S-BRANES, NEGATIVE TENSION BRANES, AND COSMOLOGY

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A general class of solutions of string background equations is studied and its physical interpretations are presented. These solutions correspond to generalizations of the standard black p-brane solutions to surfaces with curvature $k = -1, 0$. The relation with the recently introduced S-branes is provided. The mass, charge, entropy and Hawking temperature are computed, illustrating the interpretation in terms of negative tension branes. Their cosmological interpretation is discussed as well as their potential instability under small perturbations. (Work done in collaborations with C. Grojean, and with C. P. Burgess and S. J. Rey).

1. Motivations and Conclusions

One of the most interesting ideas coming out of the brane world scenario is what is known as 'mirage cosmology' [1]. A 3-brane moving in a (static) 5d anti de Sitter black hole induces, for an observer on the brane, the illusion that his 4d universe is expanding. In this case, the treatment is completely equivalent to the standard cosmological description of the brane as a fixed object embedded in a time-dependent background. The equivalence is due to the existence of Birkhoff's theorem in 5d, which essentially states that the most general spherically symmetric metric is static.

This view is very interesting, and we may wonder if it also applies to backgrounds more stringy than just 5d anti de Sitter space. The generalization can go in several directions. First, we should start with an action including the typical bulk fields of string theory: the graviton, dilaton and antisymmetric RR or NS-NS fields in d-dimensions. Second, we have to include solutions that go beyond the spherical symmetric black hole solution, which would include only FRW cosmologies with curvature $k = 1$. We then need to look for black hole-like solutions with curvatures $k = -1, 0$. Third, we have to see if Birkhoff's theorem holds in these more general settings.

Another, *a posteriori*, motivation for the analysis of this kind of solutions, is that they present alternative cosmologies to the standard big-bang, in which, extrapolating the scale factor to the past, it does not meet a singularity but a regular horizon for the metric. The possible realisation of these geometries would have very important implications for early universe cosmology [2,3,4].

Moreover, there has been recent interest in the study of space-like or S-branes in string theory [5], as time-dependent solutions of supergravity equations. Their interpretation offers important challenges, like the understanding of what kind of physical objects these space-times correspond to. In particular, in their original definition, there was no reference to the concept of conserved charges, and the study of their stability under perturbations was not considered.

We will start by presenting the analysis of solutions to the Einstein-dilaton-antisymmetric tensor equations with any curvature $k = 1, -1, 0$. The $k = 1$ case corresponds to standard black p-branes. For $k = -1, 0$ we have a geometry with past and future cosmological regions, separated, by regular horizons, from static regions. The analysis of these static regions reveal the presence of time-like singularities: we will provide a physical interpretation for the singularities in terms of a pair of negative tension branes, with opposite charge. The solutions describe also S-brane–like objects, sited on the horizons. The similarity with the Schwarzschild black hole allows us to compute the Hawking temperature and entropy density. The stability of the whole solution and the horizons is discussed: the past horizons appear to be unstable, due to infinitely blue-shifted energy flux coming from the past cosmological region.

We finally note that the mirage cosmology cannot have the level of generality as in the simple 5d case, since Birkhoff's theorem does not hold in the presence of a dilaton. In particular, this implies that the cosmology of a brane world will have two different sources, the time dependence of the bulk background and the motion of the brane through this background.

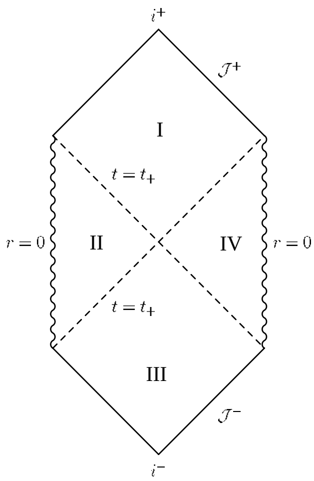

Figure 1. Penrose diagram for our time-dependent solutions. Notice that it correspond to the conformal diagram for a Schwarzschild black hole, but *rotated* by 90^o degrees.

2. General solutions

We now turn to the description of various space-time and brane's world-volume dimensions, for a system involving metric, dilaton, and $(q+1)$-form tensor fields — a system encompassing bosonic fields of diverse supergravity or superstring theories and their compactifications. The class of solutions we present, has been obtained in [2,3].

2.1. *Dilaton-Antisymmetric tensor-Einstein solutions*

We are interested in the following Einstein-frame action, in $d = (n + q + 2)$-dimensional space-time:

$$S = \frac{1}{2} \int_{\mathcal{M}_d} d^d x \sqrt{g} \left[R - (\nabla \phi)^2 - \frac{1}{(q+2)!} e^{-\sigma\phi} F_{q+2}^2 \right], \tag{1}$$

where $g_{\mu\nu}, \phi, F$ denotes metric, dilaton field, and $(q+2)$-form tensor field strength, respectively. Eq. (1) includes supergravity, and so also low-energy string theory, for specific choices of d, σ and q.

