Orientifolds with Intersecting D-branes

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Abstract

Intersecting branes provide a framework to engineer semi-realistic gauge interactions with massless fermion spectra identical to those of the Standard Model or grand unified theories. We describe the most general setting where these models can preserve supersymmetry in the context of perturbative string theory. It is defined by Calabi-Yau and K3 compactifications of type II strings supplemented by a modified world sheet projection, which leaves $\mathcal{N}=1$ supersymmetry in the gravity sector. The setting further involves D-branes wrapped on cycles of middle dimension in the internal geometry, intersecting in points. The topological data of these configurations determine the chiral matter spectrum of the effective theory, while the issues of supersymmetry breaking and stability also involve geometrical properties of the branes and background space. As an example of the construction with nice phenomenological properties we briefly discuss a model with D6-branes on special Lagrangian cycles in the quintic Calabi-Yau that produces the a Standard Model gauge group and matter spectrum. The presented techniques also offer a novel approach to six-dimensional orbifold vacua and provide a very efficient method to obtain their chiral spectra.

1. Introduction

A central object of string phenomenology is to provide an existence proof for a string vacuum whose low energy approximation is reproducing the known physics of the Standard Model or of its supersymmetric and grand unified extensions. As a first approach one may concentrate on finding models with the correct light degrees of freedom, the right gauge group and chiral fermion spectra, leaving the details of their dynamics aside for the moment. Intersecting brane worlds [1] have proven to be a candidate framework of model building which offers excellent opportunity to meet this requirement [2-9]. In these string compactifications, the fermion spetrum is determined by the intersection numbers of certain 3-cycles in the internal space, as opposed for instance to the older approaches involving heterotic strings, where the number of generations was given by the Euler characteristic in the simplest case. Beyond these topological data also some more geometrical issues have been adressed, which provide access to computing the leading order perturbative scalar potential and determining the dynamics at least at the classical level [8,10]. Up to now, the construction has been limited to toroidal backgrounds and orbifolds thereof for the sake of simplicity. This has actually put severe contraints on the genericity of the examples obtained and prevented to get any possibility of supersymmetric models, except for the one case of the $T^6/\mathbb{Z}_2^2$ Calabi-Yau-orbifold [11,9], generalizing the former work on the six-dimensional $T^4/\mathbb{Z}_2$ K3-orbifold [6]. In the present work, we describe the general framework of intersecting brane world constructions on any smooth background Calabi-Yau space. This extends the range of accessible background spaces to include basically all geometrical string compactifications with supersymmetry in the bulk gravity sector. Therefore obstacles to finding supersymmetric Standard Model or GUT compactifications may possibly be overcome.

2. Intersecting brane worlds on Calabi-Yau 3-folds

In the brane world scenarios we are going to consider here, there are D-branes filling out the entire four-dimensional space-time providing the degrees of freedom for an effective gauge theory. The overall transverse six-dimensional space is compact, such that the internal excitations decouple from the effective theory below the string scale. The global consistency conditions in string models with D-branes that fill out the non-compact space-time involve the cancellation of the Ramond-Ramond (RR) charges. Furthermore, supersymmetry requires the cancellation of the brane tensions and the corresponding Neveu-Schwarz-Neveu-Schwarz (NSNS) tadpoles as well. If the latter is neglected, one can achieve the RR charge cancellation within type II vacua by including anti-branes, but these vacua usually suffer from run-away instabilities, if not even tachyons. The only setting in which objects with negative tension arise naturally in string theory are orientifolds, where the orientifold O-planes can balance the charge and tension of the D-branes. Therefore, orientifolds provide the framework where supersymmetric brane worlds may be found within string theory.

2.1. Definition

According to the above reasoning we will consider orientifold compactifications, where the ten-dimensional space-time $\mathcal{X}$ is of the kind

$$\mathcal{X} = \mathbb{R}^{3,1} \times \frac{\mathcal{M}^6}{\Omega\overline{\sigma}}. \tag{2.1}$$

Here $\mathcal{M}^6$ is a Calabi-Yau 3-fold with a symmetry under $\overline{\sigma}$, the complex conjugation

$$\overline{\sigma} : z_i \mapsto \bar{z}_i, \ i = 1, \ldots, 3, \tag{2.2}$$

in local coordinates. It is combined with the world sheet parity Ω to form the orientifold projection $\Omega\overline{\sigma}$. Orientifold O6-planes are defined as the fixed locus $\mathrm{Fix}(\overline{\sigma})$ of $\overline{\sigma}$, which is easily seen to be a supersymmetric 3-cycle in $\mathcal{M}^6$. It is special Lagrangian (sLag) and calibrated with respect to the real part of the holomorphic 3-form Ω_3. This orientifold projection truncates the gravitational bulk theory of closed strings down from a theory with 16 supercharges in ten dimensions to 4 supercharges and $\mathcal{N} = 1$ in four dimensions, after compactifying on the Calabi-Yau. In order to cancel the RR charge of the O6-planes it is required to introduce D6-branes into the theory as well, which will provide the gauge sector of the theory. If we label the individual stacks of D6_a-branes with multiplicities N_a by a label a, the gauge group of the effective theory will be given by $G = \prod_a U(N_a)$.

