# D-branes, fluxes, and chirality

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**Abstract**

We describe a topological effect of certain combinations of NS-NS and RR field strength fluxes on D-branes. Such fluxes induce the appearance of chiral anomalies on lower dimensional submanifolds of the D-brane worldvolume. This anomaly is not associated to a dynamical chiral fermion degree of freedom, but rather should be regarded as an explicit flux-induced anomalous term (Wess-Zumino term) in the action. In compactifications to four dimensions, such terms modify the familiar anomaly cancellation patterns. We describe cancellation of diverse anomalies in four-dimensional compactifications with field strength fluxes and D-branes.

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# 1 Introduction

Compactifications of type II string theory / M-theory with field strength fluxes turned on (see e.g. [1, 2]) provide an interesting class of constructions which may shed new light on several questions of phenomenological relevance. For instance, construction of vacua with warped internal dimensions, or introduction of fluxes as a canonical mechanism to stabilize a large numbers of moduli.

Despite the recent progress in understanding the effect of such fluxes on the closed string sector of these constructions, their effect on open strings and D-branes are poorly understood. In this lecture we discuss one particularly easy and interesting effect. It is important to distinguish 'dilutable' effects, which completely go away in the large volume limit (for instance, effects of fluxes turned on in directions in which D-branes are localized at a point e.g. Myers' dielectric effect [3]), and 'topological' effects, which do not. The latter are discrete and arise when fluxes are turned on cycles on which D-branes wrap. This kind of effects are the ones discussed in the present talk.

# 2 Chirality from fluxes

In this section we discuss the effect of flux-induced anomaly on D-branes in the presence of fluxes. Instead of following the original discussion in [4], we follow a more direct route.

The Chern-Simons piece of the D-brane world-volume action is roughly speaking of the form

$$S_{Dp} = \int_{Dp} \mathcal{C} \wedge e^F \hat{A}(R)^{1/2} \tag{2.1}$$

where $\mathcal{C} = \sum_p C_p$ is a formal sum over RR forms of all degrees, $F$ is the world-volume gauge field strength, and $\hat{A}$ is the A-roof genus of the (pullback of the) spacetime curvature. Integration selects the top-form terms in the above expression.

In the presence of RR field strengths, these terms are more properly written as

$$S_{Dp} = \int_{Dp} \mathcal{G} \wedge [\, e^F \hat{A}(R)^{1/2} \,]^{(0)} \tag{2.2}$$

where $\mathcal{G}$ is a formal sum of RR fields strength tensors, and we are using Wess-Zumino descent notation. Namely, for a closed gauge invariant form $Y$, we define $Y = dY^{(0)}$, $\delta Y^{(0)} = dY^{(1)}$, where $\delta$ represents a gauge variation.

Expansion of the above expression leads to several important terms, which are of topological nature, and play a fundamental role in anomaly cancellation in the presence of D-branes. For instance, the anomaly inflow mechanism for intersecting D-branes [5], or the Green-Schwarz mechanism in compactifications with D-branes.

However, we should note that there exist terms beyond those written above. In particular, there are modifications to (2.1) in the presence of NS-NS 2-form field. Requiring invariance under gauge transformations of the NS-NS form potential

$$B_{NS} \to B_{NS} + d\Lambda \quad ; \quad F \to F - \Lambda \tag{2.3}$$

shows the modification is simply to replace $F$ by the gauge invariant field strength $\mathcal{F} = F + B_{NS}$ in (2.1).

Expansion of the terms linear in $B_{NS}$ in (2.1) leads to an action, which written as in (2.2), reads

$$S_{Dp} \,=\, \int_{Dp} \mathcal{G} \,\wedge\, B_{NS}\,[\,e^F\,\hat{A}(R)^{1/2}\,]^{(0)} \tag{2.4}$$

Our purpose is to discuss these terms and their role in diverse chiral anomalies. Upon world-volume gauge (or spacetime diffeomorphism) transformation, the above terms suffer a non-zero variation

$$\begin{aligned} \delta S_{Dp} \,&=\, \textstyle\int_{Dp} \mathcal{G} \,\wedge\, B_{NS}\,\delta\,[\,e^F\,\hat{A}(R)^{1/2}\,]^{(0)} \,= \\ =\, \textstyle\int_{Dp} \mathcal{G} \,\wedge\, B_{NS}\,d\,[\,e^F\,\hat{A}(R)^{1/2}\,]^{(1)} \,&=\, \textstyle\int_{Dp} \mathcal{G} \,\wedge\, H_{NS}\,[\,e^F\,\hat{A}(R)^{1/2}\,]^{(1)} \end{aligned} \tag{2.5}$$

