Beautified with Goodly Shape: Rethinking the Properties of Large Extra Dimensions

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Abstract

Much recent attention has focused on theories with large extra compactified dimensions. However, while the phenomenological implications of the volume moduli associated with such compactifications are well understood, relatively little attention has been devoted to the shape moduli. In this talk, I demonstrate that non-trivial shape moduli can lead to a number of effects which are relevant not only for experimental searches for extra dimensions, but also for the interpretation of experimental data if such extra dimensions are found. This talk reports on work done partly in collaboration with Arash Mafi.

1 Introduction: Beautified with goodly shape

Over the past several years, there has been an explosion of interest in theories with large extra spacetime dimensions [1, 2, 3, 4]. Much of this interest stems from the realization that large extra dimensions have the potential to lower the fundamental energy scales of physics, such as the Planck scale, the GUT scale, and the string scale. Indeed, the degree to which these scales may be lowered depends on the volume of the compactified dimensions.

However, compactification geometry is not merely endowed with volume; it is also "beautified with goodly shape."* Indeed, both volume (or "Kähler") moduli and shape (or "complex") moduli are necessary in order to fully describe the geometry of compactification. This distinction has phenomenological relevance because the shape moduli also play a significant role in determining the experimental bounds on such scenarios. Unfortunately, in most previous discussions of large extra dimensions, relatively little attention has been paid to the implications of these moduli.

In this talk, I shall discuss some of the phenomenological implications of non-trivial shape moduli [5, 6, 7]. First, as we shall see, shape moduli can have dramatic effects on the corresponding Kaluza-Klein (KK) spectrum [5]: they can induce level-crossings and varying mass gaps, they can help to eliminate light KK states, and they can alter experimental constraints in such scenarios to the extent that the bounds on the largest extra dimension can be completely eliminated. In other words, we shall see that large extra dimensions can be essentially invisible (even if they are flat).

*W. Shakespeare, *The Two Gentlemen of Verona*: IV, i.

Second, we shall see that shape casts "shadows" [6]. Specifically, we shall show that non-trivial shape moduli can distort our experimental perceptions of compactification geometry at low energies. Indeed, we shall find that spacetime geometry is essentially "renormalized" as a function of the energy scale with which it is probed.

Finally, we shall briefly discuss the effects that non-trivial shape moduli can have on string winding modes. We shall show, in particular, that toroidal compactifications exist for which all KK states as well as all winding modes are heavier than the string scale [7]. Thus, in the context of low-scale string theories, it is possible to cross the string scale without detecting *any* string states associated with spacetime compactification.

2 Shape matters: A bit of background

In order to illustrate these points, let us begin by considering the case of compactification on a general two-torus, as illustrated in Fig. 1. Such a torus is realized as the space $\mathbb{R}^2$ subject to the two discrete identifications

$$\begin{cases} y_1 \;\to\; y_1 + 2\pi R_1 \\ y_2 \;\to\; y_2 \end{cases} \qquad\qquad \begin{cases} y_1 \;\to\; y_1 + 2\pi R_2 \cos\theta \\ y_2 \;\to\; y_2 + 2\pi R_2 \sin\theta \;. \end{cases} \tag{2.1}$$

Thus, the flat two-torus is specified by three parameters: R_1, R_2, and θ. The volume of the torus is $V \equiv 4\pi^2 R_1 R_2 \sin\theta$, while the shape parameters are R_2/R_1 and θ.

Most previous studies have focused on the effects of V and R_2/R_1, essentially fixing $\theta = \pi/2$. But what are the phenomenological implications of non-trivial θ?

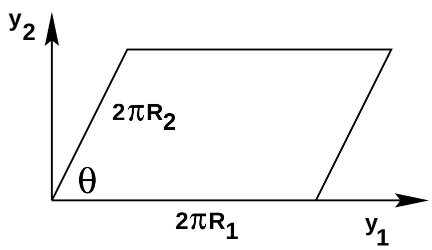

Figure 1: General two-dimensional torus with shift angle θ.

Clearly, we need to analyze the KK wavefunctions on this space. Demanding invariance under the torus periodicities in Eq. (2.1), we find that the KK wavefunctions take the form

$$\Psi_{n_1,n_2} \;\sim\; \exp\left[i\frac{n_1}{R_1}\left(y_1 - \frac{y_2}{\tan\theta}\right) \;+\; i\frac{n_2}{R_2}\frac{y_2}{\sin\theta}\right] \;, \qquad n_i \in \mathbb{Z} \;. \tag{2.2}$$

Applying the $(\text{mass})^2$ operator $-(\partial^2/\partial y_1^2 + \partial^2/\partial y_2^2)$, we then find the KK masses

$$\mathcal{M}^2_{n_1,n_2} \;=\; \frac{1}{\sin^2\theta}\left(\frac{n_1^2}{R_1^2} + \frac{n_2^2}{R_2^2} - 2\frac{n_1 n_2}{R_1 R_2}\cos\theta\right) \;. \tag{2.3}$$

Note that these masses are no longer invariant under $n_1 \to -n_1$ or $n_2 \to -n_2$ individually unless combined with $\theta \to \pi - \theta$. We can therefore restrict our attention to values of θ in the range $0 < \theta \le \pi/2$ without loss of generality.

