Custodial supersymmetry in non-supersymmetric quivers

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Abstract

Quiver theories which result from a nonsupersymmetric orbifolding of the $\mathcal{N} = 4$ $U(K)$ gauge theories exhibit an improved UV behaviour - the quadratically divergent contributions to the effective potential vanish at the one-loop level. If the gauge group resulting from orbifolding becomes broken down to the diagonal subgroup by universal vevs, then the resulting low-energy theory exhibits custodial supersymmetry and theory space extra dimensions.

1 Introduction

Higher-dimensional brane worlds lead to interesting extensions of the Standard Model, pertaining to novel approaches to the hierarchy problem. One of the most fascinating features of these theories is a nontrivial interplay between gauge and gravity sectors. In fact, their generic feature is the non-decoupling of gravity. The two places where this non-decoupling is easily seen are the issues of the stabilization of extra dimensions and of supersymmetry breakdown [1]. However, there exist models where the decoupling of gravity from extra dimensions is natural, and can be arranged by standard methods known from four-dimensional field theory. Such are the models of deconstructed dimensions [2],[3], where gravity is taken to be four-dimensional, and extra dimensions are fictitious and fully contained within the 4d gauge sectors. The example of such a situation is provided by quivers with the custodial supersymmetry [4]. The UV properties of these models are better than these of a generic non-supersymmetric model, and a separation between the vevs on the gauge sector and the cut-off scale, which may be taken to be the 4d Planck scale can be arranged. This is accompanied by a rather soft breakdown of supersymmetry.

2 Orbifolding and supersymmetry breaking

Consider the type IIB string theory with a stack of n coinciding D3 branes. It is well known that the gauge bosons and fermions living on the worldvolume of the D branes form a 4d $N = 4$ supersymmetric Yang-Mills model with gauge group $U(n)$. The six transverse dimensions form, from the point of view of the 4d theory living on the worldvolume, six extra nongravitational dimensions. One can obtain a theory with fewer supersymmetries than $N = 4$ $U(n)$ by dividing the extra dimensions by a discrete group Z_Γ and embedding this orbifold group into the gauge group $U(n\Gamma)$. The resulting theory is called a quiver theory. We will focus on non-supersymmetric quiver theories. They are obtained by retaining in the spectrum only the fields which are invariant under the combined geometric and gauge actions of Z_Γ [5]. Their interactions are consistently truncated to yield a smaller daughter gauge theory. The truncation process breaks the gauge group and some (or all) supersymmetries. The gauge symmetry breaking is dictated by the embedding of the generator of Z_Γ into $U(n\Gamma)$. The matrix γ that represents the gauge action of Z_Γ is chosen to be of the form of a direct sum of Γ unit matrices of dimensions $n \times n$, , each multiplied respectively by ω^i with $\omega = e^{\frac{2\pi}{\Gamma} i}$. Then the invariant components of the gauge fields fulfill the condition

$$A = \gamma A \gamma^{-1} \tag{1}$$

where A is a matrix in the adjoint representation of $U(n\Gamma)$. This leaves invariant the subgroup $U(n)^\Gamma$. There are four generations of Weyl fermions, each in the adjoint of $U(n\Gamma)$, whose invariant components must obey the condition

$$\psi^i = \omega^{a_i} \gamma \psi^i \gamma^{-1} \tag{2}$$

where $i = 1, .., 4$ and

$$a_1 + a_2 + a_3 + a_4 = 0. \tag{3}$$

The invariant fermions transform in the bifundamental representations of the broken gauge group $(\mathbf{n}_l, \bar{\mathbf{n}}_{l+a_i})$ where l numbers blocks of the original $n\Gamma \times n\Gamma$ matrices. Furthermore, one obtains three generations of complex bosons ϕ^i, $i = 1, 2, 3$, in the adjoint of $U(K)$, whose invariant components fullfil the condition

$$\phi^i = \omega^{\tilde{a}_i} \gamma \phi^i \gamma^{-1}. \tag{4}$$

The invariant scalars transform as $(\mathbf{n}_l, \bar{\mathbf{n}}_{l+\tilde{a}_i})$ under the broken gauge group. The truncated fields have a block structure in the $U(n\Gamma)$ mother gauge group

$$\phi^i_{lp} = \phi^i_l \delta_{p,l+\tilde{a}_i}, \ \psi^i_{lp} = \psi^i_l \delta_{p,l+a_i} \tag{5}$$