The meaning of this is that in the presence of NS-NS and RR field strength fluxes a gauge variation of roughly the form of a chiral anomaly develops on the D-brane world-volume. This anomaly arises on a submanifold of the D-brane world-volume, localized at the core of the fluxes (for fluxes without well-define core, we may say the density of anomaly is given by the density of field strength). For instance, a coupling to be used below is that of a D6-brane to the RR 0-form field strength $\lambda$ and $B_{NS}$, which leads to a gauge variation

$$\delta S_{D6} \,=\, \int_{D6} \lambda\, H_{NS} - NS\,[\,e^F\,\hat{A}(R)^{1/2}\,]^{(1)} \tag{2.6}$$

The field strengths induce an anomal similar to that of a four-dimensional chiral fermion. The argument here (see also [4]) makes clear though that the origin of the anomaly is not an underlying chiral fermion, but rather an explicit anomalous term of the D-brane action in the presence of field strength fluxes.

In non-compact situations, the anomaly is canceled by an inflow coming from the volume of the D-brane towards the submanifold described above. The mechanism works as follows. Type II supergravity contains a ten-dimensional Chern-Simons interaction

$$\int_{\mathbf{X}_{10}} H_{NS}\, G_{6-p}\, C_{p+1} \tag{2.7}$$

with $C_{p+1}$, $G_{6-p}$ are a RR $(p+1)$-form potential and $(6-p)$-form field strength. It leads to a equation of motion

$$dG_{8-p} \,=\, G_{6-p}\, H_{NS} \tag{2.8}$$

for the field strength dual to $C_{p+1}$.

Now using another of the couplings in (2.2), namely

$$S_{Dp} \,=\, \int_{Dp} G_{6-p} \,\wedge\, [\, e^F\, \hat{A}(R)^{1/2}\, ]^{(0)} \tag{2.9}$$

there is a gauge variation of the latter induced by the equation of motion (2.8), given by

$$\begin{aligned} \delta S_{Dp} \,&=\, \textstyle\int_{Dp} G_{8-p} \,\wedge\, \delta\,[\, e^F\, \hat{A}(R)^{1/2}\, ]^{(0)} \,= \\ =\, \textstyle\int_{Dp} dG_{8-p} \,\wedge\, [\, e^F\, \hat{A}(R)^{1/2}\, ]^{(1)} \,&=\, \textstyle\int_{Dp} G_{6-p}\, H_{NS} \,\wedge\, [\, e^F\, \hat{A}(R)^{1/2}\, ]^{(1)} \end{aligned} \tag{2.10}$$

This is interpreted [5] as an inflow of anomaly from the D-brane world-volume, of the right form to cancel the anomaly (2.6)

# 3 Compactification and anomaly cancellation

In this Section we study the above anomalous term in type II compactifications with field strength fluxes and D-branes. For simplicity we center on four-dimensional compactifications. Also, we consider compactifications containing chiral fermions, and study the intricate pattern of anomaly cancellation in such models.

For concreteness we center on compactifications of type IIA theory on a Calabi-Yau threefold $\mathbf{X}_3$ with $N_a$ D6-branes wrapped on homology 3-cycles $[\Pi_a]$, and NS flux $H_{NS}$ turned on with total homology class $[H_{NS}]$, and in the presence of a cosmological constant $\lambda$. Compactifications of this kind in the absence of fluxes have been considered in [6, 7, 8, 9, 10]. Notice that these compactifications are typically non-supersymmetric, but supersymmetric configurations could be obtained by introducing O6-planes [10]. Study of supersymmetric models lies beyond the scope of the present paper, which centers on more topological aspects.

