CP Phenomenology in Heterotic String Models

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Based on work done
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Introduction

- CP violation is one of the central aspects of low energy phenomenology

- CP is violated in
  - K–system
  - B–system
  - during the evolution of the universe

- String theory must explain its origin

- In string theory CP is a gauge symmetry
  $\longrightarrow$ has to be broken spontaneously

- Natural candidates: S, T_i , other moduli

Phenomenological Constraints

- Low–energy physics requires the Cabibbo-Kobayashi-Maskawa CP phase to be order one

- If we are to have low energy supersymmetry, one must make sure that the SUSY CP phases are **very small** as required by the fermion EDMs

- Other constraints:
  - dilaton is stabilized at Re$S \simeq 2$
  - TeV SUSY particle masses
  - no FCNC

- Observation: very difficult to reconcile the first two requirements.

- The "String CP Problem":

  EDMs appear if the soft breaking parameters (or the μ-term) are complex:

$$\Delta\mathcal{L}_{\text{soft}} = \frac{1}{2}M_a\lambda^a\lambda^a - m_\alpha^2\hat{\phi}^{*\alpha}\hat{\phi}^\alpha - \frac{1}{6}A_{\alpha\beta\gamma}\hat{Y}_{\alpha\beta\gamma}\hat{\phi}^\alpha\hat{\phi}^\beta\hat{\phi}^\gamma - B\hat{\mu}\hat{H}_1\hat{H}_2$$

  Some of them can be made *real* if the SUSY breaking fields do not break CP (Im$F_{S,T}$=0):

$$M_a = \frac{1}{2}(\text{Re}f_a)^{-1}F^m\partial_m f_a\ ,$$
$$A_{\alpha\beta\gamma} = F^m\left[\hat{K}_m + \partial_m \ln Y_{\alpha\beta\gamma} - \partial_m \ln(\tilde{K}_\alpha\tilde{K}_\beta\tilde{K}_\gamma)\right]$$

  Here K and W are the Kähler potential and the superpotential:

$$K = \hat{K} + \tilde{K}_\alpha\phi^{*\alpha}\phi^\alpha + (ZH_1H_2 + \text{h.c.})$$
$$W = \hat{W} + \frac{1}{6}Y_{\alpha\beta\gamma}\phi^\alpha\phi^\beta\phi^\gamma$$

- Yet, even if all of the SUSY breaking parameters are real while the A-terms are non-universal ($A_{\alpha\beta\gamma} \neq 1$), the problem still exists:

- A quark superfield rotation to the physical basis (where the quark masses are diagonal) necessarily contains complex phases,

$$\widehat{U}_{L,R} \to V^u_{L,R}\ \widehat{U}_{L,R}\ \ ,\ \ \widehat{D}_{L,R} \to V^d_{L,R}\ \widehat{D}_{L,R}\ ,$$
$$Y^u \to V^{u\dagger}_L\ Y^u\ V^u_R = \mathrm{diag}(h_u, h_c, h_t)\ ,$$
$$Y^d \to V^{d\dagger}_L\ Y^d\ V^d_R = \mathrm{diag}(h_d, h_s, h_b)\ ,$$

- This induces CP phases in the A-terms in the physical basis since they transform as

$$\widehat{A}^u \to V^{u\dagger}_L\ \widehat{A}^u\ V^u_R\ ,$$
$$\widehat{A}^d \to V^{d\dagger}_L\ \widehat{A}^d\ V^d_R\ .$$

- Thus, CP violation leaks from the SM Yukawa couplings into the SUSY sector.

- To suppress it, we need **flavor–universal A-terms.**

Heterotic Orbifold Models

- Modular symmetry:

$$T \longrightarrow \frac{aT - ib}{icT + d},$$
$$S \longrightarrow S + \frac{3}{4\pi^2}\delta_{GS} \ln(icT + d)$$

- Gaugino condensation:

$$W = d\frac{e^{\frac{-3S}{2\beta}}}{\eta(T)^{6-\frac{9\delta_{GS}}{4\pi^2\beta}}}$$

- Kähler potential:

$$K = -\ln Y - 3\ln(T + \overline{T}),$$

where $Y = S + \overline{S} + \frac{3}{4\pi^2}\delta_{GS} \ln(T + \overline{T})$.

