5}, 1, 0)$
$\bar{\Phi}_1$	$(\bar{5}, 5, N-1, 0)$
Φ_2	$(5, \bar{5}, (N-3)/2, 0)$
$\bar{\Phi}_2$	$(\bar{5}, 5, (N+3)/2, 0)$
A_1	$(24, 1, (N-5)/2, 1)$
A_2	$(24, 1, (N+5)/2, 1)$
A_3	$(24, 1, 0, 1)$
S	$(1, 1, 0, M-1)$
h	$(5, 1, 1, 1)$
$\bar{h}'$	$(1, \bar{5}, 0, 0)$
h	$(\bar{5}, 1, 0, 0)$
h'	$(1, 5, N-1, 1)$

Table 1: The fields of the basic model

symmetries allow the following superpotential

$$W = \lambda_{12}\Phi_1 A_1 \bar{\Phi}_2 + \lambda_{21}\Phi_2 A_2 \bar{\Phi}_1 + \lambda_{11}\Phi_1 A_3 \bar{\Phi}_1 + \lambda_{22}\Phi_2 A_3 \bar{\Phi}_2 + \eta_{12} S A_1 A_2 + \eta_{33} S A_3 A_3$$
$$+ \quad \frac{1}{M_{\mathrm{Pl}}^{M-4}} S^{M-1} + \frac{1}{M_{\mathrm{Pl}}^{M-2}} S^{M-1}\Phi_i \bar{\Phi}_i + \cdots \tag{1}$$

where we do not show non-renormalizable terms that do not contribute to the scalar potential, and the dots stand for higher-dimension terms.

We will be interested in the direction

$$\begin{aligned}
\langle \Phi_1 \rangle = \langle \bar{\Phi}_1 \rangle &= v_1 \times \mathrm{diag}(1,1,1,0,0), \\
\langle \Phi_2 \rangle = \langle \bar{\Phi}_2 \rangle &= v_2 \times \mathrm{diag}(0,0,0,1,1), \\
\langle S \rangle &= s, \\
\langle A_i \rangle &= 0.
\end{aligned} \tag{2}$$

Along this direction, it is easy to see that with $\lambda_{11} v_1^2 = \lambda_{22} v_2^2$, the only contribution to the scalar potential is from the non-renormalizable terms appearing in (1). The bi-fundamental VEVs indeed break $SU(5) \times SU(5)$ down to the standard model gauge group. The three adjoints A_i and the singlet are required in order to give mass to all the Goldstone bosons.

2.1   Splitting doublets from triplets

Let us first discuss how doublet-triplet splitting works here. The bi-fundamental VEVs of (2) preserve a discrete symmetry Z_N' which is a combination of the original Z_N and a discrete subgroup of hypercharge of, say, the first $SU(5)$. (The latter is of the form

$$\mathrm{diag}(\alpha^{-1}, \alpha^{-1}, \alpha^{-1}, \alpha^{(N+3)/2}, \alpha^{(N+3)/2})$$

for N even.) This unbroken symmetry distinguishes between the Higgs doublets and triplets. Furthermore, if the Higgses are charged under *different* $SU(5)$ factors, their mass terms have different Z_N' charges. Thus, the doublet mass term can be forbidden while the triplet mass term is allowed. This is not the case if the Higgses are a 5 and a $\bar{5}$ of the same $SU(5)$. We then see why this idea only works in the context of semi-simple GUTs.

Imagine then, that the standard model Higgses are the fields h and $\bar{h}'$ of Table 1. We may want to add the remaining fields h' and $\bar{h}$ to cancel anomalies (we will discuss different possibilities shortly). The most general renormalizable superpotential that involves the h fields is:

$$W_1 = h\bar{\Phi}_1 \bar{h}' + h'\Phi_1 \bar{h}. \tag{3}$$

Since the h fields do not couple to the Φ_2 and S fields, only the triplets acquire masses.

2.2   GUT breaking and supersymmetry breaking

The symmetries of our model can ensure an almost flat potential for the GUT fields. Roughly, the potential scales as

$$V \sim \frac{v^{2n-2}}{M_{\text{Pl}}^{2n-6}} \ , \qquad (4)$$

where v is the typical VEV and in our example n is either $M-1$ or $M+1$ (M is associated with the discrete symmetry Z_M^R as we will now see, we will want to take $M \sim 10$, so terms with $n = M+1$ of $n = M-1$ will give similar results). If some or all of the GUT breaking fields get a negative soft mass-squared $m^2 < 0$ of order the weak scale, the minimum of the potential is at

$$v \sim \left(\frac{m}{M_{\text{Pl}}}\right)^{\frac{1}{n-2}} M_{\text{Pl}} \ . \qquad (5)$$

This is around 10^{16}GeV for $n \sim 10$.

3   Standard model Higgses and Yukawa couplings

So far, we concentrated on the GUT breaking fields. We saw that we can break the GUT symmetry to the standard model gauge group, and give mass to all GUT breaking fields. We could also generate the GUT scale from supersymmetry breaking. Finally, the discrete symmetry of the model allowed mass terms for all Higgs triplets. The unbroken discrete symmetry further forbids mass terms for all Higgs doublets. Thus, if we have two pairs of 5 and $\bar{5}$ Higgses as in Table 1, we are left with four light doublets.

There are then three options:

- **A**: The theory does not contain h' and $\bar{h}$. $SU(5)$ anomalies are canceled by appropriate choices of $SU(5) \times SU(5)$ representations for the standard model matter fields. The standard model Higgses come from fields charged under different $SU(5)$'s. Some standard model Yukawa couplings arise from non-renormalizable terms. Z'_N can be broken by supersymmetry breaking effects to generate the μ term.

- **B**: The theory does contain h' and $\bar{h}$, but these remain massless[1] [2]. Witten speculates that these could be the messengers of supersymmetry breaking. The heavy triplets are from h and $\bar{h}'$.

  - **B1**: The standard model Higgses come from fields charged under different $SU(5)$'s, say, h and $\bar{h}'$, so that, again, some standard model Yukawa couplings arise from non-renormalizable terms.

  - **B2**: The standard model Higgses come from fields charged under a single $SU(5)$, say, h and $\bar{h}$. Then, standard model fields can all be charged under the same $SU(5)$, and all Yukawa couplings are renormalizable.

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[1] It is easy to forbid the relevant mass terms by choosing appropriate charges for h' and $\bar{h}$.

- **C**: The theory does contain h' and $\bar{h}$. All triplets gain mass through the couplings (3). The Z'_N is broken at a high scale, so that one doublet pair also gets mass around M_{GUT}. It is possible to arrange for an acceptable μ term for the remaining two doublets, for example, through the mechanisms proposed in [6] or in [4]. This is most easily done by adding a gauge-singlet, S_H, charged under the Z_N, with a GUT scale VEV. Then, in order to have masses of order M_{GUT}, the doublets that couple to S_H must be charged under the same $SU(5)$, and the Higgs doublets are charged under the second $SU(5)$.

This clearly leads to interesting predictions for quark and lepton masses. Some of the Yukawa terms can only arise from higher-dimension operators involving powers of the bifundamental fields and suppressed by M_{Pl}. In addition, because of the discrete symmetry, when some doublet Yukawa couplings are allowed, the corresponding triplet coupling is forbidden. This gives rise to triplet "Yukawa splitting". For further details, we refer the reader to reference [7].

References

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