

A New Cosmological Scenario in String Theory

Lorenzo Cornalba¹, Miguel S. Costa^{2,3}, and Costas Kounnas³

¹ Instituut voor Theoretische Fysica, Universiteit van Amsterdam

Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands

² Departamento de Física, Faculdade de Ciências da Universidade do Porto

Rua do Campo Alegre 687, 4169-007 Porto, Portugal

³ Laboratoire de Physique Théorique, Ecole Normale Supérieure

24 rue Lhomond, F-75231 Paris Cedex 05, France

In this note we show how the presence of a negative cosmological constant localized on a time-like codimension one hypersurface avoids the conventional space-like singularity of the big-crunch/big-bang scenarios in Type II supergravity. This localized energy density arises naturally in string theory as branes with negative tension like orientifolds. The key idea is that the existence of a cosmological horizon flips the would be space-like singularity to a time-like singularity leading to a brane resolution of the singularity.

1 Introduction

In the standard cosmological models, a space-like big-bang singularity is present in the past. The existence of an initial singularity is, to a large extent, independent of the model, and in fact one can show that, under general assumptions, a singularity in the cosmological past is inevitable [1]. Despite these problems, the standard cosmological models (far) below the Planck scale are extremely successful in explaining the present experimental data. Attempts to solve the cosmological singularity problem lead to the pre big-bang scenario conjecture [2], motivated by the existence of a minimum distance in string theory. However, either the exact string backgrounds are not realistic [3], or they show time-like singularities whose nature has remained unclear [4, 5, 6].

The simplest way to avoid the cosmological singularity is to introduce a bulk positive cosmological constant, which introduces a uniform negative pressure. In the absence of other contributions to the stress-energy tensor one obtains a de Sitter space-time. A de Sitter cosmology is problematic both from the string theory point of view and from a phenomenological view-point. Although problematic, de Sitter space avoids the

cosmological singularity by introducing an effectively *repulsive* part to the gravitational interaction due to the negative pressure induced by the positive cosmological constant.

Another natural way to avoid the big-bang singularity is the introduction of a cosmological constant localized on hyperplanes of lower dimension [7], more precisely on domain walls of codimension one. It was indeed shown in [8] that, in pure gravity, domain walls *with positive tension* generate a repulsive gravitational field, for reasons similar to the ones which drive the dynamics of de Sitter space. We will review these results in the next section. We will also show that these results, albeit true in pure gravity, are no longer true in Type II supergravity, where one has to take into account the effects of the dilaton. Nonetheless we will show that a repulsive background can be produced with domain walls of *negative tension*. Even though it is problematic in pure gravity to consider objects with negative mass, they appear quite naturally in string theory. Orientifold planes are, for example, hypersurfaces of arbitrary dimension, charged under the Ramond–Ramond fields, with negative tension and no localized dynamical degrees of freedom [9, 10]. From the point of view of gravity, *O*-planes act as sources of matter and gravity fields by adding a local term in the supergravity action

$$|T| \int_{\Gamma} d^d x e^{-\phi} \sqrt{-\det G} \pm Q \int_{\Gamma} A_d ,$$

on the d -dimensional hypersurface Γ , where T and Q are the tension and the charge respectively. This source term acts as a negative cosmological term localized on the surface Γ , together with the coupling to the other supergravity fields.

The basic idea that leads to the resolution of the cosmological singularity with negative tension branes is to start by replacing the big-bang singularity at $t = 0$ by a cosmological horizon and to continue space-time across this horizon. To understand the existence of this horizon it is convenient to consider the description of $(d + 1)$ -dimensional flat space-time using the Milne and Rindler ‘polar’ coordinates. Along the ‘cosmological’ Milne wedge the metric depends on the cosmological coordinate t , however, as one crosses the horizon the metric depends on a space-like coordinate x , and it is foliated by d -dimensional de Sitter slices. For a general cosmological background one can start by integrating the classical equations of motion starting from the horizon [6]. Then the geometry develops a time-like singularity in the Rindler patch which is resolved by the presence of an orientifold [7], i.e. a negative cosmological constant localized at the boundary of space-time. Since the boundary conditions on the orientifolds for fields propagating in the bulk are well defined, the past/future transition amplitudes for quantum fields are well defined in the geometry. In particular, assuming the trivial vacuum in the past, we showed in [7] that the particle production rate in the far future is thermal.