- Scalar potential:

$$V = e^G \left(G_i \left(G^i_j\right)^{-1} G^j - 3\right),$$

where $G = K + \ln(|W|^2)$.

- SUSY breaking F-terms:

$$F_j = e^{G/2} \left(G^i_j\right)^{-1} G_i$$

- The vacuum values of the modulus T and the dilaton S are found by minimizing the scalar potential

- Need CP–violating T and S, and ReS ~2

- Problem: a single gaugino condensate leads to $S = \infty$

- There are a few ways to fix that:
  - multiple gaugino condensates:

$$W = \tilde{d}_1 \frac{e^{\frac{-3S}{2\tilde{\beta}_1}}}{\eta(T)^{6-\frac{9\delta_{GS}}{4\pi^2\tilde{\beta}_1}}} + \tilde{d}_2 \frac{e^{\frac{-3S}{2\tilde{\beta}_2}}}{\eta(T)^{6-\frac{9\delta_{GS}}{4\pi^2\tilde{\beta}_2}}}$$

  - nonperturbative Kähler potential:

$$K_S = \ln\left(\frac{1}{2\mathrm{Re}S} + d(\mathrm{Re}S)^{\frac{p}{2}} e^{-b\sqrt{\mathrm{Re}S}}\right)$$

  - S–duality:

$$W = -\frac{2\beta\mu^3}{3e\eta(S)^2\eta(T)^6(j(S)-744)^{\frac{3}{4\pi\beta}}}$$

- With an appropriate choice of the gauge group, etc., each of these options leads to Re$S \sim 2$

- Another problem: T at the minimum is always real

- Need to generalize the superpotential. One of the possibilities:

$$W \to W \times H(T)$$
$$H(T) = [j(T) - 1728]^{\frac{m}{2}} j(T)^{\frac{n}{3}} P\,[j(T)]$$

- Complex T becomes possible

- Does it lead to the CKM phase ?

The CKM Phase

- We will require the CKM phase to appear at the renormalizable level

- Complex T or S and Yukawa couplings do not imply a non-vanishing CKM phase

- Yukawa couplings obey string selection rules. $Y_{f_1 f_2 f_3}$ is allowed only if

  — $\theta_1\theta_2\theta_3 = \mathbf{I}$

  — $(\mathbf{I} - \theta_1)f_1 + (\mathbf{I} - \theta_2)f_2 + (\mathbf{I} - \theta_3)f_3 = 0$

- These are very restrictive. For *prime* orbifolds, given two fixed points, the third one is found uniquely. Thus, the textures are

$$Y^\alpha = \begin{pmatrix} a^\alpha & 0 & 0 \\ 0 & b^\alpha & 0 \\ 0 & 0 & c^\alpha \end{pmatrix}, Y^\alpha = \begin{pmatrix} a^\alpha & a^\alpha & a^\alpha \\ a^\alpha & a^\alpha & a^\alpha \\ a^\alpha & a^\alpha & a^\alpha \end{pmatrix}$$

$$Y^u = \begin{pmatrix} a & 0 & a \\ 0 & b & 0 \\ a & 0 & a \end{pmatrix} \quad , \quad Y^d = \begin{pmatrix} c & c & 0 \\ 0 & 0 & d \\ c & c & 0 \end{pmatrix}$$

- In $prime$ orbifolds the CKM phase is zero at the renormalizable level

- The situation is better in $non-prime$ order orbifolds since the space group selection rule is not diagonal

- Example: Z_6-I, $\theta\theta^2\theta^3$ coupling is allowed if $f_1|_{SU(3)} = f_2|_{SU(3)}$

- A non-trivial CKM phase can be produced if the **Jarlskog invariant** does not vanish

$$J = \mathrm{Im}\left(\det\left[Y^u Y^{u\dagger}, Y^d Y^{d\dagger}\right] \right)$$

- The Yukawas are calculated in terms of T via

$$Y_{\theta\theta^2\theta^3} = N\sqrt{l_2 l_3} \sum_{\vec{u}\in Z^4} \exp[-4\pi T \left(\vec{f_{23}} + \vec{u}\right)^T M \left(\vec{f_{23}} + \vec{u}\right)]$$

