Counting the Entropy of Schwarzschild Black Holes

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Overview:

- Introduction
- BH-Entropy and Dual Brane Doublets
- Chain-States and their Entropy
- Corrections to BH-Entropy
- Summary and Conclusion

Introduction

Thermodynamics of Black Holes associates a BH-entropy

$$\mathcal{S}_{BH} = \frac{A_H}{4G_4}\frac{k_B c^3}{\hbar}$$

with event horizon. To be more than just a formal analogy with conventional thermodynamics, Black Holes must emit thermal radiation. This was indeed found by Hawking and fixed proportionality constant.

Q: But what are the microscopic d.o.f. leading to this entropy?

(Main) String-Theory approaches:

- Counting microscopic d.o.f. in weakly coupled regime and export result to strongly coupled regime (supersymmetry!)

- Map spacetime to another spacetime (T- and S-Dualities and/or boost transformations) whose entropy is microscopically understood, e.g. D=3 BTZ black hole or near-extremal branes

Unfortunately indirect counting makes it difficult to see what the degrees of freedom are that make up the black hole. Are they located at the horizon or outside?

Quantum Geometry:

- led to derivation of BH-entropy for D=4 Schwarzschild case but had to fix undetermined proportionality constant (Barbero-Immirzi parameter) appropriately

Subsequently we want to describe an approach which is rooted in String-/M-Theory as it exploits dual brane doublets but which does not exploit stringy dualities or relies on supersymmetry. It is meant as an approach to deal directly with the generic strongly coupled ($g_s \simeq 1$) regime.

BH-Entropy and Dual Brane Doublets

Consider type II String-Theory on D=10 Lorentzian spacetime $\mathcal{M}^{1,3} \times \mathcal{M}^{p-1} \times \mathcal{M}^{7-p}$ ($p = 1, \ldots, 5$) with D=4 spherical symmetry

$$\begin{aligned} ds^2 = & g^{(2)}_{\alpha\beta}(t,r) dx^\alpha dx^\beta + r^2 d\Omega^2 + g^{(p-1)}_{ab}(x^c) dx^a dx^b \\ & + g^{(7-p)}_{kl}(x^m) dx^k dx^l \end{aligned}$$

where $x^\alpha, x^\beta = \{t, r\}$ and $d\Omega^2$ is the line-element of a unit two-sphere S^2.

D=4 Newton's Constant related to Regge slope α' and string coupling constant g_s through

$$G_4 = \frac{G_{10}}{V_{p-1} V_{7-p}} = \frac{(2\pi)^6 \alpha'^4 g_s^2}{8 V_{p-1} V_{7-p}} \, ,$$

where $V_i = vol(\mathcal{M}^i) \equiv \int_{\mathcal{M}^i} d^i x \sqrt{g^{(i)}}$.

Now wrap two orthogonal Euclidean "electric-magnetic" dual branes, Dp and $D(6-p)$, around $S^2 \times \mathcal{M}^{p-1}$ and $\mathcal{M}^{7-p}$, resp. such that they cover the whole internal space plus the external sphere.

Dirac-quantisation condition demands

$$\tau_{Dp}\tau_{D(6-p)} = \frac{1}{(2\pi)^6 \alpha'^4 g_s^2} .$$

Thus we can write

$$\frac{1}{G_4} = 8(\tau_{Dp} V_{p-1})(\tau_{D(6-p)} V_{7-p}) .$$

Q: But what do these dual branes have to do with a specific D=4 spacetime with (spherical) event horizon S_H^2 whose BH-entropy we want to understand?

A: Dual Branes act as supergravity sources which lead to the geometry

(D=4 Spacetime) $\times$ (Compact internal Space)

with the identification

$$S^2 \equiv S_H^2$$

With this identification we can rewrite D=4 BH-entropy as

$$\mathcal{S}_{BH} = \frac{A_H}{4G_4} = 2 S_{Dp}^{NG} S_{D(6-p)}^{NG} ,$$

where

$$S_{Dp}^{NG} = \tau_{Dp} \int_{S^2 \times \mathcal{M}^{p-1}} d^{p+1}x \sqrt{\det g}$$

$$S_{D(6-p)}^{NG} = \tau_{D(6-p)} \int_{\mathcal{M}^{7-p}} d^{7-p}x \sqrt{\det g}$$

are the Nambu-Goto actions.