2 Domain Walls in Supergravity

The physics of domain walls in gravity is rather non-intuitive, and it was first explored in some detail in [8]. One of the interesting results is that, *in pure gravity, positive tension domain walls repel*. This fact could be of potential use in cosmology, where one might use this dynamical mechanism to avoid the big-bang singularity. The basic facts of domain-wall physics can be obtained using the linearized theory of gravity, even though the complete results must be derived in the full non-linear setting. Let us review the basic considerations of [8]. Consider flat D -dimensional space-time, and let us consider a gravitational source localized on a $(p + 1)$ -dimensional hyperplane. Denoting $x^\perp$ the Euclidean directions transverse to the hyperplane, the linearized equations for the Einstein metric $g_{ab} = \eta_{ab} + h_{ab}$ read

$$-\Delta h_{ab} = \tau_{ab} \delta(x^\perp) ,$$

where τ_{ab} is related to the stress-energy tensor matrix T_{ab} by

$$\tau_{ab} = T_{ab} - \frac{1}{D-2} \eta_{ab} T.$$

Now let us consider a source with energy density $T_{00} = \mathcal{T}$ and pressure $T_{ii} = \mathcal{P}$ ($i = 1, \dots, p$). If $\mathcal{T} > 0$ and $\mathcal{P} \geq -\mathcal{T}$, the effective gravitational mass τ_{00} is negative whenever

$$(D-3)\mathcal{T} + p\mathcal{P} < 0.$$

In particular, for a brane with $\mathcal{P} = -\mathcal{T}$, the above implies that either $p = D-1$ (de Sitter space) or $p = D-2$ (a domain wall).

From now on we will concentrate on the case with $D = 10$ and $\mathcal{P} = -\mathcal{T}$, which is the relevant case for branes in string theory. The tensor τ_{ab} is given by

$$\begin{aligned} -\tau_{00} &= \tau_{ii} = \frac{p-7}{8} \mathcal{T}, & (i \leq p) , \\ \tau_{mm} &= \frac{p+1}{8} \mathcal{T}, & (m > p) . \end{aligned}$$

We see that probes which couple *only to the gravitational field* will be attracted by a p -brane for $p \leq 6$, and will be repelled by a 8-brane with *positive tension* $\mathcal{T}$. On the other hand, in Type II string theory, all brane-like sources, as well as all massive probes, have non-trivial couplings with other massless SUGRA fields, in particular with the dilaton field. For a Dp -brane source we must add the source equations for the dilaton and the RR field

$$\begin{aligned} -\Delta \Phi &= \frac{3-p}{4} \mathcal{T} \delta(x^\perp) , \\ -\Delta A_{0\dots p} &= \mathcal{T} \delta(x^\perp) . \end{aligned}$$

Let us then consider, for simplicity, a brane probe with the same dimension as the source, and governed by the action¹ (in this section, we are working in the Einstein frame)

$$- \int d^{p+1}x e^{\frac{p-3}{4}\Phi} \sqrt{-\det g_{ab}} \pm \int A .$$

If we consider the function $h(x^\perp) = -\Delta^{-1} \delta(x^\perp)$, which always *decreases away from the source*, then the potentials seen by the probe due to the graviton, dilaton and RR couplings are given by

$$\begin{aligned} V_g &= \frac{(p-7)(p+1)}{16} \mathcal{T} h(x^\perp) , \\ V_\Phi &= -\frac{(p-3)^2}{16} \mathcal{T} h(x^\perp) , \\ V_A &= \pm \mathcal{T} h(x^\perp) . \end{aligned}$$

Therefore, even though for $p = 8$ the gravitational potential V_g is repulsive (as in the analysis above), the combined gravi-dilaton potential

$$V_g + V_\Phi = -\mathcal{T} h(x^\perp) ,$$

is always attractive *for any value of p* , as long as $\mathcal{T} > 0$. This seems to suggest that, in supergravity theory, it is not possible to use the dynamical repulsion of domain walls in gravity to construct non-trivial cosmological scenarios. On the other hand, in string theory, there exist objects which, although non-dynamical, effectively act as sources for the gravity fields with negative tension $\mathcal{T} < 0$. The above analysis then suggests that orientifolds generically are gravitational repellers, and in particular orientifold domain walls can be used in cosmological settings. This is the subject of our proposal.