The case with $\theta = \pi/2$ corresponds to the usual "rectangular" torus, but we are interested here in the behavior of the KK spectrum as θ is varied while holding the radii (R_1, R_2) fixed. We then find dramatic changes in the KK spectrum, as illustrated in Fig. 2 (upper plots) for the cases with $R_1 = R_2$ and $R_1 = 4R_2$. We observe that in general, the *mass gap* μ (defined as the mass of the first excited KK state) depends on θ, and tends to *increase* as $\theta \to 0$. In the case of KK gravitons, this increase in the mass gap implies that the deviations from Newtonian gravity will be *exponentially suppressed* as $\theta \to 0$, even though the radii of the extra dimensions are held fixed.

Another possibility, of course, is to hold the *volume* of the torus fixed (as well as the ratio R_2/R_1) while we vary θ. We then find the results illustrated in Fig. 2 (lower plots). In this case, the mass gap can vary, but need not necessarily grow as $\theta \to 0$.

3 The collapsing torus: A closer look

The limit as $\theta \to 0$ is interesting and requires a closer look. It turns out [5] that the resulting behavior of the lightest KK modes depends critically on whether the ratio R_2/R_1 is rational or irrational.

Let us first consider the case when the radii are held fixed. If R_2/R_1 is rational, so that we can write $R_2/R_1 = p/q$ where $p, q \in \mathbb{Z}$ in lowest form, then in the $\theta \to 0$ limit we find that the two torus periodicities in Eq. (2.1) collapse to form a single periodicity corresponding a circle of radius $R \equiv R_1/q = R_2/p$. Indeed, the states with $(n_1, n_2) = k(p, q)$ where $k \in \mathbb{Z}$ remain finitely massive in this limit, asymptotically becoming the circle KK states with masses k/R, while all other KK states become infinitely heavy and decouple from the spectrum. This behavior is shown in Fig. 2 (upper plots). If R_2/R_1 is *irrational*, however, then p and q are essentially infinite, resulting in a circle with vanishing radius; *all* KK states become infinitely heavy as $\theta \to 0$. This reflects the incompatibility of the two toroidal periodicities which grows increasingly severe as the torus collapses.

When the volume is held fixed, by contrast, the radii must grow to compensate as $\theta \to 0$. These growing radii therefore provide an extra tendency towards making the KK states lighter than they would have been if the radii (and not the volume) had been held fixed. Indeed, if R_2/R_1 is rational, we see from Fig. 2 (lower plots) that the circle-compactified states now become *massless* as $\theta \to 0$.

A more interesting situation occurs when R_2/R_1 is irrational. In this case, the incompatibility of the toroidal periodicities which forces the KK states to become infinitely heavy is balanced against the growing radii which tend to push the KK states to become massless. The outcome of this competition turns out to depend [5] on whether R_2/R_1 is transcendental (such as π or e) or merely algebraic [such as $\sqrt{2}$ or the Golden Mean $\gamma \equiv (1 + \sqrt{5})/2$]. If R_2/R_1 is transcendental, the tendency towards masslessness wins, and we again obtain massless KK states as $\theta \to 0$.

However, if R_2/R_1 is algebraic, it turns out that there is a *lower bound* on the masses of the KK states as $\theta \to 0$ with the compactification volume V held fixed [5]. As $\theta \to 0$, one finds that $V\mathcal{M}^2 \geq 4\pi^2 A$ where the lower bound A depends on certain number-theoretic properties of the ratio R_2/R_1, but does not depend directly on its actual value. Specifically, there exists an infinite set of algebraic irrational values of R_2/R_1 which all lead to the same value of A; moreover, for any value of A, the corresponding values of