Supersymmetry is preserved when the group Z_Γ is embedded in $SU(3)$

$$\tilde{a}_1 + \tilde{a}_2 + \tilde{a}_3 = 0 \tag{6}$$

In that case $a_4 = 0$ and at least one of the fermions can be paired with the gauge bosons, i.e. becoming a gaugino of $N = 1$ supersymmetry. We focus on the non-supersymmetric case $a_4 \neq 0$.

Let us move a stack of n $D3$ branes from the origin. From the field theory point of view, moving the stacks of n $D3$ branes from the origin is equivalent to going to the Higgs branch of the theory where all the off-diagonal scalars with $\tilde{a}_i \neq 0$ take a vev

$$\phi^i_l = v^i 1_{n\times n} \tag{7}$$

Due to the Z_Γ action the stacks have Γ copies around the fixed point. The gauge group is broken to the diagonal subgroup $U(n)_D$. This is the deconstructed phase.

We are interested in the structure of one-loop divergences of the non-supersymmetric quivers. The vanishing of quadratic divergences in models based upon the low energy dynamics of branes in string theory turns out to be a general phenomenon.

3 Orbifolding of the field-theoretical Lagrangian and custodial supersymmetry [4]

To find out more about non-supersymmetric quivers it is very useful to study the effective Lagrangian of such theories. Inserting the block decomposition into the $\mathcal{N} = 4$ lagrangian we find the daughter theory lagrangian

$$\begin{aligned}\mathcal{L} = Tr \Big\{ &-\tfrac{1}{2} F_{\mu\nu,p} F_{\mu\nu,p} + i\overline{\lambda_p}\gamma^\mu D_\mu \lambda_p + 2 D_\mu \phi^\dagger_{i,p} D_\mu \phi_{i,p} + i\overline{\psi_{i,p}}\gamma^\mu D_\mu \psi_{i,p} \\ &- g_0 \left[2i\sqrt{2}(\overline{\psi_{i,p}} P_L \lambda_{p+a_i} \phi^\dagger_{i,p} - \overline{\psi_i} \phi^\dagger_{i,p-a_4} P_L \lambda_{p-a_4}) + \text{h.c.} \right] \\ &- g_0 \left[i\sqrt{2}\epsilon_{ijk} (\overline{\psi_{i,p}} P_L \psi_{j,p+a_i} \phi_{k,p-\tilde{a}_k} - \overline{\psi_{i,p}} \phi_{k,p+a_i} P_L \psi_{j,p-a_j}) + \text{h.c.} \right] \\ &- g_0^2 (\phi_{i,p}\phi^\dagger_{i,p} - \phi^\dagger_{i,p-\tilde{a}_i}\phi_{i,p-\tilde{a}_i})(\phi_{j,p}\phi^\dagger_{j,p} - \phi^\dagger_{j,p-\tilde{a}_j}\phi_{j,p-\tilde{a}_j}) \\ &+ 4 g_0^2 (\phi_{i,p}\phi_{j,p+\tilde{a}_i}\phi^\dagger_{i,p+\tilde{a}_j}\phi^\dagger_{j,p} - \phi_{i,p}\phi_{j,p+\tilde{a}_i}\phi^\dagger_{j,p+\tilde{a}_i}\phi^\dagger_{i,p}) \Big\} .\end{aligned} \tag{8}$$

where $\lambda \equiv \psi_4$. The covariant derivative acting on scalars is $D_\mu \phi_{i,p} = \partial_\mu \phi_{i,p} + ig_0 A_p \phi_{i,p} - ig_0 \phi_{i,p} A_{p+\tilde{a}_i}$. It is then a tedious exercise to obtain the mass matrices and compute the super-trace