Extending the analysis in [8], we now derive the RR tadpole cancellation conditions. The action for the RR 7-form field is

$$S \,=\, \int_{M_4\times X_3} dC_7 * dC_7 \,+\, \sum_a N_a \int_{M_4\times\Pi_a} C_7 \,+\, \int_{M_4\times X_3} \lambda\, H_{NS}\, C_7 \tag{3.1}$$

The equation of motion is

$$dH_2 \,=\, \sum_a N_a\, \delta(\Pi_a) \,+\, \lambda\, H_{NS} \tag{3.2}$$

where $H_2$ is the field strength of the RR 1-form, and $\delta(\Pi_a)$ is a bump 3-form on $\mathbf{X}_3$ with support on $\Pi_a$. The equation in homology reads

$$\sum_a N_a\,[\,\Pi_a\,] \,+\, \lambda\,[\,H_{NS}\,] \,=\, 0 \tag{3.3}$$

The low energy four-dimensional theory is generically chiral, with chiral fermions arising from D6-brane intersections. The gauge group is $\prod_a U(N_a)$, and the chiral fermion content is given by

$$\sum_{a<b} I_{ab}(\Box_a, \overline{\Box}_b) \tag{3.4}$$

where $I_{ab} = [\Pi_a]\cdot[\Pi_b]$ is the intersection number, with its sign specifying the fermion chirality.

The anomalous terms (2.6) appear in the low-energy effective field theory as explicit Wess-Zumino terms in the four-dimensional action. Let us be a bit more concrete about these. A Wess-Zumino term is an explicit non gauge invariant interaction whose variation has the structure of a chiral gauge anomaly. Since an anomaly is a gauge variation which cannot be canceled against a local counterterm, it is clear that a four-dimensional Wess-Zumino term is non-local (although its gauge variation is local) [1]. The simplest way to write such terms (see e.g. [11]) is to pick a five-dimensional manifold $\mathbf{X}_5$ whose boundary is four-dimensional spacetime $M_4$. The Wess-Zumino terms we need are of the form

$$S_{WZ} \,=\, \int_{\mathbf{X}_5} [\, e^F \hat{A}^{1/2}(R)\,]^{(0)} \tag{3.5}$$

By default we consider Wess-Zumino terms with gauge fields traced in the fundamental representation. The gauge variation of (3.5) gives

$$\delta S_{WZ} \,=\, \int_{\mathbf{X}_5} d\,[\, e^F \hat{A}^{1/2}(R)\,]^{(1)} \,=\, \int_{M_4} [\, e^F \hat{A}^{1/2}(R)\,]^{(1)} \tag{3.6}$$

[1]From the ten-dimensional viewpoint, though,a non-local four-dimensional Wess-Zumino term may arise from local higher-dimensional interactions of the kind discussed above.

In the particular configuration of D6-branes given above, the gauge factor $U(N_a)$ associated to the $a^{th}$ stack of D6-branes, has a WZ term given by

$$S_{WZ,a} = I_{a,H} \int_{\mathbf{X}_5} [\, e^{F_a} \hat{A}^{1/2}(R) \,]^{(0)} \tag{3.7}$$

with $I_{a,H} = \lambda\, [\Pi_a] \cdot [H_{NS}]$.

In the following we study diverse kinds of anomalies and the different kinds of contributions. There are three main contribution to a four-dimensional anomaly: i) The familiar triangle diagram, with chiral fermions running in the loop, which contributes to cubic non-Abelian and mixed anomalies. ii) The Green-Schwarz contribution to $U(1)$ - non-Abelian and $U(1)$ - gravitational mixed anomalies, given by a diagram where the $U(1)$ gauge boson couples to a 2-form field which subsequently couples to two non-Abelian gauge bosons or two gravitons. iii) The explicit Wess-Zumino terms (3.7). This may contribute to cubic non-Abelian or mixed anomalies. We discuss the different anomalies in turn

**Cubic non-Abelian anomalies**

This receives contributions from the triangle diagrams and the Wess-Zumino terms. The total contributions to the $U(N_a)^3$ cubic anomaly are

$$\begin{aligned} \textstyle\sum_b N_b\, I_{ab} + I_{a,H} &= \textstyle\sum_b N_b\, [\Pi_a] \cdot [\Pi_b] + \lambda\, [\Pi_a] \cdot [H_{NS}] \\ &= [\Pi_a] \, ( \textstyle\sum_b N_b\, [\Pi_b] + \lambda\, [H_{NS}] \,) = 0 \end{aligned} \tag{3.8}$$

which vanishes by using the RR tadpole cancellation condition (3.3).