3 Space-time Global Structure

In this section and the next we will show how the results of the previous section, obtained in the linearized approximation, can be generalized to the full non-linear SUGRA. We start quite generally by considering $(d+1)$ -dimensional gravity coupled to a scalar field ψ

$$S = \frac{1}{2\kappa^2 \mathcal{E}^{d-1}} \int d^{d+1}x \sqrt{|g|} \left[R - \frac{\beta}{2} (\nabla\psi)^2 - V(\psi) \right] , \quad (3.1)$$

¹We are careless with the absolute normalization of the probe action. This does not affect the equations of motion. On the other hand, a correct treatment of a negative-tension probe cannot be analyzed with a dynamical action (O -planes do not have dynamical degrees of freedom), and can be analyzed fully by considering the interaction of *two sources in the non-linear theory*, a problem which will be addressed in the rest of this note.

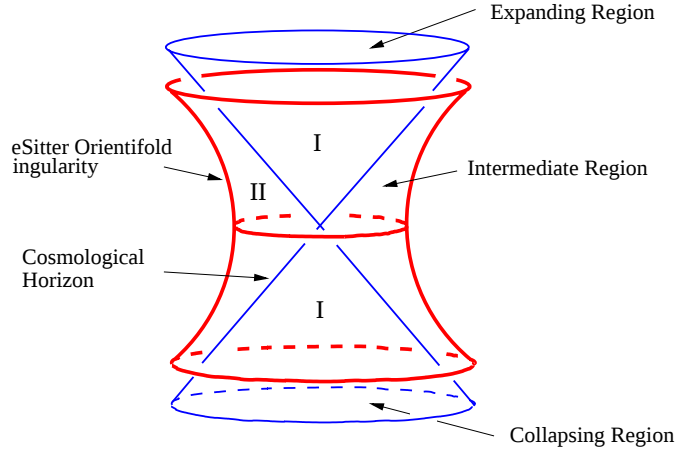


Figure 1: The space-time global structure. We shall see that the geometry develops a time-like singularity in region *II* interpreted as a negative tension brane at the boundary of space-time. This geometry describes a transition from a contracting to an expanding cosmological phase. Clearly, with this space-time global structure the standard cosmological horizon problem, associated with the conventional big-bang space-like singularity, does not arise.

where $\mathcal{E}$ is an energy scale. In this equation, $V(\psi)$ is the potential for the scalar field, β is a dimensionless constant and the coordinates are dimensionless in units of $1/\mathcal{E}$. Following [6], we are interested in cosmological solutions to the equations of motion of (3.1) with a contracting and an expanding phase (which we call region *I*), together with an intermediate region (denoted by region *II*). The expanding phase is the standard Robertson-Walker geometry for an open universe

$$\begin{aligned} ds_{d+1}^2 &= -dt^2 + a_I^2(t) ds^2(H_d), \\ \psi &= \psi_I(t), \end{aligned} \quad (3.2)$$

where H_d is the d -dimensional hyperbolic space with unit radius. Therefore, the dynamics of the system is described by the scalar and Friedman equations

$$\begin{aligned} \ddot{\psi}_I + d\dot{\psi}_I \frac{\dot{a}_I}{a_I} &= -\frac{1}{\beta} \frac{\partial V}{\partial \psi_I}, \\ \left(\frac{\dot{a}_I}{a_I}\right)^2 - \frac{1}{a_I^2} &= \frac{1}{d(d-1)} \left[\frac{\beta}{2} \dot{\psi}_I^2 + V(\psi_I) \right], \end{aligned} \quad (3.3)$$

where dots denote derivatives with respect to the cosmological time t . The contracting phase is nothing but the time-reversed solution, where we replace in (3.2) t by $-t$.

