only 3 light neutrinos: $\nu_1, \nu_2, \nu_3 \leftrightarrow Z^0$
and the mass pattern?

(1) Flavor problem: Why THREE families? How they get masses?

Unanswered theoretical problems

Why go beyond the SM
or create more problems...

Three may ask if SUSY or extra dimensions, in fact, solve

Transform the hierarchy into geometry stabilization.

Recent developments in string theories and extra dimensions.

Super-symmetry.

New physics is needed to stabilize the hierarchy, e.g.,

- Large Hierarchy: $\frac{M_{\text{Planck}}}{M} \ll 1$
- To unify with gravity we need string theories.
- Super-symmetric GUT theorems

So far the EM, weak, and strong couplings can unify in a number of

Grand Unification...

Theoretical problems...
Extra Dimensions
New ideas using extra dimensions can bring $M_{Pl}$ down to $\text{TeV}$.

\begin{itemize}
  \item New physics at $\sim \text{TeV}$, e.g., SUSY.
  \item Huge difference between $M_{EW}$ and $M_{Pl}$.
\end{itemize}

Desert

$M_{Pl} \sim 10^{19}$ GeV

$M_{GUT} \sim 10^{16}$ GeV

New Physics $\sim \text{TeV}$

Extra dimensions

Gravity $\sim 10^{-4}$ eV

New Physics $\sim \text{TeV}$
$z^+ uW \frac{u \delta}{u (u z)} = z^+ u W \quad \text{ instead of } \quad u W$.

Some conventions used instead of $u W$:

$$u - u I \quad * W \sim \frac{1}{z} z W$$

$$u - u \left( \frac{* W}{u - u I} \right) = u - u I = u - u \Lambda \quad \text{ and } \quad \frac{u W}{u I} = u I = u \Lambda$$

$$u - u \Lambda u \Lambda u + z W = \frac{1}{z} z W$$

The observed AD Planck scale $\Lambda \sim 10^{19} \text{ GeV}$ is a derived quantity.

Let $n$ total extra dimensions, in small, in large.

R. Chneur
\( M_{2+1} \)  

\( M_{3+1} \)  

\( R \)  

\( R \gtrsim 1 \text{ mm} \)

Proposed by Arkani et al., the size of the extra dimensions can be as large

Gravity only

Our World 3+1 dim.

\( \frac{R_c}{R} = 1 \)

Branes separated by

ADD model

K. Cheung
SM particles confined to a brane.

\[ \frac{H}{\mu} \stackrel{\text{2}}{\sim} \mu \]

In 4-D, the graviton becomes towers of Kaluza-Klein states.

Allow only gravity in the bulk.

Solve the hierarchy.

\[ M^* \text{ is the fundamental Planck scale, as low as TeV.} \]
\[ \text{Effectively, interaction } (1/\text{TeV})^{-1} \Rightarrow \frac{\mu}{1} \sim \frac{1}{1\text{GeV}} \]

Each couples to the SM field with a strength by separated each.

\[ R = \frac{\mu c}{1} \]

Our World 3+1 dim.
Sub-Planckian Physics

- Graviton emission into extra dimensions.
- Graviton exchange processes: interference with the SM.
- Exponential suppression to scattering amplitudes when close to the string scale.

Trans-Planckian Physics

- Black holes
- String balls
- p-branes

The energy scale is $\gg M_D, M_s$
Limits on Large Spatial Extra Dimensions

\[ F = 2 \log \left( \frac{0.6 \text{ TeV}}{M_S} \right), \quad n = 2 \]
\[ F = 2/(n-2), \quad n < 2 \]

\[ M_S, \text{ TeV} \]
\[ n = 2, M_S > 1.37 \text{ TeV} \]
\[ n = 3, M_S > 1.44 \text{ TeV} \]
\[ n = 4, M_S > 1.21 \text{ TeV} \]
\[ n = 5, M_S > 1.10 \text{ TeV} \]
\[ n = 6, M_S > 1.02 \text{ TeV} \]
\[ n = 7, M_S > 0.97 \text{ TeV} \]

\[ \mathcal{D}\ (2000) \]

95\% CL Upper Limit on \( F/M_S^4 \)

\( F/M_S^4 \)

\( M_S, \text{ TeV} \)
\[ M_{\text{BH}} \lesssim 5 M \] for \( S_{\text{BH}} \gtrsim 25 \]

\[ \left( \frac{1+u}{1+u} \right) \left( \frac{1+u}{1+u} \right) \left( \frac{a_{\text{BH}}}{a_{\text{BH}}} \right) \frac{2+u}{1+u} = S_{\text{BH}} \]

\[ \left( \frac{1+u}{1+u} \right) \left( \frac{1+u}{1+u} \right) \left( \frac{a_{\text{BH}}}{a_{\text{BH}}} \right) \frac{a_{\text{BH}}}{1} = R_{\text{BH}} \]

Minimum \( M_{\text{BH}} \) a few \( \sim M_{\text{D}} \), in order that large entropy is fulfilled.