**Mixed $U(1)$ - non-Abelian anomalies**

This receives contributions from triangles, Wess-Zumino terms and Green-Schwarz diagrams. The latter are mediated by the 2-forms arising from compactification of the RR 5-form integrated over a basis of 3-cycles. The computation goes as in [8] and leads to the result below. The three contributions to the mixed $U(1)_a - SU(N_b)^2$ anomaly are

$$\begin{aligned} A_{ab}^{\text{triangle}} &= N_a\, I_{ab} + \delta_{ab} \sum_c N_c I_{ac} \\ A_{ab}^{\text{WZ}} &= \delta_{ab}\, I_{a,H} \\ A_{ab}^{\text{GS}} &= -N_a\, I_{ab} \end{aligned} \tag{3.9}$$

The total contribution vanishes.

**Mixed anomalies involving gauge fields from the RR closed string sector**

As noticed in [12], in compactifications with field strength fluxes there exist $BF$ couplings the mix gauge bosons $U(1)_{RR}$ from the closed string RR sector with RR with RR 2-forms. If the latter couple to two gauge bosons on D-brane world-volumes, there exists a Green-Schwarz contribution to the mixed $U(1)_{RR}$ - non-Abelian anomaly. Since there are no chiral fermions charged under both gauge symmetries, there is no triangle contribution to such anomaly. Cancellation is hence achieved via the contribution of a suitable Wess-Zumino term, which was there described.

In the particular example of our above compactification, there exist couplings of the RR 1-form field with 2-forms arising from integration of the RR 5-form

$$\int_{10d} = \int_{X_{10}} H_{NS}\, F_2\, C_5 \;\rightarrow \sum_i n_i \int_{M_4} B_2^i\, F_2 \tag{3.10}$$

where $n_i = \int_{\Sigma_i} H_{NS}$ are periods of the NS 3-form over a basis of 3-cycles. This implies that the scalars dual to those 2-forms shift under $U(1)_{RR}$ gauge transformations. Since they couple to the D6-brane non-Abelian gauge bosons as

$$\sum_i [\Sigma_i]\cdot[\Pi_a] \int_{M_4} \phi_i \,\mathrm{tr}\, F_a^2 \tag{3.11}$$

one obtains a total $U(1)_X - SU(N_a)^2$ mixed anomaly proportional to $n_a = \int_{\Pi_a} H_{NS}$.

The WZ term required to cancel this contribution arises from the coupling

$$S_{CS} = \int_{D6_a} B_{NS}\, C_1 \,\mathrm{tr}\, F_a^2 \; = \int_{D6_a} B_{NS}\, [\, F_2 \,\mathrm{tr}\, F_{a_0}^2\, ]^{(0)} \tag{3.12}$$

Indeed, its gauge variation is

$$\delta S_{CS} = \int_{D6_a} H_{NS}\, [\, F_2 \,\mathrm{tr}\, F_{a_0}^2\, ]^{(1)} \; = \; n_a \int_{M_4} [\, F_2 \,\mathrm{tr}\, F_{a_0}^2\, ]^{(1)} \tag{3.13}$$

and provides the required term to cancel Green-Schwarz contribution.

This completes the discussion of anomaly cancellation in the present setup. Note that there may exist mixed anomalies with more gauge fields from the RR sector, obtained from integration of higher-degree forms over submanifolds of the internal space.

# 4 Conclusions

We have discussed how field strength fluxes induce interesting topological effects on D-branes. In particular they lead to an intricate pattern of anomaly cancellation in compactifications leading to chiral spectra. We also expect and hope many more surprises to show up in future research on these interesting vacua.

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# References

[1] J. Polchinski, A. Strominger, 'New vacua for type II string theory', Phys. Lett. B388 (1996) 736, hep-th/9510227;
K. Becker, M. Becker, 'M theory on eight manifolds', Nucl. Phys. B477 (1996) 155, hep-th/9605053;
J. Michelson, 'Compactifications of type IIB strings to four-dimensions with nontrivial classical potential', Nucl. Phys. B495 (1997) 127, hep-th/9610151;
S. Gukov, C. Vafa, E. Witten, 'CFT's from Calabi-Yau four folds', Nucl. Phys. B584 (2000) 69, Erratum-ibid. B608 (2001) 477, hep-th/9906070;
K. Dasgupta, G. Rajesh, S. Sethi, 'M theory, orientifolds and G-flux', JHEP 9908 (1999) 023, hep-th/9908088;
T. R. Taylor, C. Vafa, 'R R flux on Calabi-Yau and partial supersymmetry breaking', Phys. Lett. B474 (2000) 130, hep-th/9912152;
B. R. Greene, K. Schalm, G. Shiu, 'Warped compactifications in M and F theory', Nucl. Phys. B584 (2000) 480, hep-th/0004103;
G. Curio, A. Klemm, D. Lust, S. Theisen, 'On the vacuum structure of type II string compactifications on Calabi-Yau spaces with H fluxes', Nucl. Phys. B609 (2001) 3, hep-th/0012213;
S. B. Giddings, S. Kachru, J. Polchinski, 'Hierarchies from fluxes in string compactifications', hep-th/0105097;