$$\begin{aligned} STr(\mathcal{M}^2) &= 4g_0^2 \sum_k \sum_p \delta_{\tilde{a}_k,0} \Bigg[\Big(Tr(\phi^\dagger_{k,p}) Tr(\phi_{k,p+a_4}) \\ &\qquad + Tr(\phi^\dagger_{k,p+a_4}) Tr(\phi_{k,p}) - 2Tr(\phi^\dagger_{k,p}) Tr(\phi_{k,p}) \Big) \\ &+ \sum_i \Big(Tr(\phi^\dagger_{k,p}) Tr(\phi_{k,p+a_i}) + Tr(\phi^\dagger_{k,p+a_i}) Tr(\phi_{k,p}) \\ &\qquad - Tr(\phi^\dagger_{k,p}) Tr(\phi_{k,p+\tilde{a}_i}) - Tr(\phi^\dagger_{k,p+\tilde{a}_i}) Tr(\phi_{k,p}) \Big) \Bigg]. \end{aligned} \tag{9}$$

One can check that (9) vanishes identically if at least one of the following conditions is satisfied:

- $a_4 = 0$ or $a_i = 0$, that is when at least $\mathcal{N} = 1$ supersymmetry is preserved by the orbifolding,
- $\tilde{a}_1 \neq 0$, $\tilde{a}_2 \neq 0$ $\tilde{a}_3 \neq 0$, that is when there are no scalars in adjoint representation of $U(n)$ group.

In the first case the vanishing of the supertrace is of course guaranteed by unbroken supersymmetry of the daughter theory. Surprisingly, the absence of quadratic divergences can also occur if the daughter theory is completely non-supersymmetric, the only condition being that all scalars are in bifundamental representations of the $U(n)^\Gamma$ gauge group. This result is valid for any background value of the six scalar fields.

Let us now come back to the deconstructed case. One can explicitly diagonalize the mass matrices. For instance the gauge bosons acquire mass terms:

$$\mathcal{L} = \sum_p \sum_{k=1}^{3} g_0^2 v_k^2 (A^a_p - A^a_{p+\tilde{a}_k})^2. \tag{10}$$

(We have rewritten the gauge fields as $A = A^a T^a$ and evaluated the trace over generators. In the following we often omit the adjoint index a.) These mass terms are diagonalized by the following mode decomposition [1]:

$$A_p = \sqrt{\frac{2}{\Gamma}} \left(\sum_{n=0}^{(\Gamma-1)/2} \eta_n \cos\left(\frac{2n\pi}{\Gamma} p\right) A^{(n)} + \sum_{n=1}^{(\Gamma-1)/2} \sin\left(\frac{2n\pi}{\Gamma} p\right) \tilde{A}^{(n)} \right). \tag{11}$$

where $\eta_0 = 1/\sqrt{2}$ and $\eta_n = 1,\ n \neq 0$. Plugging in this decomposition we get:

$$\mathcal{L} = \frac{1}{2} \sum_n \sum_k (m_k^{(n)})^2 (A^{(n)} A^{(n)} + \tilde{A}^{(n)} \tilde{A}^{(n)}) \qquad m_k^{(n)} \equiv 2\sqrt{2} g_0 v_k \sin\left(\frac{n\pi}{\Gamma} \tilde{a}_k\right), \tag{12}$$

[1] The decomposition is given for odd Γ. For even Γ the first sum goes to $\Gamma/2$ and the second to $\Gamma/2 - 1$.

so that the n-th level gauge bosons have masses $(m^{(n)})^2 = \sum_k m_k^2$. Similar calculations can be done for the other fields resulting in the fact that the spectrum is perfectly boson-fermion degenerate. We already know that this degeneracy can be traced back to a custodial supersymmetry. Let us now investigate it further. We define the vector superfields in the Wess-Zumino gauge as:

$$V^{(n)}(y,\theta) = \frac{i}{2}(\overline{\theta}\gamma_5\gamma_\mu\theta)A^{(n)} - i(\overline{\theta}\gamma_5\theta)(\overline{\theta}\lambda^{(n)}) - \frac{1}{4}(\overline{\theta}\gamma_5\theta)^2 D^{(n)}. \tag{13}$$

Similarly we define chiral superfields:

$$\Phi_i^{(n)}(y,\theta) = X_i^{(n)} - \sqrt{2}(\overline{\theta}P_L\psi_i^{(n)}) + F_i^{(n)}(\overline{\theta}P_L\theta). \tag{14}$$

Analogous expressions for the tilded fields hold.