- **Modular properties of the Jarlskog invariant J:**

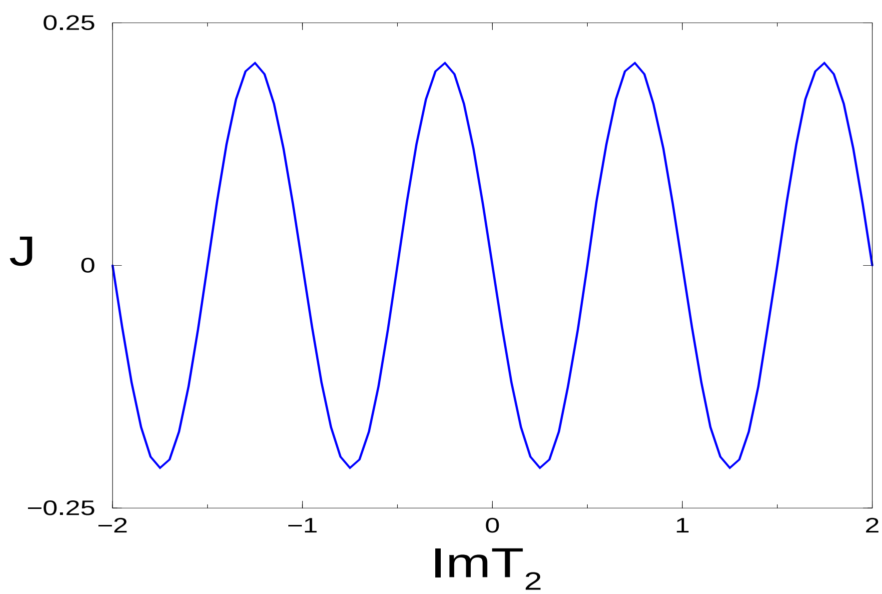
  Since

  $$Y(f_2 - f_3; T + i) = Y(f_2 - f_3; T)\, e^{i\phi(f_2)} e^{i\phi(f_3)}$$

  J is invariant under the axionic shift. Then, if $T_i^* = T_i \pm i$,

  $$J[Y(T_i)] = -J[Y^*(T_i)] = -J[Y(T_i^*)] = -J[Y(T_i)]$$

  So, J vanishes at the fixed points and at $\text{Im}T_i = \pm 1/2$.

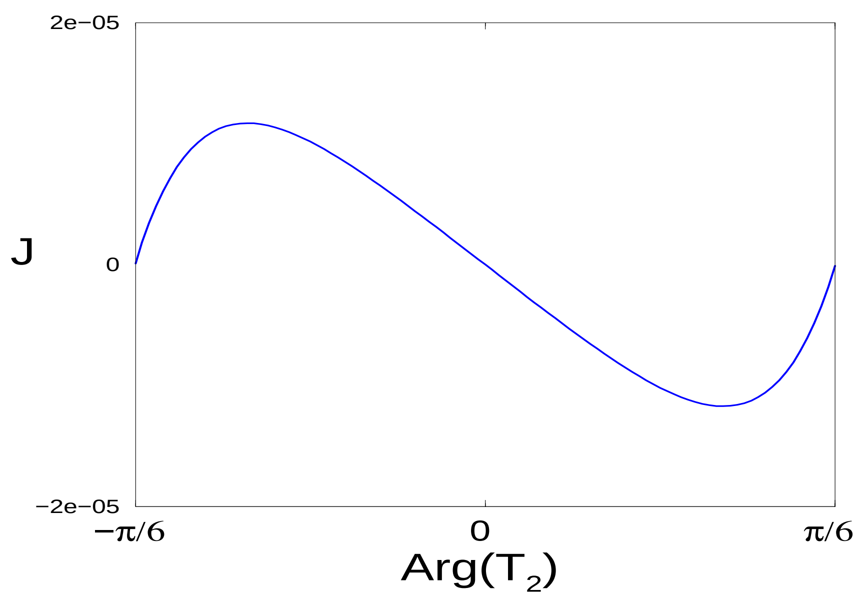


- **Thus, one must avoid the fixed points in realistic models.**

- This argument does not apply to the duality transform $T \to 1/T$ since it is not an explicit symmetry of the SM sector