We are interested in classical solutions whose space-time geometry takes the form of an asymmetrically warped product between q-dimensional flat space-time and n-dimensional maximally-symmetric space, characterized by its curvature $k = 0, \pm 1$. That is, solutions that depend only on one warping variable — either t or r. The Ansatz is suitable to describe a flat q-brane propagating in $d = (n+q+2)$-dimensional ambient space-time, where n-dimensional transverse hyper-surface is a space of maximal symmetry. The solutions satisfying these requirements are given by

$$ds^2 = h_-^A \left(-h_+ h_-^{1-(n-1)b} dt^2 + h_+^{-1} h_-^{-1+b} dr^2 + r^2 h_-^b dx_{n,k}^2\right) + h_-^B dy_q^2\,, \tag{2}$$

$$\phi = \frac{(n-1)\sigma b}{\Sigma^2} \ln h_-\,, \tag{3}$$

$$F_{try_1\ldots y_q} = Q\epsilon_{try_1\ldots y_q} r^{-n}, \qquad \epsilon_{try_1\ldots y_q} = \pm 1\,. \tag{4}$$

The notations are as follows. The metric of the n-dimensional maximally symmetric space, whose Ricci scalar is equal to $n(n-1)k$ for $k = 0, \pm 1$, is denoted with $dx_{n,k}^2$. The harmonic functions $h_\pm(r)$ depend on two first-integral constants, $r_\pm$, and are given by:

$$h_+(r) = \pm\left(1 - \left(\frac{r_+}{r}\right)^{n-1}\right), \qquad h_-(r) = \left|k - \left(\frac{r_-}{r}\right)^{n-1}\right|, \tag{5}$$

where $\pm$ refers to the sign of h_-. The constant Q is given by

$$Q = \left(\frac{n(n-1)^2 (r_+ r_-)^{n-1}}{n\Sigma^2 + 4(n-1)}\right)^{1/2}, \tag{6}$$

where Σ and b are:

$$\Sigma^2 = \sigma^2 + 4\frac{q(n-1)^2}{n(n+q)}\,, \qquad b = \frac{2n\Sigma^2}{(n-1)(n\Sigma^2 + 4(n-1))}\,.$$

Likewise, the exponents A, B in eq. (2) are given by:

$$A = -\frac{4q(n-1)^2 b}{n(n+q)\Sigma^2}\,, \qquad \text{and} \qquad B = -\frac{n}{q} A\,.$$

The two integration constants, $r_\pm$, are related intimately to two conserved charges associated with the solution. One of these is the q-form electric charge Q — see eq. (4) — acting as the source of the $(q+2)$-form tensor field strength, while the other will correspond to the mass of the object in a form that we will discuss later.

Let us now describe the geometrical characteristics of these solutions. First of all, it is not hard to find that, for any k, r_+ is always a horizon. The other properties depend on the value of the constant curvature k:

- ***k=1:*** By computing the scalar curvature, it is possible to realize that $r = r_-$ is a scalar singularity for any positive value of b.[a] Then in order to avoid the presence of naked singularities, one requires that $r_- < r_+$ and, moreover, we should also require $(r_- r_+)^{n-1} \geq 0$ in order to have a real valued charge. The background is asymptotically flat and corresponds (for $d = 10$), to the black q-brane solutions constructed by Horowitz and Strominger [6].

[a]For $b = 0$, $r = r_-$ is actually another horizon as in the Reissner–Nordström solution.

• ***k=0 and k=−1:*** One finds, from the expressions for h_- above, that $r = r_-$ is a regular point for the metric. The coordinate r becomes time-like in the region $r > r_+$ while remains space-like for $r < r_+$, exactly the opposite of Schwarzschild black hole. The q-dimensional singularity at $r = 0$ is then time-like. The Penrose diagram is presented in Fig. (1): each point correspond to an n dimensional hyperboloid (for $k = -1$) or an n dimensional flat space (for $k = 0$), cross a q dimensional flat surface[b]. These solutions do not correspond to black objects, but then, what are they? We will discuss this point in what follows.

3. Properties and Interpretation

It is possible to interpret our solutions, for $k = 0, -1$ and $q = 0$, as particular examples of space-like branes (S-branes), as described recently in [5]. An S-brane is a brane-like object for which the world-volume is space-like. S-branes can be found as time-dependent solutions of supergravity equations.