First, we note that the self-couplings in the zero-mode sector are those of the $\mathcal{N}=4$ supersymmetric theory. Indeed, the interactions of the zero-modes can be found by making in (8) the replacement $\phi_{i,p} \to \frac{1}{\sqrt{\Gamma}}\phi_i^{(0)}$ (and similarly for fermion and gauge fields). Since all memory of the block indices is lost, as a result we obtain the $\mathcal{N}=4$ lagrangian with gauge coupling $g = \frac{g_0}{\sqrt{\Gamma}}$. Second, we have already shown that the mass pattern in the deconstruction phase is supersymmetric. It turns out that the custodial supersymmetry has a much wider extent and all the terms quadratic in the heavy modes (including triple and quartic interactions with the zero-modes) match the structure of a globally supersymmetric theory! As an example we present a superfield lagrangian which reproduces the Yukawa terms and the scalar potential of the daughter theory:

$$\begin{aligned}\mathcal{L} = \textstyle\sum_n \sum_k \mathrm{Tr}\Big[&4g_0 v_k \sin\left(\tfrac{n\pi\tilde{a}_k}{\Gamma}\right)\left(\tilde{V}^{(n)}\Phi_k^{(n)} - V^{(n)}\tilde{\Phi}_k^{(n)}\right)\\ &+2g\cos\left(\tfrac{n\pi\tilde{a}_k}{\Gamma}\right)\left([\Phi_k^{(0)\dagger},\Phi_k^{(n)}]V^{(n)} + [\Phi_k^{(0)\dagger},\tilde{\Phi}_k^{(n)}]\tilde{V}^{(n)}\right)\\ &+2g\sin\left(\tfrac{n\pi\tilde{a}_k}{\Gamma}\right)\left(\{\Phi_k^{(0)\dagger},\Phi_k^{(n)}\}\tilde{V}^{(n)} - \{\Phi_k^{(0)\dagger},\tilde{\Phi}_k^{(n)}\}V^{(n)}\right) + \mathrm{h.c.}\Big]_D\\ &+[W]_F + [W^*]_F,\end{aligned} \tag{15}$$

where the superpotential is:

$$\begin{aligned}W = -i\sqrt{2}\textstyle\sum_n \sum_{ijk}\epsilon_{ijk}\mathrm{Tr}\Big[&4g_0 v_k \sin\left(\tfrac{n\pi\tilde{a}_k}{\Gamma}\right)\Phi_i^{(n)}\tilde{\Phi}_j^{(n)}\\ &-g\cos\left(\tfrac{n\pi\tilde{a}_k}{\Gamma}\right)\left([\Phi_k^{(0)},\Phi_i^{(n)}]\Phi_j^{(n)} + [\Phi_k^{(0)},\tilde{\Phi}_i^{(n)}]\tilde{\Phi}_j^{(n)}\right)\\ &+g\sin\left(\tfrac{n\pi\tilde{a}_k}{\Gamma}\right)\left(\{\Phi_k^{(0)},\Phi_i^{(n)}\}\tilde{\Phi}_j^{(n)} - \{\Phi_k^{(0)},\tilde{\Phi}_i^{(n)}\}\Phi_j^{(n)}\right)\Big].\end{aligned} \tag{16}$$

Supersymmetry is explicitly violated by triple and quartic self-interactions of the heavy modes. Nevertheless, the presence of the custodial supersymmetry in the lagrangian is sufficient to ensure the vanishing of one-loop corrections to the zero-mode masses. A mass-splitting of the zero-mode multiplets can appear only at the two-loop level and we expect the supersymmetry breaking scale to be suppressed $M_{\mathrm{SUSY}} \ll v \ll \Lambda$.