- This can be shown by Poisson resummation of the Yukawas

$$\sum_{m \in Z^d} \exp[-\pi \left(m+\delta\right)^T \mathsf{A} \left(m+\delta\right)]$$
$$= \frac{1}{\sqrt{\det\ \mathsf{A}}} \sum_{m \in Z^d} \exp[-\pi m^T \mathsf{A}^{-1} m - 2\pi i\ \delta^T m]$$



- Thus J generally does not vanish on the unit circle

SUSY Breaking

- The SUSY breaking parameters are given by

$$
\begin{aligned}
M_a &= \frac{1}{2}(\mathrm{Re} f_a)^{-1} F^m \partial_m f_a \,, \\
m_\alpha^2 &= m_{3/2}^2 + V_0 - \bar{F}^{\bar{m}} F^n \partial_{\bar{m}} \partial_n \ln \tilde{K}_\alpha \\
A_{\alpha\beta\gamma} &= F^m [\hat{K}_m + \partial_m \ln Y_{\alpha\beta\gamma} \\
&- \partial_m \ln(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)] \\
B &= \hat{\mu}^{-1} \left(\tilde{K}_{H_1} \tilde{K}_{H_2}\right)^{-1/2} [(2 m_{3/2}^2 + V_0) Z \\
&- m_{3/2} \bar{F}^{\bar{m}} \partial_{\bar{m}} Z \\
&+ m_{3/2} F^m \left(\partial_m Z - Z \ \partial_m \ln(\tilde{K}_{H_1} \tilde{K}_{H_2})\right) \\
&- \bar{F}^{\bar{m}} F^n (\partial_{\bar{m}} \partial_n Z - \partial_{\bar{m}} Z \ \partial_n \ln(\tilde{K}_{H_1} \tilde{K}_{H_2}))] \\
\hat{\mu} &= \left(m_{3/2} Z - \bar{F}^{\bar{m}} \partial_{\bar{m}} Z\right) \left(\tilde{K}_{H_1} \tilde{K}_{H_2}\right)^{-1/2}
\end{aligned}
$$

where

$$
\tilde{K}_\alpha = (T + \overline{T})^{n_\alpha} \,, \ Z = \frac{1}{(T + \overline{T})}
$$

- The physical SUSY CP phases are $\mathrm{Arg}((B\hat{\mu})^* \hat{\mu} M)$ and $\mathrm{Arg}(A^* M)$

- The physical phases Arg$((B\widehat{\mu})^*\widehat{\mu}M)$ and Arg(A^*M) are modular invariant since

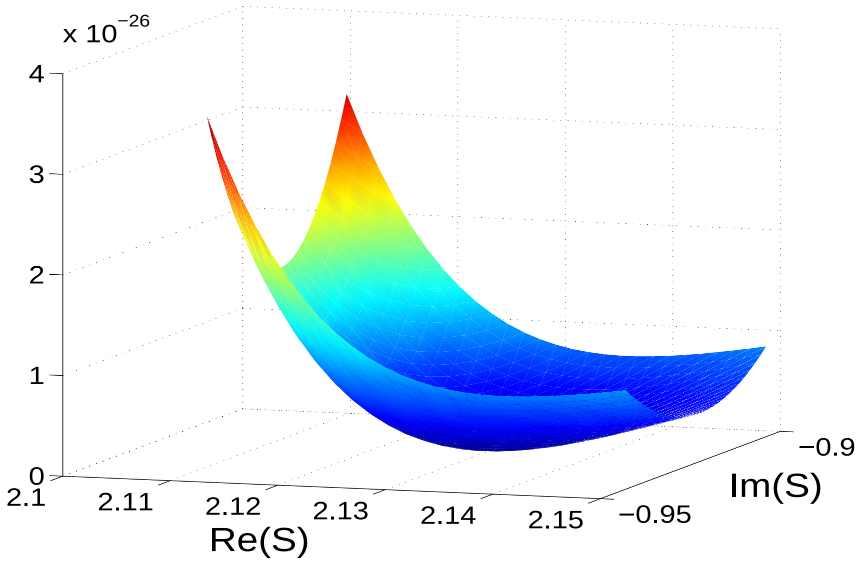
$$
\begin{aligned}
\widehat{\mu} &\rightarrow -\frac{T}{\bar{T}}\widehat{\mu} \\
B\widehat{\mu} &\rightarrow -\frac{T}{\bar{T}}B\widehat{\mu} \\
M_a &\rightarrow M_a \\
A &\rightarrow A
\end{aligned}
$$

under the duality transform.