2.2. RR charges and brane tension

The general Chern-Simons action for Dp-branes and Op-planes was first given in [12,13]. It measures the RR charge of these objects, the relative normalization of the charge of an orientifold Op-plane given by $Q_p = -2^{p-4}$. The situation simplifies drastically for D6-branes on sLag 3-cycles, where the characteristic classes that classify the topology of the gauge, tangent and normal bundles become trivial and the only contribution in the CS-action then comes from the RR 7-form potential C_7.

In the following we denote the homology class of $\mathrm{Fix}(\overline{\sigma})$ by $\pi_{\mathrm{O6}} = [\mathrm{Fix}(\overline{\sigma})] \in H_3(\mathcal{M}^6)$ and the homology class of any given brane stack D6_a-brane by π_a. By our assumptions the π_a are never invariant under $\overline{\sigma}$ but mapped to image cycles π'_a. Therefore, a stack of D6-branes is wrapped on that cycle by symmetry, too. The RR charge cancellation can now easily be deduced by looking at the equation of motion of C_7

$$\frac{1}{\kappa^2}\, d \star dC_7 = \mu_6 \sum_a N_a\, \delta(\pi_a) + \mu_6 \sum_a N_a\, \delta(\pi'_a) + \mu_6 Q_6\, \delta(\pi_{\mathrm{O6}}), \tag{2.3}$$

where $\delta(\pi_a)$ denotes the Poincaré dual form of π_a, $\mu_p = 2\pi(4\pi^2\alpha')^{-(p+1)/2}$, and $2\kappa^2 = \mu_7^{-1}$. Upon integrating over $\mathcal{M}^6$ the RR-tadpole cancellation condition becomes a relation in homology

$$\sum_a N_a\, (\pi_a + \pi'_a) - 4\pi_{\mathrm{O6}} = 0. \tag{2.4}$$

Similarly one can determine the disc level tension by integrating the Dirac-Born-Infeld effective action. Whenever the potential is non-vanishing, supersymmetry is broken and a classical vacuum energy generated by the net brane tension. If we demand that any single D6_a-brane conserves the same supersymmetries as the orientifold plane the cycles π_{O6} and π_a must all be calibrated with respect to $\Re(\Omega_3)$. In this case, the RR charge and NSNS tension cancellation is equivalent.

2.3. Massless modes and chiral spectra

The action of $\Omega\overline{\sigma}$ on the cohomology determines the spectrum of the closed string bulk modes as usual. The most important input for constructing intersecting brane world models of particle physics are the formulae that determine the spectrum of the chiral fermions of the effective theory in terms of topological data of the brane configuration and the Calabi-Yau manifold. Roughly speaking, at any intersection point of two stacks of D6-branes a single chiral fermion is localized [14], transforming in the bifundamental representation of the two respective gauge groups. As was mentioned already, the search for a viable model close to the Standard Model particle content boils down to looking for Calabi-Yau spaces with an involution $\overline{\sigma}$ and an intersection form for its 3-cycles that allows to realize the desired particle spectrum at the intersections.

To obtain the chiral spectrum of a given set of D6-branes wrapped on cycles π_a with their images on π'_a and the O6-planes wrapped on $\pi_{\rm O6}$ few considerations are necessary. The only novelty is that in addition to the standard operation by Ω a permutation of the branes and intersection points by $\overline{\sigma}$ occurs, formally encoded in acting by a permutation matrix on the Chan-Paton labels that determine the representation under the gauge group. According to these rules, the spectrum of left-handed massless chiral fermions is shown in table 1.

Representation	Multiplicity
$[\mathbf{A}_a]_L$	$\frac{1}{2}\left(\pi'_a \circ \pi_a + \pi_{\rm O6} \circ \pi_a\right)$
$[\mathbf{S}_a]_L$	$\frac{1}{2}\left(\pi'_a \circ \pi_a - \pi_{\rm O6} \circ \pi_a\right)$
$[(\overline{\mathbf{N}}_a, \mathbf{N}_b)]_L$	$\pi_a \circ \pi_b$
$[(\mathbf{N}_a, \mathbf{N}_b)]_L$	$\pi'_a \circ \pi_b$

Table 1: *Chiral fermion spectrum in $d = 4$*

Due to the topological nature of the chiral spectrum table 1 should hold for every smooth Calabi-Yau manifold and even the six-dimensional torus [15]. Little can be said about the fate of the D-brane setting away from the limit of classical geometry, when venturing into the interior of the Kähler moduli space, where potentials may be generated. Therefore, the configuration will in general not be stable, but the important point is, whenever the setting is describable purely in terms of D6-branes on sLag 3-cycles table 1 may apply.