In the original S-brane solutions, the static regions were ignored: it was then not possible to define conserved quantities such as charge or mass for those objects. We will see, in what follows, how the inclusion of those regions allows us to define various properties for these solutions and, moreover, give us an interesting interpretation for this geometry in terms of negative tension objects.

It is also interesting to write the solutions in terms of the conformal time in which the nature of a time-like sort of kink, associated to the S-branes, becomes evident and, moreover, a cosmological bouncing evolution is also clear. Let us consider for concreteness the case $k = -1$ and $q = 0$, in the absence of scalar and antisymmetric form. In this case, the metric in the original coordinates is

$$ds^2 = -\frac{1}{h_+}dt^2 + h_+\, dr^2 + t^2 dx^2_{n,-1}\,, \tag{7}$$

where $h_+ = 1 - (r_+/t)^{n-1}$. We now rewrite this metric in terms of the conformal time η as

$$ds^2 = C^2(\eta)\left[-d\eta^2 + dx^2_{n,-1}\right] + D^2(\eta)dr^2\,, \tag{8}$$

where the conformal time is defined by

$$C(\eta) = t(\eta) = r_+ \cosh^{2/(n-1)}\left[\frac{(n-1)}{2}\eta\right] \geq r_+\,, \tag{9}$$

and so η ranges over $-\infty < \eta < \infty$. The scale factor for the r-direction becomes

$$D(\eta) = \tanh\left[\frac{(n-1)}{2}\eta\right] \tag{10}$$

and has the same functional dependence for any value of n. These expressions exhibit a bouncing structure of the $(n+1)$-dimensional space and a (time-like) kink structure for the radial dimension.

3.1. *Conserved quantities*

We now identify two conserved quantities as Noether charges, carried by the branes at the singularities, whose metric, dilaton field, and $(q+2)$-form field strength are given in eqs. (2)–(4).

[b] It is interesting to observe that the solution for $k = -1$ can be obtained by an analytical continuation of the $k = 1$ case [3].

Electric charge. From the field equation eq. (4) of the $(q+2)$-form tensor field strength, a conserved charge density can be defined through $d^*F_{q+2} = {}^*J$. This leads to the following expression for the electric charge:

$$Q = \int_\Sigma d\Sigma_{\mu i \ldots} \nabla_\nu \left(e^{-\sigma\phi} F^{\mu\nu i \ldots} \right) = \int_{\partial\Sigma} d\Sigma_{\mu\nu i \ldots} e^{-\sigma\phi} F^{\mu\nu i \ldots} , \qquad (11)$$

where Σ refers to any $(n+1)$-dimensional space-like hyper-surface transverse to the q-brane. The main advantage of the above expression for the electric charge lies in the observation that the integrand vanishes almost everywhere by virtue of the field equation for F_{q+2}. It does not vanish literally everywhere, because the integrand behaves like a delta function at each of the two time-like singularities. Conservation of Q is also clear in this formulation, as the second equality of eq. (11) shows that Q does not depend on Σ as long as the boundary conditions on $\partial\Sigma$ are not changed.

We are led in this way to identify the conserved quantities, $\pm Q$, with electric charges carried by each of the two q-branes located at the time-like singularities (which, unlike the horizons, *are not* S-branes). Which brane carries which sign of the electric charge may be determined as follows. As eq. (4) defines the constant Q relative to a coordinate patch labeled by r and t, the key observation is that the coordinate t can increase into the future only for one of the two regions, II or IV. Then, the charge $+Q$ applies to the brane whose static region t increases into the future, and $-Q$ applies to the brane whose t increases into the past.

Gravitational mass. We can also associate a mass to these objects by adopting the *Komar integral* formalism, which cleanly associates a conserved quantity with a Killing vector field, ξ^μ, by defining a flux integral (see however [7]):

$$K[\xi] := \frac{c}{16\pi G} \oint_{\partial\Sigma} dS_{\mu\nu} D^\mu \xi^\nu . \qquad (12)$$

Here, c denotes a normalization constant, and Σ is again an $(n+1)$-dimensional space-like hyper-surface transverse to the q-brane, and $\partial\Sigma$ refers to the boundary of Σ. We have also recovered units by reintroducing Newton's constant, $8\pi G$. The Komar charge K is manifestly conserved, since it is invariant under arbitrary deformations of the space-like hyper-surface Σ for a fixed value of the fields on the boundary $\partial\Sigma$.