[2] S. Kachru, M. Schulz, S. Trivedi, 'Moduli stabilization from fluxes in a simple iib orientifold', hep-th/0201028;
A. R. Frey, J. Polchinski, 'N=3 warped compactifications', hep-th/0201029.

[3] R. C. Myers, 'Dielectric branes', JHEP 9912 (1999) 022, hep-th/9910053.

[4] A. M. Uranga, 'D-brane, fluxes and chirality, JHEP 0204 (2002) 016, hep-th/0201221.

[5] M. B. Green, J. A. Harvey, G. W. Moore, 'I-brane inflow and anomalous couplings on d-branes', Class. Quant. Grav. 14 (1997) 47, hep-th/9605033.

[6] R. Blumenhagen, L. Gorlich, B. Kors, 'Supersymmetric 4-D orientifolds of type IIA with D6-branes at angles', JHEP 0001 (2000) 040, hep-th/9912204;
S. Forste, G. Honecker, R. Schreyer, 'Supersymmetric Z(N) x Z(M) orientifolds in 4-D with D branes at angles', Nucl. Phys. B593 (2001) 127, hep-th/0008250.

[7] R. Blumenhagen, L. Goerlich, B. Kors, D. Lust, 'Noncommutative compactifications of type I strings on tori with magnetic background flux', JHEP 0010 (2000) 006, hep-th/0007024.

[8] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan, A. M. Uranga, 'D=4 chiral string compactifications from intersecting branes', J. Math. Phys. 42 (2001) 3103, hep-th/0011073; 'Intersecting brane worlds', JHEP 0102 (2001) 047, hep-ph/0011132.

[9] R. Blumenhagen, B. Kors, D. Lust, 'Type I strings with F flux and B flux', JHEP 0102 (2001) 030, hep-th/0012156;
L. E. Ibanez, F. Marchesano, R. Rabadan, 'Getting just the standard model at intersecting branes', JHEP 0111 (2001) 002, hep-th/0105155;
S. Forste, G. Honecker, R. Schreyer, 'Orientifolds with branes at angles', JHEP 0106 (2001) 004, hep-th/0105208;
R. Blumenhagen, B. Kors, D. Lust, T. Ott, 'The standard model from stable intersecting brane world orbifolds', Nucl. Phys. B616 (2001) 3, hep-th/0107138;
D. Cremades, L. E. Ibanez, F. Marchesano, 'SUSY Quivers, Intersecting Branes and the Modest Hierarchy Problem', hep-th/0201205; 'Intersecting brane models of particle physics and the Higgs mechanism', JHEP 0207 (2002) 022, hep-th/0203160;
C. Kokorelis, 'GUT model hierarchies from intersecting branes', hep-th/0203187; 'New standard model vacua from intersecting branes', hep-th/0205147; 'Exact standard model compactifications from intersecting branes', hep-th/0206108;
R. Blumenhagen, V. Braun, B. Körs, D. Lüst, 'Orientifolds of K3 and Calabi-Yau manifolds with intersecting D-branes', JHEP 0207 (2002) 026, hep-th/0206038;
A. M. Uranga, 'Local models for intersecting brane worlds', hep-th/0208014.

[10] M. Cvetic, G. Shiu, A. M. Uranga, 'Chiral four-dimensional N=1 supersymmetric type 2A orientifolds from intersecting D6 branes', Nucl. Phys. B615 (2001) 3, hep-th/0107166; 'Three family supersymmetric standard - like models from intersecting brane worlds', Phys. Rev. Lett. 87 (2001) 201801, hep-th/0107143.

[11] J. L. Mañes, 'Differential geometric construction of the gauged Wess-Zumino action', Nucl. Phys. B250 (1985) 369.

[12] G. Aldazabal, L. E. Ibanez, A. M. Uranga, 'Gauging away the strong CP problem', hep-ph/0205250.