2.4. The Quintic

Now that we have collected the machinery to construct intersecting brane worlds on general Calabi-Yau 3-folds, we proceed to discuss the example of the quintic. One defines the Fermat quintic by the following hypersurface in $\mathbb{CP}^4$

$$\mathcal{Q}: \sum_{i=1}^{5} z_i^5 = 0 \quad \subset \mathbb{CP}^4. \tag{2.5}$$

It has the obvious involution from the complex conjugation of the coordinates $z_i \to \overline{z}_i$ as a symmetry. The fixed points of $\overline{\sigma}$ are the real quintic $\sum_{i=1}^{5} x_i^5 = 0 \subset \mathbb{RP}^4$, topologically a sLag $\mathbb{RP}^3$. Upon applying the $\mathbb{Z}_5^5$ symmetry of the quintic one can generate a whole class of $5^4 = 625$ different minimal $\mathbb{RP}^3$ defined via

$$|k_2, k_3, k_4, k_5\rangle \stackrel{\text{def}}{=} \Big\{[x_1 : \omega^{k_2} x_2 : \omega^{k_3} x_3 : \omega^{k_4} x_4 : \omega^{k_5} x_5]\Big| x_i \in \mathbb{R},\ \sum_{i=1}^{5} x_i^5 = 0\Big\}. \tag{2.6}$$

The only information further needed is their intersection form, determined in [16]. Using the intersection matrix for the 125 sLags that are calibrated by the same 3-form as the O6-plane one can check that they generate a 101-dimensional subspace of $H_3(\mathcal{Q})$. In order to construct any supersymmetric brane world model, it would be necessary to use D6-branes wrapping these 125 cycles only. Only in this case the scalar potential generated by the tension of the branes would be balanced by the negative tension of the O6-planes. Unfortunately, this turns out to be impossible with the present class of sLags, since it is found that the intersections among themselves all vanish. Therefore, a chiral spectrum cannot be reconciled with a supersymmetric groundstate. However, it is indeed possible to construct a model with the correct intersection numbers by dropping the requirement of supersymmetry, as we shall demonstrate in the following. The breaking of supersymmetry is still of a special and somehow weak nature, since the individual stacks still respect some supersymmetry generators, just not all the same. This can have interesting consequences for the dynamics of the effective gauge theory.

In order to realize the three generation Standard Model spectrum on an intersecting brane world on the Fermat quintic with an O6-plane on the cycle $\pi_{\mathrm{O6}} = |0,0,0,0\rangle$ and additionally wrap D6-branes on the following 3-cycles

$$\begin{aligned}
\pi_a &= |0,0,3,1\rangle & & & \pi_a' &= |0,0,2,4\rangle \\
\pi_b &= |4,3,0,3\rangle & & & \pi_b' &= |1,2,0,2\rangle \\
\pi_c &= |3,0,1,1\rangle - 2\,|4,3,0,3\rangle & &\Rightarrow & \pi_c' &= |2,0,4,4\rangle - 2\,|1,2,0,2\rangle \\
\pi_d &= |4,2,4,4\rangle - 2\,|0,0,3,1\rangle & & & \pi_d' &= |1,3,1,1\rangle - 2\,|0,0,2,4\rangle\,.
\end{aligned} \tag{2.7}$$

They just reproduce the “intersection numbers of the Standard Model” as proposed in [7]: If one wraps 3 branes on π_a, 2 branes on π_b, and a single brane on π_c and π_d, the gauge group is $U(3) \times U(2) \times U(1)^2$ before performing any anomaly analysis. The latter is actually of great importance since the anomalous Abelian factors decouple through a Green-Schwarz mechanism. Furthermore, there can appear Stückelberg mass terms for gauge bosons even in the absence of anomalies. The two effects together can be combined to leave exactly the hypercharge gauge boson massless, while the other three Abelian factors get massive at the string scale.

3. Intersecting brane worlds in six dimensions

The methods developed above for constructing four-dimensional intersecting brane world models on smooth Calabi-Yau backgrounds can also be applied to orbifolds. In this case one first needs to resolve the singular geometry in order to be able to compare to the classical data encoded in the intersection numbers. In this section we demonstrate the elegance and technical simplicity of the construction for six-dimensional K3-orbifolds, which is slightly simpler than Calabi-Yau-orbifolds, and via the six-dimensional constraints on anomaly cancellation offers an excellent check on the consistency of the results. Of course, some modifications need to be applied to the four-dimensional prescriptions in order to adapt to the K3. We are not going to explain everything in detail, but refer the reader to [17] for more instructions and proper definitions.