In order to have an intermediate region, we are interested in solutions of (3.2) where $a_I(t)$ and $\psi_I(t)$ are, respectively, odd and even functions of t with initial conditions

$$a_I(t) = t + \mathcal{O}(t^3), \quad \psi_I(t) = \psi_0 + \mathcal{O}(t^2), \quad (3.4)$$

for small cosmological time t . This implies that the $t = 0$ surface does not correspond to a big-bang singularity, but represents a null cosmological horizon [6]. In this case, the space-time can be extended across the horizon to an intermediate region II , where the solution takes the form

$$\begin{aligned} ds_{d+1}^2 &= a_{II}^2(x) ds^2(dS_d) + dx^2, \\ \psi &= \psi_{II}(x), \end{aligned}$$

with dS_d the d -dimensional de Sitter space. The equations of motion which determine a_{II} and ψ_{II} are again the equations (3.3), with the potential V replaced by $-V$. Given the boundary conditions (3.4), regions I and II can be connected along the null cosmological horizon, just as a Milne universe can be glued to a Rindler wedge to form flat Minkowski space. Moreover, it is clear that the solution possesses a global $SO(1, d)$ symmetry, which acts both on H_d and on dS_d (see figure 1).

4 Embedding in String Theory

Now let us consider a particular case of the construction of the previous section which can be embedded in Type II string theory. We consider backgrounds with a non-trivial RR field. The corresponding ten-dimensional Type II effective action has the form ($l_s = 2\pi\sqrt{\alpha'}$)

$$S = \frac{2\pi}{l_s^8} \left[\int d^{10}x \sqrt{|g|} e^{-2\phi} \left(R + 4(\nabla\phi)^2 \right) - \frac{1}{2} \int F \wedge \tilde{F} \right],$$

where F is the RR $(d+1)$ -form field strength and $\tilde{F} = \star F$ is the dual $\tilde{d}$ -form field strength, with $\tilde{d} = 9 - d$. A family of solutions, parameterized by an arbitrary constant g_s , can be constructed by considering the following ansatz

$$\begin{aligned} \mathcal{E}^2 ds^2 &= \Lambda^{-\frac{1}{2}\frac{d+1}{d-1}} ds_{d+1}^2 + \Lambda^{\frac{1}{2}} ds^2(\mathbb{T}^{\tilde{d}}), \\ e^\phi &= g_s \Lambda^{\frac{4-d}{4}}, \quad \tilde{F} = \frac{1}{g_s \mathcal{E}^{\tilde{d}-1}} \epsilon(\mathbb{T}^{\tilde{d}}), \end{aligned}$$

where we have defined

$$\Lambda = e^{2\beta\psi},$$

and the line element ds_{d+1} is defined in the previous section. The constants $\mathcal{E}$ and g_s are related to the electric field and the string coupling at the horizon, respectively. Following [6], the equations of motion for the metric ds_{d+1}^2 and scalar field are those presented in the previous section with

$$V = \frac{1}{2} e^{-\psi}, \quad \beta = \frac{1}{2} \frac{d-1}{3d-2}.$$

To find the solution for the line element ds_{d+1} and scalar field ψ we can solve the equations of motion as a power expansion in the dimensionless coordinate t around the cosmological horizon. The scale factor starts with an early acceleration phase and then, at late times, the universe becomes curvature dominated. The scalar field will roll down the potential from an initial value ψ_0 to $\psi \rightarrow +\infty$. An observer that is comoving with the expansion could think that at an earlier time the matter density blows up and there is a cosmological space-like singularity. This is the usual understanding in cosmology. However, in the picture we are proposing here, there is a past cosmological horizon where the geometry is perfectly smooth. Of course, there is also a collapsing region I , where the comoving observer sees a future horizon.