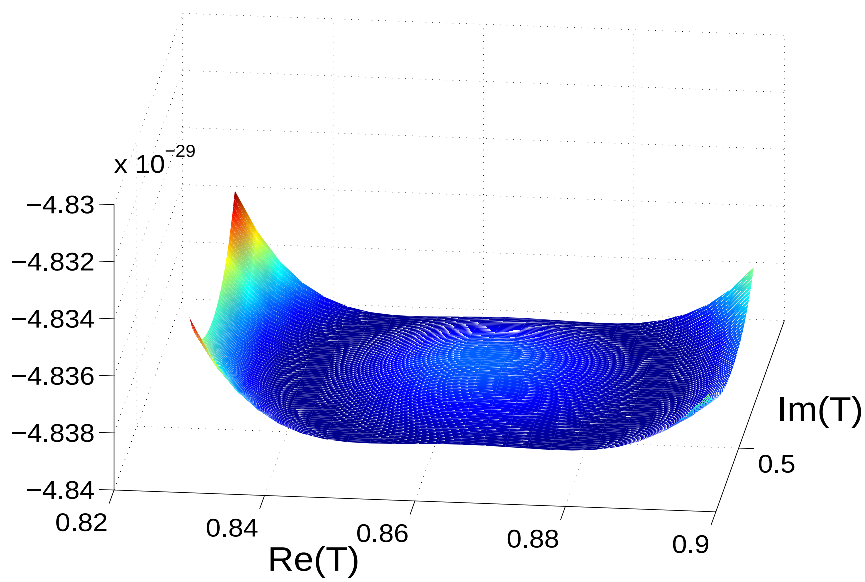
- We need to make sure that
  - these phases are (almost) zero
  - A–terms have no flavor dependence

- At the same time, T must be complex and away from the fixed points

- This can only be achieved if
  - $F_T = 0$
  - $F_S =$real



Racetrack scalar potential with H and m=1, n=0. T is set to its minimum value, $T_{min} = 0.9850e^{0.5471i}$. The minimum in S is at $S_{min} = 2.13 - 0.92i$.



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- For $m, n \geq 0$, if we require CP violation, we get

$$
\begin{aligned}
M_a &\sim 10^{-1} - 1\, GeV \,, \\
m_\alpha &\sim \mathrm{i} \times 10^4\, GeV \;\; \text{(tachyonic)} \,, \\
A_{\alpha\beta\gamma} &\sim 10^3\, GeV \,, \\
\widehat{\mu} &\sim 10^4\, GeV \,, \\
\sqrt{B\widehat{\mu}} &\sim 10^4 \,, GeV \\
\mathrm{Arg}(M_a) &= 2.147 \,, \\
\mathrm{Arg}(A_{\alpha\beta\gamma}) &= -1.387 \,, \\
\mathrm{Arg}(\widehat{\mu}) &= -0.041 \,, \\
\mathrm{Arg}(B\widehat{\mu}) &= 0
\end{aligned}
$$

- This is unacceptable

- However, for $m, n < 0$ the situation improves. The superpotential is singular at the fixed points in this case

- m, n are not arbitrary

- Since at $T \to \infty$

$$
\begin{aligned}
\eta(T)^{-1} &\to e^{\pi T/12}\,, \\
j(T) &\to e^{2\pi T}\,,
\end{aligned}
$$