To evaluate the tension $\mathcal{T} = K[\partial_t]$ in the patch for which ∂_t is future-directed, we will choose for the hyper-surface Σ a constant-t spatial slice and for the boundary $\partial\Sigma$ a $r = r_0$ (viz. a constant radius) slice in the regions II and IV, respectively. It turns out that, if $Q \neq 0$, the expression for the tension depends on the value r at which the boundary $\partial\Sigma$ is defined[c]. Explicitly, we find the tension is given by:

$$\frac{\mathcal{T}(r)}{V} = -\frac{(n-1)}{8\pi G} \left[r_-^{n-1} - k r_+^{n-1} + \frac{2Q^2}{(n+q)(n-1)} \left(\frac{1}{r^{n-1}} - \frac{1}{r_+^{n-1}} \right) \right] \qquad (13)$$

Negative tension, $\mathcal{T} < 0$, for both branes is in accord with the form of the Penrose diagram of Fig.(1), which, in the static regions, II and IV, is similar to the Penrose diagram for a negative-mass Schwarzschild black-hole, or to the over charged region of the Reissner-Nordström black-hole. Negative-valued gravitational mass or tension is also borne out by the behavior of time-like geodesics in these regions [3].

[c]The same happens for the radius-dependent mass in Reissner-Nordström black-holes.

3.2. *Thermodynamics*

Given the explicit time dependence of the space-time in the time-dependent regions I and III, one would expect that particle production takes place in these regions. One can show that a Hawking temperature can be associated to the horizons with the static regions II and IV of the space-time.

Hawking temperature. To estimate the temperature we proceed in the usual way, performing a Euclidean continuation of the metric in the static region by sending $t \to i\tau$, and then demanding not to have a conical singularity on the horizon in this Euclidean space-time. This condition requires the Euclidean time coordinate to be periodic $\tau \sim \tau + 2\pi/\kappa$, and so implicitly defines a temperature as:

$$T = \frac{\kappa}{2\pi} = \frac{n-1}{4\pi r_+} \left| k - \left(\frac{r_-}{r_+}\right)^{n-1} \right|^{1-nb/2} . \tag{14}$$

This reduces to previously obtained expressions for the special cases where these metrics agree with those considered elsewhere. In particular, it vanishes for extremal black-branes, for which $k = 1$ and $r_- = r_+$.

Entropy. The possibility to define the temperature of a space-time involving horizons immediately suggests that it may also be possible to associate it an entropy. In fact by following a standard procedure computing the Euclidean action, one arrives to a simple expression for the entropy[3]

$$s = \frac{S}{V} = \frac{r_+^n}{4G} \left| k - \left(\frac{r_-}{r_+}\right)^{n-1} \right|^{nb/2} = \frac{1}{4G} \sqrt{g_{nn}} \,\big|_{r_+} , \tag{15}$$

It is remarkable that, for any k, the expression for the entropy does not depend on the coordinate $r : g_{nn}$ corresponds to the determinant of the induced metric on the n spatial dimensions, calculated at the horizon r_+.

In the case $k = 1$, we obtain the well-known relation

$$S = \frac{\mathcal{A}}{4G} , \tag{16}$$

where $\mathcal{A}$ is the area of the black-hole horizon. For $k = -1$ and 0, the area of the horizon is infinite, but we can still consider the entropy per unit volume and the same formula holds.

3.3. *Stability*

The stability of the whole solution is difficult to establish. Considering the simple case of a massive scalar field embedded in the space-time and satisfying the Klein Gordon equation, we have checked that it does not present fast growing modes: this indicates that the time-dependent regions are stable under perturbations. However, the energy flux of the modes of the scalar blows up at the past Cauchy horizon, suggesting a possible instability similar to the one of the Reissner-Nordström black hole.

4. Realistic Cosmology?

These solutions, with a contracting phase followed by an expanding one, without meeting a singularity, constitute interesting cosmological backgrounds. However, there are obstacles to promote this kind of solutions to a realistic cosmological setting. First, the instability of the Cauchy horizon may re-introduce the initial singularity. Furthermore, in 4D these solutions are not isotropic. To have 4D isotropy we have to consider higher dimensional solutions, and some of the spatial dimensions need to be compactified. This procedure generally implies the presence of regions with closed time-like curves, and/or some instability under small perturbations [9].

On the other hand, we may construct brane world models in this background. Introducing a moving brane in the asymptotically flat regions, we have an induced expansion on the brane coming both from the cosmological expansion and the motion of the brane. In particular, the induced Friedmann's equation on the brane will present the usual brane-world form $H^2 \sim \rho^2 + \cdots$. The procedure of adding a constant Λ to the energy density, and then expand the square to get a linear term in ρ plus small corrections, does not work here. This because, in the time dependent region, the Λ^2 term can not be compensated by any bulk contribution. The resulting model describes then an accelerating universe with positive cosmological constant. As a final comment, notice that Birkhoff's theorem does not hold [2,8], due to the presence of the scalar field: the most general maximally symmetric metric will depend (at least) on two coordinates. Therefore, to study the most general cosmological configuration, solutions depending on r and t should be found. We can see there are many interesting challenges in this field.

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