A New Cosmological Scenario
in String Theory

Miguel S. Costa  (École Normale Supérieure)

hep-th/0203031 with Lorenzo Cornalba
hep-th/0204261 with Lorenzo Cornalba & Costas Kounnas
Introduction

- Cosmological Singularity Problem

\[
\rho - p = 3\rho > 0
\]

Cannot then reverse from contraction to expansion.

[Singularity Theorems]: basic assumptions are global structure and reasonable matter \( \rho + 3p > 0 \)

[Horizon Problem]

- DeSitter Space evades these problems with a positive cosmological constant \( \Lambda \)

\[
p = -\rho
\]
New Space-Time Global Structure

- Consider the effective d+1-dimensional gravitational action

\[
\int d^{d+1}x \sqrt{g} \left[ R - \frac{\beta}{2} (\nabla \psi)^2 - V(\psi) \right]
\]

- General solution with a cosmological horizon at \( t = 0 \) and SO(1,d) symmetry:

\[
\begin{align*}
\text{Region I - Open Cosmology} & & \text{Region II - 'Static' Region} \\
d s_{d+1}^2 &= -dt^2 + a_I^2(t) ds^2(H_d) & d s_{d+1}^2 &= d\kappa^2 + a_H^2(x) ds^2(dS_d) \\
\psi &= \psi_I(t) & \psi &= \psi_H(x)
\end{align*}
\]

- Gluing conditions on the horizon and analytic continuation

\[
\begin{align*}
a_I(t) &= t + o(t^3) & \psi_I(t) &= \psi_0 + o(t^2) \\
a_H(x) &= ia_I(ix) & \psi_H(x) &= \psi_I(ix)
\end{align*}
\]

Gluing conditions on the horizon and analytic continuation

Expanding

Intermediate: A singularity will have a \textit{branelike} interpretation

Contracting
Embedding in String Theory

- **Toroidal Compactification of Type II** \((\tilde{d} = 9 - d)\)

\[
E^2 ds^2 = \Lambda^\frac{1}{2d-1} ds_{d+1}^2 + \Lambda^{\frac{1}{2}} ds^2 (\mathbf{E}^d)
\]

\[
e^\phi = g_s \Lambda^{\frac{4-d}{4}}
\]

\[
\tilde{F} = \frac{1}{g_s E^{d-1}} \epsilon (\mathbf{E}^d)
\]

- As before \(d+1\)-dimensional action with scalar potential

\[
V(\psi) = \frac{1}{2} e^{-\psi}
\]

- Behavior of scale factor

- Carter-Penrose diagram

- **Embedding in String Theory**

- **Toroidal Compactification of Type II** \((\tilde{d} = 9 - d)\)

\[
E^2 ds^2 = \Lambda^\frac{1}{2d-1} ds_{d+1}^2 + \Lambda^{\frac{1}{2}} ds^2 (\mathbf{E}^d)
\]

\[
e^\phi = g_s \Lambda^{\frac{4-d}{4}}
\]

\[
\tilde{F} = \frac{1}{g_s E^{d-1}} \epsilon (\mathbf{E}^d)
\]

- As before \(d+1\)-dimensional action with scalar potential

\[
V(\psi) = \frac{1}{2} e^{-\psi}
\]

- Behavior of scale factor

- Carter-Penrose diagram

- **Embedding in String Theory**

- **Toroidal Compactification of Type II** \((\tilde{d} = 9 - d)\)

\[
E^2 ds^2 = \Lambda^\frac{1}{2d-1} ds_{d+1}^2 + \Lambda^{\frac{1}{2}} ds^2 (\mathbf{E}^d)
\]

\[
e^\phi = g_s \Lambda^{\frac{4-d}{4}}
\]

\[
\tilde{F} = \frac{1}{g_s E^{d-1}} \epsilon (\mathbf{E}^d)
\]

- As before \(d+1\)-dimensional action with scalar potential

\[
V(\psi) = \frac{1}{2} e^{-\psi}
\]

- Behavior of scale factor

- Carter-Penrose diagram
• No Horizon Problem

• Structure of the singularity

$$E^2 ds^2 \cong \Lambda^{-\frac{1}{2}} \left[ \mu ds^2(dS_d) \right] + \Lambda^{2} \left[ d\Lambda^2 + ds^2(\tilde{E}^d) \right]$$

$$e^\phi = g_s \Lambda^{-\frac{4-d}{4}}$$

$$F = \frac{1}{g_s E^d} \frac{1}{\Lambda^2} d\Lambda \wedge \epsilon(dS_d)$$

Similar to D(d-1)-brane metric delocalized along the $\tilde{E}^d$. Harmonic function

$$H(\Lambda, \tilde{E}^d) = \Lambda$$

But

$$\text{Tension} \propto -\nabla^2 H < 0$$

Negative tension O(d-1)-plane, smeared over the transverse 9-d directions, with a deSitter world-volume.

- Solution of SUGRA with negative tension brane source on the O-plane world-volume $\Gamma$

$$|T| \int_{\Gamma} d^d x e^{-\phi} \sqrt{-\det G} \pm O \int_{\Gamma} A$$

- Near singularity orientifold is locally near flat and BPS

- Radius

$$L \approx \frac{1}{E}$$

- Number of O-planes per unit transverse volume

$$n \approx \frac{l_s E}{g_s}$$
Two Dimensional Toy Model

- The case of the O-particles can be obtained as the M-theory compactification

\[ M^3 / \Gamma \times \mathbb{E}^8 \]

where \( \Gamma \) is Boost & Translation.