Angular Momentum \( J(0) \) = 0.

Charge \( Q \) = 0.

Mass \( M_{\text{BH}} \leq M_{\text{BH}}, S_{\text{BH}} \) – all BHs is characterized by

Black Holes
\( (QW^{\text{BH}}) \) versus BH entropy \( S_{\text{BH}} \)
\[ H^\mu \left( \frac{s}{H^2 W} \right) \int_1^\infty = (s) \rho : \gamma \leftarrow \bar{z} \]

\[ H^\mu = \left( \frac{s}{H^2 W} - 1 \right) \rho H^\mu \left( \frac{s}{H^2 W} \right) \int_1^\infty = (s) \rho : \bar{z} \leftarrow 1 \]

\[ H^\mu \rho \approx (H^\rho W)^\rho \]

Production cross sections

K. Cheung
BH \gg R

\begin{itemize}
\item s-wave point source. Equally into brane and bulk modes. No reason to expect a correspondence to the Hawking temp. Much larger than RBH, BH evaporates like a
\item \( R_{BH} \gg R \)
\\end{itemize}
A spherical geometry like TeV decays into $30 \sim 50$ particles. Each has about a few $100$ GeV, a nonrotating BH decays isotropically. E.g., a BH with $M \sim$ a few

\begin{align*}
\text{Hadronic : Leptonic} & \sim 5 : 1 \\
18 : 72 & = 30 : 72
\end{align*}

according to the degrees of freedom.

A BH decays "bimdly". The main phase is the Hawking Evaporation,
$^{20}$ the BH decay, a boosted "fireball•

Two sides: $^{10}$ a few energetic partons $\rightarrow$ provide tags.

A typical BH event with a large $p_T$. 
\[ \frac{s_b}{s} W^H_B W = (H_B) \circ = \frac{s_b}{s} W^H_B W = \frac{s_b}{s} W^H_B W \]

match those of a string ball with a string theory with a mass \( m_B \).

- BH Correspondence Principle: properties of a BH with mass \( m_B \)
- SBS are the string properties of BHs.

\[ \frac{s_b}{s} W = \frac{H_B}{m_B} W \]

...strings - string balls (SBS).

minimum mass, it transits into a state of highly excited and jagged

Dimopoulos and Emparan pointed out that when a BH reaches a

String Balls
When the energy reaches saturation of unitarity sets a constant.

When the energy ... becomes a constant, the scattering is

\[ \frac{s_B}{s_W} \gtrless s_B W \gg s_W \]

\[ \frac{s_B}{s_W} \gtrless s_B W \gtrless \frac{s_B}{s_W} \]

\[ \frac{a_W}{s_B/s_W} \]

\[ \frac{a_W}{s_B/s_W} \]

\[ = (HB/\Sigma B) \varphi \]

Production cross sections
$M^s = 1 \text{ TeV}$. Correspondence point at $M^s/\bar{\rho} = 5M_{W}$

$\sigma_{\text{SB/SH}} (x10^{-6} \text{ GeV}^{-2})$ versus $M_{\text{SB/SH}} (\text{TeV})$ for different values of $n$. The graph shows the relationship between the cross-section and the mass for $n=2$, $n=3$, $n=4$, $n=5$, $n=6$, and $n=7$. The scale on the y-axis ranges from 0 to 30, while the x-axis ranges from 14 to 2 TeV.
Most of time, SB decays into visible quanta.

When the SB decays further, its shrinking back to $L$, and emits as a point

momentum states provided by the extra dim.

size the $\chi$ of the emissions. Therefore, it makes more use of the angular

When MSB gets below $M_s^s/\sqrt{g_s}$, the SB likely to pulls up to a random walk

equally into brane and bulk modes.

At $M_{SB} \sim M_s^s/\sqrt{g_s}$, wavenlength $\chi \equiv \sqrt{L}$ is larger than $R_{SB}$.