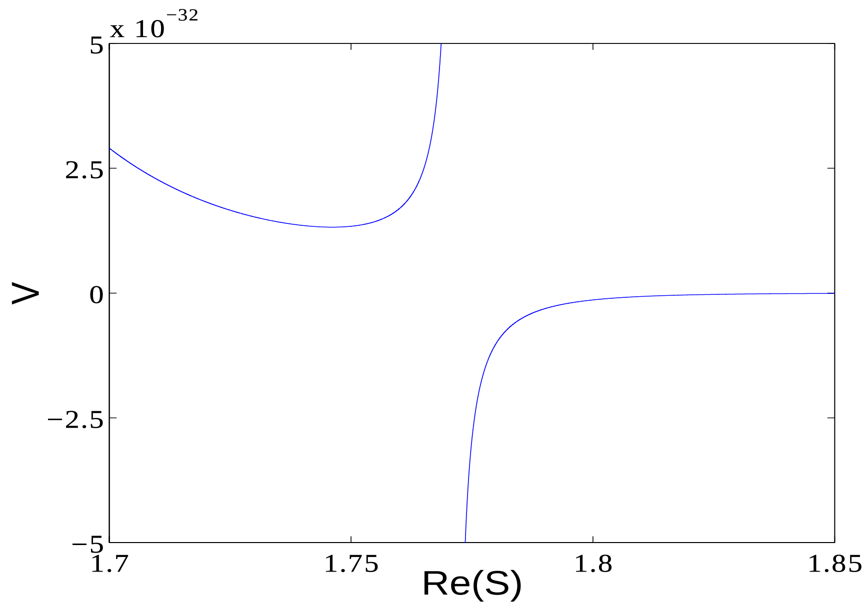
  if we are to achieve modulus stabilization, we need

$$
\frac{m}{2} + \frac{n}{3} > -\frac{1}{4}
$$

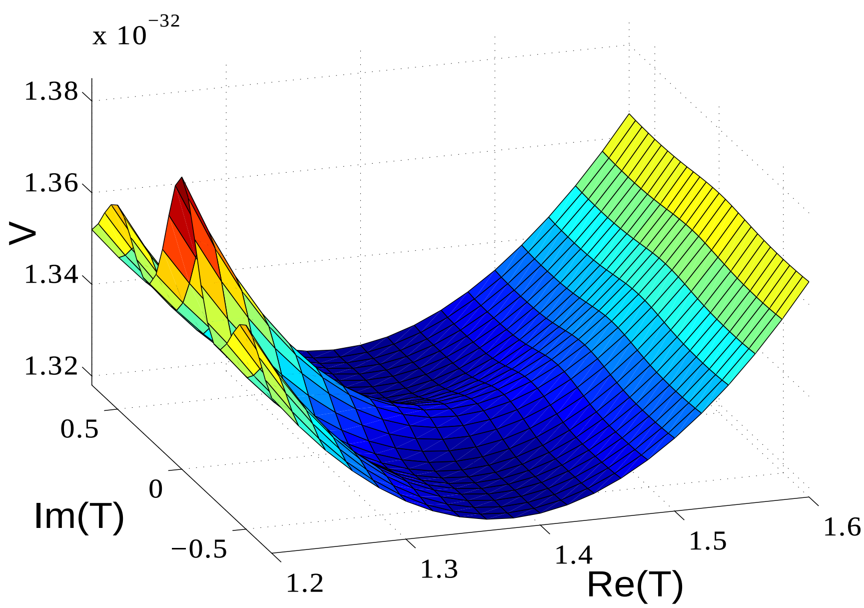
- The superpotential has poles at $T = 1, e^{\pm i\pi/6}$, such that the minima are repelled from the fixed points

- To achieve dilaton stabilization and dilaton dominated SUSY breaking, we use the non-perturbative Kähler potential

- This set-up possesses an axionic symmetry $S \to S + i\alpha$ which means that the CP phases of S and F_S are unphysical



Scalar potential with $\delta_{GS} = 0$ and $m = -\frac{1}{15}, n = -\frac{2}{15}$. T is set to its minimum value, $T_{min} = 1.38 + 0.36i$. The minimum in S is at $S_{min} = 1.75$.



Scalar potential with $\delta_{GS} = 0$ and $m = -\frac{1}{15}, n = -\frac{2}{15}$. S is set to its minimum value, $S_{min} = 1.75$. The minimum in T is at $T_{min} = 1.38 + 0.36i$.

- **Up's:**

- order 1 CKM phase

- no SUSY CP phases

- TeV SUSY breaking masses

- dilaton is stabilized at Re$S \sim 2$

- **Down's:**

- tree level μ is too small

- possible charge breaking minima

- hard to achieve radiative EW symmetry breaking

Conclusions

- heterotic orbifolds can provide a semi–realistic picture of CP violation

- they also provide important clues concerning a solution to the SUSY CP problem

- yet there are phenomenological issues to be addressed