4 Theory space dimensions

From the previous discussion we know that the daughter theory is the low-energy field theory of branes located at the fixed point of an orbifold. The low energy degrees of freedom on a brane are those combinations of the open string states that are invariant under the action of Z_Γ. When one moves a stack of n $D3$ branes at a distance d away from the fixed point, due to the Z_Γ symmetry there appear Γ copies of the stack, spaced symmetrically in the transverse directions around the fixed point, see [6]. The custodial supersymmetry implies an extension of the results of Arkani-Hamed et. al. to nonsupersymmetric orbifoldings. For instance, it was shown that in the large Γ limit, when the distances between images of the stack are much smaller than d, one can redefine the orbifold metric in such a way, that consecutive boson-fermion degenerate mass levels correspond to open strings winding around a circular direction of the transverse geometry. This geometric picture allows for the straightforward computation of the massive string spectrum:

$$m_n^2 = 4\frac{d^2}{l_s^4}\sum_{i=1}^{3}\sin^2\left(\frac{n\pi\tilde{a}_i}{\Gamma}\right),$$

where l_s is the string scale and the shifts $\tilde{a}_i$ represent the action of Z_Γ on the three complex coordinates. When all vevs are equal, this is precisely the field theoretical spectrum in the deconstruction phase of the nonsupersymmetric model. In fact one can forget about the underlying stringy picture, and view the additional dimensions as fictitious, theory space, dimensions[2]. The ladder of scales which appears in a deconstructed field theoretical quiver model is as follows. The first scale one encounters, taking the bottom-up direction in available energy, is the fictitious compactification scale $1/R_5 = agv/\Gamma$, where $a^2 = \sum_i \tilde{a}_i^2$. At this scale a seeming fifth dimension opens up and one sees the tower of Kaluza-Klein states with masses of order $1/R_5$. Hence above this scale the theory looks five-dimensional. Moreover the spectrum of massive states is determined by the custodial supersymmetry. This picture holds up to the deconstruction scale v where non-diagonal gauge bosons become massless again. Above the deconstruction scale the theory is explicitly four-dimensional, nonsupersymmetric and renormalizable. Quadratic divergences are absent at the one-loop level. Also at one-loop the deconstruction scale is a flat direction of this four dimensional theory, hence it stays decoupled from any UV cut-off scale, including the 4d Planck scale. Moreover the compactification scale $1/R_5$ can be arbitrarily smaller than the deconstruction scale and it is determined by the discrete parameter which is the order Γ of the orbifold group Z_Γ.

5 Summary

A possibility of a natural separation between the gauge and gravity sectors, achieved by field-theoretical methods known from four-dimensional theories, appears in models with deconstructed dimensions. As the example of deconstruction we have discussed quiver theories which result from a nonsupersymmetric orbifolding of the $\mathcal{N} = 4$ $U(K)$ gauge theories. In a generic situation these non-supersymmetric models exhibit an improved UV behaviour - the quadratically divergent contributions to the effective potential vanish at the one-loop level. If the gauge group resulting from orbifolding becomes broken down to the diagonal subgroup by universal vevs, then the resulting low-energy theory exhibits custodial supersymmetry and theory space extra dimensions. The hierarchy $v \ll M_{cut-off}$

[2]Actually, each allowed set of shifts defines a closed subset of links in quiver diagrams, which can be interpreted as an iternal dimension

is protected at the one-loop level, and at one-loop universal vevs remain a flat direction and zero-mode multiplets do not suffer a mass splitting. Of course, the situation becomes even better in $N = 1$ supersymmetric orbifoldings. The deconstructed extra dimensions are fictitious, and belong to a renormalizable 4d gauge model. On the other hand gravity is four-dimensional at all scales. Hence, while retaining at low energies signatures of extra dimensions, these models simplify the physics of the gauge sector/gravity interface.

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