The string Haweordon temp. or an excited string matches the Hawking temp. of a BH at $M_{BH}^H$.

Highly excited long strings

entit massless quanta with a thermal specrum at the Haweordon temperture.

Decay Signature of SB
\[ R_{BH} \rightarrow \mathbb{R}^d \]

\[ (d - u + \varepsilon)(\varepsilon + u) \left[ \frac{d}{d + 1} \left( \frac{\varepsilon}{d - u + \varepsilon} \right) \right] = (d, u) \wedge \]

\[ \left( \frac{\varepsilon W}{d!} \right) \approx \frac{u}{d} \frac{u - u}{d - d} = \mathbb{R}^d \Lambda \\
\]

\[ \frac{d - u + 1}{1} \left( \frac{\varepsilon W}{d!} \right) \frac{d - u + 1}{1} \Lambda (d, u) \wedge \frac{\varepsilon W}{1} = \mathbb{R}^d \]

The radius of the \( d \)-brane is \( \mathbb{R}^d \).

The \( d \)-brane wraps on \( (m - u \geq 0) \), and on \( m \leq u \) small extra dim.

Consider as uncharged, static \( d \)-brane with mass \( W \).

A \( d \)-brane is considered a \( 0 \)-brane, so \( d \)-branes, in principle, can also be

\[ \text{Brane} \]
\[
\left( \frac{(0', u)}{d^2} \left( \frac{d^2}{u^2} \right) \right) \left( \frac{\delta W}{W} \right) \left( \frac{d^2}{(u^2 - u) e^{\frac{d^2 W}{W}}} \right) = \left( \frac{W = \text{planar}}{W = \text{planar}} \right) \phi \equiv \mathcal{H}
\]

Compare \( \phi \) with \((d)\phi \) \( \text{BH} \) \( \phi \).

\( d = \mathcal{I} \)

Wraps entirely on the small extra dimensions only.

Volumetric \( \mathcal{I} \) occurs when the \( d \)-brane \( \mathcal{I} \)-brane \( \mathcal{I} \) wraps by the \( d \)-brane. Minimum \( \mathcal{I} \) is suppressed by some powers of the volume \( \mathcal{I} \)-brane of a \( d \)-brane is suppressed by \( \mathcal{I} \).

\[
\mathcal{I} \phi = \left( \frac{\delta W}{\mathcal{I}} \phi \right)
\]

Production of \( d \)-branes.
\[ m = d = \lambda, \quad \text{and} \quad \mathcal{W}^{B\Phi} = \mathcal{W} \quad \text{The ratio} \quad \mathcal{H} \]
Decay mainly into brane particles.

Like a point source in s-waves.

Size $R^d$ is much smaller than the extra dim. We expect it decays.

Cascade of branes.

One possibility is decay into lower dimensional branes, leading to

Not well understood.

Decay of $p$-branes.
$M_B = 10 \text{ TeV}$, $M_{\text{SB}}^{n=3}$, $M_{\text{SB}}^{n=6}$

![Graph showing the relationship between $\sqrt{s}$ (TeV) and $\sigma$ (pb) for different models: BH, p-brane, and SB. The graph compares the cross-sections for $n=3$ and $n=6$ configurations.]
$M_D = 1.5 \text{ TeV}, \ M_{\text{min}}^{SB} = 3 \ M_D, \ M_{\text{min}}^{BH} = 5 \ M_D$.

![Graph showing differential cross section $d\sigma/dM$ for $M_D = 1.5 \text{ TeV}$ with lines for $BP$, $BH$, and $SB$.]
$W_D = 1.5 \text{ TeV}$, $W_{\text{min}}^B = 5 W_D$, $W_{\text{min}}^H$}

**Graph**

- **$pT(p_T|p_T)$ (GeV)**
- **$d\sigma/dp_T (pb/GeV)$**

**Legend**

- $SB$
- $BH$
- $pB$ (r=p=m)

**Conditions**

- $n=4$
- $LHC \ M^D = 1.5 \text{ TeV}$

**Equation**

$\frac{d\sigma}{dp_T}$ (pb/GeV)
Decay spectrum of the BH (spin-mine) deserves more studies.

Inclusion of black hole the production increases by about a factor of 2.

LHC and LHC will be ideal places to study BH, p-branes.

Trans-Planckian physics becomes very interesting, e.g., black holes.

Present limits are around $M_p \sim 1.4 \text{ TeV}$ for the fundamental Plank scale.

Conclusions