Similarly, we can start from the cosmological horizon and integrate the equations of motion into region II . Then we see that the geometry develops a singularity where the scale factor and the scalar field have the behavior

$$\begin{aligned} a_{II}(x) &= a_s (x_s - x)^\gamma, \\ \psi_{II}(x) &= \log [\eta (x_s - x)]^2, \end{aligned} \quad (4.1)$$

with the constants γ and η given by

$$\gamma = \frac{2\beta}{d-1}, \quad \eta^{-2} = 4\beta(1-d\gamma).$$

The dimensionless constants a_s and x_s can be determined numerically, and are fixed by the boundary conditions imposed at the horizon. To investigate the geometry near the singularity it is convenient to use Λ as a radial variable instead of x . From the behavior of the scalar field we see that Λ is a growing function as one moves from the singularity ($\Lambda = 0$) to the horizon ($\Lambda = \Lambda_0 \equiv e^{2\beta\psi_0}$). Near the singularity, in the limit $\Lambda \ll \Lambda_0$, we obtain the following ten-dimensional background fields [7]

$$\begin{aligned} \mathcal{E}^2 dS^2 &= \Lambda^{-\frac{1}{2}} \mu ds^2(dS_d) + \Lambda^{\frac{1}{2}} \left[d\Lambda^2 + ds^2(\mathbb{T}^{\tilde{d}}) \right], \\ e^\phi &= g_s \Lambda^{\frac{4-d}{4}}, \quad F = -\frac{1}{g_s \mathcal{E}^d} d\Lambda^{-1} \wedge \left[\sqrt{\mu^d} \epsilon(ds_d) \right], \end{aligned} \quad (4.2)$$

where μ is a constant determined from the constant a_s in the expansion (4.1). If we substitute the de Sitter space dS_d by the Minkowski space $\mathbb{M}^d$ we obtain naively the solution for Dp-branes ($p = d-1$) localized at $\Lambda = 0$ and uniformly smeared along $\mathbb{T}^{\tilde{d}}$. The usual harmonic function H is simply $H = \Lambda$ because there is a unique transverse direction. However, the tension associated with the harmonic function proportional to $-\Delta H$ is negative. Therefore the singularity is correctly interpreted as non-dynamical orientifold $(d-1)$ -planes smeared along the $\mathbb{T}^{\tilde{d}}$. When $\Lambda \rightarrow 0$ the radius of the de Sitter slice $\sqrt{\mu} \Lambda^{-1/4}$ diverges, the orientifold looks flat and one approaches the usual

supersymmetric solution. This solution is interpreted as the geometry of a de Sitter Op -plane ($p = d-1$) with radius $R_{dS} \sim 1/\mathcal{E}$ and delocalized along $\mathbb{T}^{\tilde{d}}$. Finally, if we T-dualize the solution along all the directions of the compact torus, we obtain a geometry describing the cosmological evolution of an $O8$ -plane, partially wrapped on the torus.

5 Two Dimensional Toy Model

There is a particular case of the above construction where the explicit form of the bulk fields is known. This corresponds to the case where the geometry is the M-theory compactification on the time-dependent orbifold [6]

$$(\mathbb{M}^3/\Gamma) \times \mathbb{T}^8 ,$$

where Γ is an element of the Lorentz group associated to a boost and translation. The orbifold identification is defined through $X^\pm \sim e^{\pm 2\pi\Delta} X^\pm$ and $Y \sim Y + 2\pi R$ (where $X^\pm, Y$ are coordinates on $\mathbb{M}^3$). We conveniently define the energy scale E to be $E = \Delta/R$. Then, a straightforward computation gives the background fields of the type IIA theory. In region II , this is easily done by using the Rindler ‘polar’ coordinates

$$EX^\pm = \pm x e^{\pm(s+y)} , \quad Y = y ,$$

with the result

$$\begin{aligned} E^2 dS^2 &= \Lambda^{\frac{1}{2}} \left[dx^2 + ds^2(\mathbb{T}^8) \right] - \Lambda^{-\frac{1}{2}} x^2 ds^2 , \\ e^\phi &= g_s \Lambda^{\frac{3}{4}} = g_s (1 - x^2)^{\frac{3}{4}} , \quad F = -\frac{2}{g_s E} d\Lambda^{-1} \wedge ds . \end{aligned} \tag{5.1}$$