The factor 2 tells us to repeat the procedure and employ a doublet of those Euclidean brane pairs which may differ from each other.

Therefore let us wrap ANOTHER dual Euclidean brane pair $Dp' - D(6-p')$, with Dp' and $D(6-p')$ again orthogonal, in the same manner as before around the S^2 plus the internal space.

D=4 BH-entropy becomes

$$\mathcal{S}_{BH} = S_{Dp} S_{D(6-p)} + S_{Dp'} S_{D(6-p')} \ .$$

Notice: we are free to exchange any of the appearing branes with an ANTI-BRANE and still arrive at the same rewriting (as long as the inclusion of the antibrane gives the requested D=4 spacetime)

This formula works for all dual branes of String- and M-Theory. Thus

$$\mathcal{S}_{BH} = \sum_{i=1,2} S_{E_i} S_{M_i} \ ,$$

where

$$
\begin{aligned}
(E_i, M_i) \in \{&(Dp_i, D(6-p_i)),\\
&(F1, NS5),\\
&(NS5, F1),\\
&(M2, M5),\\
&(M5, M2)\}
\end{aligned}
$$

and it is understood that any brane or string might also be replaced by its anti-brane or anti-string partner

For D=4 SCHWARZSCHILD Black Hole one would take self-dual $(D3, D3) + (\overline{D3}, \overline{D3})$ doublets:

	t	r	θ	ϕ	4	5	6	7	8	9
$D3$			•	•	•	•				
$\overline{D3}$			•	•	•	•				
$D3$							•	•	•	•
$\overline{D3}$							•	•	•	•

brane+antibrane $\to$ CHARGELESSNESS, non-dilatonic $D3$ $\to$ solution to Einstein VACUUM equations (see hep-th/0204206).

Chain-States and their Entropy

For a Lorentzian brane its tension is interpreted as

$$\tau_{Dp} = \text{mass/(unit of p-volume)}$$

For a EUCLIDEAN BRANE more natural to interpret

$$\tau_{Dp} = v_{Dp}^{-1} \equiv l_{Dp}^{-(p+1)}$$

in terms of a smallest fundamental volume unit (analogous to smallest length $\sqrt{\alpha'}$ for strings but here for chains to be introduced shortly)

→ Hence Euclidean brane consists of

$$N_{Dp} = \tau_{Dp} \int d^{p+1}x \sqrt{\det g} = S_{Dp}^{NG} \ .$$

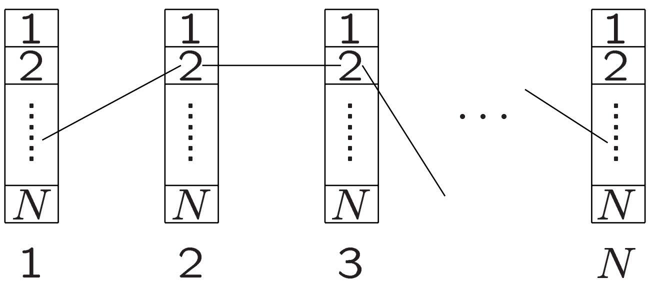
CELLS

→ D=4 BH-entropy becomes identical to a (huge) number

$$\mathcal{S}_{BH} = \sum_{i=1,2} N_{E_i} N_{M_i} =: N$$

N = total number of cells contained in the doublet of dual brane pairs whose joint worldvolume forms a LATTICE.

Let us conceive on such a lattice an $(N-1)$-CHAIN, i.e. a chain composed out of $N-1$ successive links where we allow all links to start and end on an arbitrary cell



→ Number of chains = N^N.

Motivation to consider LONG CHAINS:

uncertainty in space-resolution (cells) leads to an uncertainty in energy

$$\Delta E \simeq \Delta P \simeq \frac{1}{l_{Dp}} = (\tau_{Dp})^{\frac{1}{p+1}} = \frac{1}{\sqrt{\alpha'}(g_s(2\pi)^p)^{\frac{1}{p+1}}}\ .$$

$\rightarrow$ temperature associated with ΔE of order the HAGEDORN-temperature ($g_s \simeq 1$). From weakly coupled string-theory we know that it is entropically favourable to allocate energy of system to one single long string.