Start with flat metric on the three dimensional space \( M^3 \)

\[ ds^2 = -dX^+dX^- + dY^2 \]

Then

\[ \Gamma = e^\kappa \quad \kappa = 2\pi (\Delta J + RP) \]

\[ iJ = X^+\partial_- - X^-\partial_+ \quad iP = \partial_Y \]

No fixed points

- Orientifold singularity will appear where \( \kappa \) becomes null

\[ \kappa \cdot \kappa = 0 \quad \Rightarrow \quad X^+X^- = -\frac{1}{E^2} \quad E \equiv \frac{\Delta}{R} \]

- Compactify to IIA theory by choosing coordinates where \( \kappa = 2\pi R \partial_Y \)

Natural coordinates in Milne and Rindler wedges: Polar Coordinates

\[ E X^\pm = te^{\pm(z+iy)} \]

I: Milne

\[ y = Y \]

II: Rindler

\[ y = Y \]
• Background fields are:

**Region I**

\[ E^2 ds^2 = \Lambda^{1/2} \left[ -dt^2 + ds^2(E^8) \right] + \frac{l^2}{\Lambda^{1/2}} dz^2 \]

\[ e^\phi = g_s \Lambda^4 \]

\[ A = -\frac{1}{g_s E \Lambda} \]

\[ \Lambda = 1 + t^2 \]

**Region II**

\[ E^2 ds^2 = \Lambda^{1/2} \left[ d\kappa^2 + ds^2(E^8) \right] - \frac{x^2}{\Lambda^{1/2}} ds^2 \]

\[ e^\phi = g_s \Lambda^4 \]

\[ A = -\frac{1}{g_s E \Lambda} \]

\[ \Lambda = 1 - x^2 \]

- 2D model with contraction for \( t < 0 \) and expansion for \( t > 0 \)

- Cosmological horizons

- Singularity in Region II at \( x^2 = 1 \) \( \implies \) \( O \bar{O} \) pair delocalized on \( E^8 \)

- T-duality along all the \( E^8 \) directions gives a \( O8 - \bar{O}8 \) pair at a distance \( L \)

\[ L \approx \frac{1}{E} \]

\[ 1 = N \approx \frac{l_s E}{g_s} \]

• Carter-Penrose Diagram

Similar 2D structure:

[Kounnas, Lust]

[Grojean et al.]
• 2D geometry as a limit of the BTZ black hole

Limit of large $AdS_3$ radius $L - \infty$ such that

\[ r_- = R \quad r_+ = \Delta L - \infty \]

Region outside the black hole is removed in the limit. We have

\[ r_+ \to I^- \]

• Penrose-Simpson instability of inner horizon

These modes do not exist when

\[ r_+ \to I^- \]
Perturbative String Description

• O8-plane BPS configuration: Type IIA on

\[ S^1 / I \Omega \quad \text{World-sheet parity} \]

\[ \text{Reflection} \]

• \( O8 - \overline{O8} \) non-BPS configuration: Type IIA on

\[ S^1 / G \quad G \text{ generated by} \]

\[ g_1 = I \Omega \]

\[ g_2 = I \Omega (-)^F \delta \]

\[ \delta : x \rightarrow x + \pi R \]

\[ O8 \quad \overline{O8} \quad g_1 \text{ and } g_2 \text{ break the opposite half of SUSY} \]

- Usually one adds D-branes for \textbf{Tadpole cancellation}.

- Gravity solution (tree level string theory) corresponds to the \textbf{backreaction} of closed strings to the O-planes

\[ O8 \quad \overline{O8} \]

• Question: what is dynamics behind such configuration?
Orientifold Repulsion

Naïve point of view: the $O8 - \overline{O8}$ should attract and annihilate

Gravity solution shows repulsion

- Consider a D-brane probe between $O\overline{O}$ pair and calculate static potential.

Naïve potential without backreaction

1. $OD$ is SUSY
2. $\overline{OD}$ repel

Complete potential

D-brane attracted to core of geometry

Positive energy density at the core

Orientifolds are repelled by energy density at the core of geometry.

<table>
<thead>
<tr>
<th>Energy</th>
<th>Charge</th>
<th>Two competing forces:</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>0</td>
<td>1. $O\overline{O}$ attraction</td>
</tr>
<tr>
<td>+</td>
<td>+</td>
<td>2. Repulsion from energy density at the core</td>
</tr>
</tbody>
</table>

Second force wins!
Cosmological Thermal Radiation

- Consider minimally coupled scalar in 2D model.
  Laplace equation $\nabla^2 \chi = 0$ gives the mode decomposition
  
  Region I: $e^{ipz} H_p (\pm pt)$
  Region II: $e^{ips} H_p (\pm ipx)$

- Orientifold boundary conditions at singularity
  $\chi = 0$ or $\partial_z \chi = 0$

- Assume trivial vacuum in the far past. From Bogobulov transformations
  observer in the far future will measure a thermal spectrum with
  
  $$ T \equiv \frac{\tilde{E}}{2\pi a(i)} $$

- Temperature arises because of cosmological horizon. From surface gravity
  at the horizon this formula also holds in (d+1)-dimensional model.

  Expected for radiation in d+1 dimensional FRW geometry

  $$ \rho \approx T^{d-1} $$

  $$ \rho \approx \frac{1}{\alpha} \cdot \frac{1}{d^d} $$

  Redshift Expansion
Conclusions

1. Gravity solutions
   - Similar causal structure. How generic?
   - Gravity + negative-tension boundary conditions ↔ Presence of horizon?

2. CFT
   - Precise relation with a CFT description (rolling tachyon and Sen’s latest work).

3. Cosmology
   - Make a “realistic” model with a more complicated compactification and an exit to standard radiation domination.
   - Revisit standard literature allowing for the possibility of a cosmological horizon replacing the big-bang singularity.