The solution in region I – the Milne wedge – can be obtained by setting $x \rightarrow it$ and $s \rightarrow z$. If one analyzes the fields in (5.1) near the singularity at $x = \pm 1$, one sees that this geometry corresponds to a $O - \overline{O}$ particle pair delocalized along the 8-torus. As in the general case, we can T-dualize the above solution along the compact torus and obtain a geometry describing the interaction of two flat $O8$ -planes, which act as gravitational sources as described by the linear theory in section 2.

Let us consider a $O - \overline{O}$ pair in flat space at some large distance and a D particle near the O plane. The only force on the D particle is the gravitational (metric and dilaton) and electric repulsive force due to the $\overline{O}$ plane, since a OD pair is a BPS configuration. Hence, the D particle is pushed toward the O plane. Now let us turn on gravity and consider the back-reaction on the geometry due to the presence of the orientifolds. A calculation of the potential felt by the D particle in the full geometry shows that the D particle is pulled away from the O plane into the horizon (and away from the $\overline{O}$ plane as

before). The fields generated by the orientifolds induce an energy density between them, which interacts gravitationally with the D -particle. This gravitational attraction wins over the $D - \overline{O}$ repulsion. We conclude that the force that brings the D particle away from the O -plane is due to the back-reaction on the geometry, which is creating an extra energy density that interacts gravitationally with the probe.

Finally we can consider the propagation of a quantum field in this background. We know from string theory that quantum fields satisfy well defined boundary conditions on orientifold planes. In [7], we have been able to compute, using uniquely the boundary conditions and continuity across the horizons, the particle production in the expanding region I due to the geometry, starting with the vacuum state in the contracting region. We have concluded that the spectrum is thermal, with temperature

$$T = \frac{E}{2\pi a(\tau)} ,$$

where $a(\tau)$ is the scale factor and τ the proper cosmological time. This temperature dependence, typical of radiation domination, is associated to the surface gravity of the horizon. Therefore, one can extend the same result also to the higher dimensional cases.

Acknowledgments

This work was partially supported by European Union under the RTN contracts HPRN-CT-2000-00122 and -00131 and by CERN under contract CERN/FIS/43737/2001. The work of M.S.C. and L.C. is supported by Marie Curie Fellowships under the European Commission's Improving Human Potential programme (HPMFCT-2000-00508 and HPMFCT-2002-02016).

References

- [1] S. W. Hawking and G. F. R. Ellis, *The large scale structure of space-time*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (1973).
- [2] G. Veneziano, *String cosmology: The pre-big bang scenario*, hep-th/0002094.
- [3] E. Kiritsis and C. Kounnas, *String gravity and cosmology: Some new ideas*, gr-qc/9509017.
- [4] C. Kounnas and D. Lust, *Cosmological string backgrounds from gauged WZW models*, Phys. Lett. B **289** (1992) 56, hep-th/9205046.

-
- [5] C. Grojean, F. Quevedo, G. Tasinato, and I. Zavala C., *Branes on charged dilatonic backgrounds: Self-tuning, Lorentz violations and cosmology*, JHEP **0108**, 005 (2001), hep-th/0106120.
 - [6] L. Cornalba and M.S. Costa, *A New Cosmological Scenario in String Theory*, Phys. Rev. **D66** (2002) 066001, hep-th/0203031.
 - [7] L. Cornalba, M.S. Costa, and C. Kounnas, *Resolution of the Cosmological Singularity with Orientifolds*, Nucl. Phys. **B637** (2002) 378, hep-th/0204261.
 - [8] A. Vilenkin, *Gravitational Field Of Vacuum Domain Walls And Strings*, Phys. Rev. D **23**, 852 (1981).
 - [9] J. Polchinski, *TASI lectures on D-branes*, hep-th/9611050.
 - [10] C. Angelantonj and A. Sagnotti, *Open strings*, hep-th/0204089.