Counting of chains up to now classical: all cells distinguishable

QUANTUM feature: cells should better be regarded as indistinguishable bosonic d.o.f.

$\rightarrow$ account for this by dividing through the GIBBS-CORRECTION factor $N!$

$\rightarrow$ quantum-mechanically corrected number of different $N-1$ chains

$$\Omega(N) = \frac{N^N}{N!}$$

$\rightarrow$ Entropy of chain-states in the thermodynamic large N limit using Stirling's approximation, $\ln(N!) = N\ln N - N + \mathcal{O}(\ln N)$

$$\mathcal{S} = \ln\Omega(N) = N \equiv \mathcal{S}_{BH}\ .$$

up to $\mathcal{O}(\ln N)$

$\rightarrow$ ENTROPY OF CHAIN-STATES gives exactly the BH-ENTROPY of the D=4 spacetime

Corrections to BH-Entropy

Corrections to BH-entropy for D=4 black holes have been determined in supersymmetric cases from String-Theory while results in non-supersymmetric cases came from Quantum Geometry or the CFT approach of Carlip

General result: LOGARITHMIC CORRECTION

$$-k \ln \mathcal{S}_{BH}$$

with constant $k > 0$ (negative correction can be traced back to holographic principle)

CFT APPROACH:

By determining corrections to Cardy formula one obtains an entropy

$$\mathcal{S} = \mathcal{S}_{BH} - \frac{3}{2} \ln \mathcal{S}_{BH} + \ln c + \text{const}$$

for a class of D=4 black holes where central charge c is given by

$$c = \frac{3 A_H}{2\pi G_4} \frac{\gamma}{\kappa} \,,$$

with $\kappa =$ black hole's surface gravity, γ an undetermined periodicity parameter.

If one assumes that γ can be chosen such that c is independent of A_H then $k = 3/2$. However, it has been demonstrated (J.Jing, M.L.Yan, Phys.Rev.D63(2001) 24003) that

$$\gamma = 2\pi T_H = \kappa$$

with $T_H = \frac{\kappa}{2\pi}$ = Hawking-temperature. This gives $\ln c = \ln \mathcal{S}_{BH} + const$ and thus $k = 1/2$

$$\mathcal{S} = \mathcal{S}_{BH} - \frac{1}{2} \ln \mathcal{S}_{BH} + \text{const}$$

CHAIN-STATE APPROACH:

Let us see what corrections we obtain from counting chain-states. Corrections to chain-entropy are obtained from a more accurate approximation of $N!$ by the STIRLING-SERIES, e.g.

$$N! = \sqrt{2\pi N} N^N e^{-N} \big(1 + \frac{1}{12N} + \mathcal{O}(\frac{1}{N^2})\big) \ .$$

With $\mathcal{S}_{BH} \equiv N$ we get the corrected entropy formula

$$\begin{aligned}
\mathcal{S} =& \ln \Omega(N) \\
=& N \ln N - \ln(N!) \\
=& \mathcal{S}_{BH} - \frac{1}{2} \ln \mathcal{S}_{BH} - \ln \sqrt{2\pi} - \frac{1}{12\mathcal{S}_{BH}} + \mathcal{O}\big(\frac{1}{\mathcal{S}_{BH}^2}\big)
\end{aligned}$$

- gives expected leading logarithmic correction and agrees with $k = 1/2$

- Higher order corrections easily obtainable

Summary and Conclusion

- Long chain-states on Euclidean doublets of dual branes can account for exact BH-entropy w/o fixing overall proportionality constant or relying on supersymmetry

- Corrections easily obtained from higher orders of Stirling series and in qualitative (log. correction) and quantitative ($k = 1/2$) agreement with results from CFT approach

- Requires a "mapping" of dual brane doublets wrapped on an external S^2 to specific D=4 spacetimes via explicit supergravity solutions sourced by the branes

  (D=4 Spacetime) $\times$ (Compact internal Space)

  with

  $$S^2 \equiv S^2_H$$

  E.g. two doublets of Euclidean $D3-\overline{D3}$ pairs give the D=4 Schwarzschild Black Hole (a.k.a. the ultra non-extreme black 6-brane).