In general, the compactification of type IIB on a K3 leaves $\mathcal{N} = (0,1)$ supersymmetry in six dimensions. The involution $\overline{\sigma}$ now leaves fixed sLag 2-cycles in the K3, which are wrapped by O7-planes, whose charge is then canceled by D7-branes, according to the cancellation condition

$$\sum_a N_a\,(\pi_a + \pi_a') - 8\,\pi_{\mathrm{O7}} = 0. \tag{3.1}$$

The gauge group supported by the various stacks is again given by a product of unitary factors, while the chiral spectrum can be determined in complete analogy to the four-dimensional case and can be found in [17]. From the cancellation of the gravitational R^4 anomaly it now follows that

$$\pi_{\mathrm{O7}} \circ \pi_{\mathrm{O7}} = 2(9 - n_{\mathrm{T}}), \tag{3.2}$$

a strong consistency requirement that relates the topology of the O7-plane and the number of tensor multiplets in the effective theory.

One can demonstrate that the spectrum of table 4 reproduces essentially all known orbifold models of type IIB orientifolds on K3 [18-21], although their results are usually obtained after lengthy CFT computations and tedious Chan-Paton algebra. In this sense, the concept of intersecting branes also offers a technical short-cut to produce such supersymmetric orientifold spectra. In the following we shall now discuss just one example of a K3-orbifold.

We specialize to the $T^4/\mathbb{Z}_2$ orbifold limit of K3. The action of $\mathbb{Z}_2$ on the coordinates z_1, z_2 of the T^4 is by reflection. The homology includes some 2-cycles π_a on the K3 which are inherited from the torus 2-cycles, corresponding to massless modes in the untwisted sector of the CFT in the singular limit. They are organized in orbits under $\mathbb{Z}_2$ and the intersection form of these 2-cycles on the orbifold can be computed from the torus data.

In addition the resolution of the fixed points of Θ give rise to exceptional 2-cycles, massless fields in the twisted sectors of the orbifold CFT. For the $\mathbb{Z}_2$ orbifold there are 16 2-cycles blown-up to $\mathbb{CP}^1$ at the 16 fixed points P_{ij}. The exceptional divisors are then denoted e_{ij}. Their intersections read $e_{ij} \circ e_{kl} = -2\,\delta_{ik}\delta_{jl}$, the Cartan matrix of A_1^{16}. As can be deduced from comparing to the CFT limit [21], the O7-planes only wrap 2-cycles π_a inherited from the torus and no exceptional divisors. Its homology class is then given by $\pi_{\mathrm{O7}} = 2(\pi_{13}+\pi_{24})$. The action of $\Omega\overline{\sigma}$ on the cohomology of K3 is summarized together by

$$[\overline{\sigma}]_{\mathbf{AA}} = \mathrm{diag}\,(\mathbf{1}_2, -\mathbf{1}_{20})\,, \tag{3.3}$$

with $\mathbf{1}_n$ denoting unit matrices of rank n. From this the number of tensor multiplets follows as the number of eigenvalues $+1$ minus 1 to be $n_{\mathrm{T}} = 1$. Computing the self-intersection number of the orientifold plane we find $\pi_{\mathrm{O7}} \circ \pi_{\mathrm{O7}} = 16$, consistent with (3.2).

For the simple case of a $\mathbb{Z}_2$ orbifold group, one can directly compare the results of this procedure to the standard $\mathbb{Z}_2$ orientifold of type IIB string theory [22,18], since the projection by $\Omega\overline{\sigma}$ is equivalent to the standard projection by Ω upon performing T-dualities along the two circles parameterized by $\Im(z_i)$. One particular solution of the tadpole constraints can then be found recovering the spectrum and gauge group first discovered by Bianchi and Sagnotti and described in terms of D-branes by Gimon and Polchinski. In order to do so, we introduce just two stacks of fractional D7-branes with multiplicities $N_1 = N_2 = 16$, supporting $U(16)^2$. The cycles are

$$\begin{aligned} \pi_1 &= \frac{1}{2}(\pi_{13}) + \frac{1}{2}(e_{11} + e_{12} + e_{21} + e_{22}), \\ \pi_2 &= \frac{1}{2}(\pi_{24}) + \frac{1}{2}(e_{11} + e_{13} + e_{31} + e_{33}), \end{aligned} \tag{3.4}$$

plus their images under $\Omega\overline{\sigma}$. Their intersections can be easily determined to produce the chiral massless spectrum of [22,18].

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