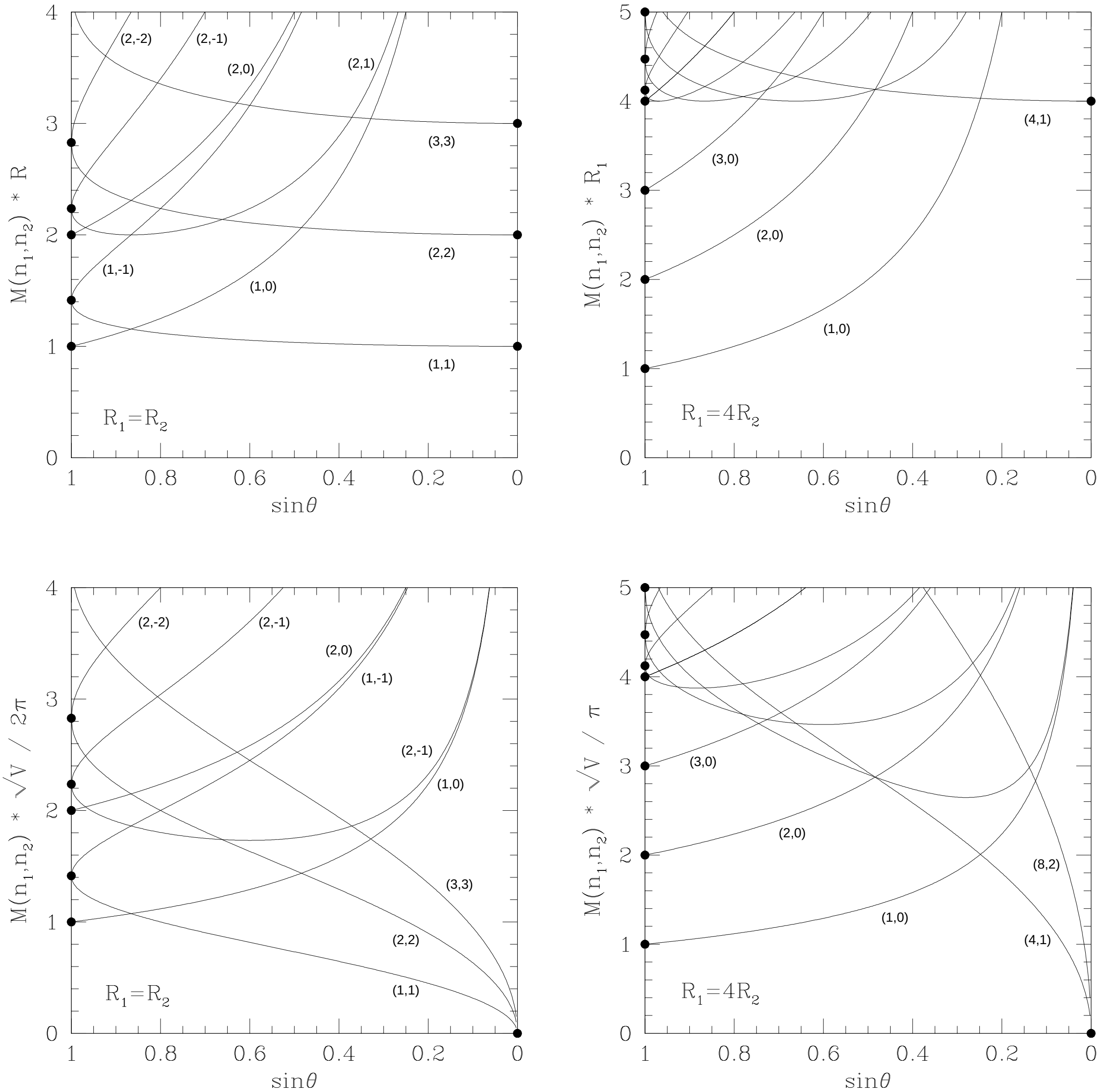

Figure 2: The masses of the lowest-lying KK states (n_1, n_2) as functions of the shape parameter θ for $R_1 = R_2 \equiv R$ (upper left plot) and $R_1 = 4R_2$ (upper right plot), holding R_1 and R_2 fixed. The lower plots represent the same scenarios where we vary θ holding both the volume of the torus and the ratio R_2/R_1 fixed.

R_2/R_1 include values with arbitrarily large magnitudes. Thus, in this sense, we see that the lower bound on the masses of the excited KK states is essentially *independent* of the value of the ratio R_2/R_1 in the $\theta \to 0$ limit, even though both the compactification volume V and the ratio R_2/R_1 are held fixed.

4 Making large, flat extra dimensions invisible

This last observation has an important consequence. Let us assume that $R_2 \geq R_1$ without loss of generality, and consider the experimental bounds that ordinarily restrict the possible sizes of extra dimensions.

In the case of a rectangular torus (for which $\theta = \pi/2$), it is straightforward to see that $V\mathcal{M}^2 \geq 4\pi^2 R_1/R_2$. However, the fact that we have not yet detected extra dimensions experimentally provides an experimental bound on the lightest possible KK state: $\mathcal{M} \geq M_{\text{expt}}$ where M_{expt} is the experimental bound. Combining these two results, and recognizing that $V \equiv 4\pi^2 R_1 R_2$, we obtain the constraint $R_2 \leq M_{\text{expt}}^{-1}$. In other words, for rectangular tori, *the experimental bounds on the non-observation of KK states become a bound on the size of the largest extra dimension*, independent of the compactification volume.

This situation changes dramatically in the $\theta \to 0$ limit. As we have seen, in this limit the KK mass gap satisfies $V\mathcal{M}^2 \geq 4\pi^2 A$ where A is independent of the specific value of R_2/R_1. Again imposing the experimental constraint $\mathcal{M} \geq M_{\text{expt}}$, we see that we now obtain a bound directly on the compactification volume: $V \leq 4\pi^2 A (M_{\text{expt}})^{-2}$. In other words, the experimental bound becomes independent of the size of the largest extra dimension! Thus, a large, flat extra dimension can be essentially *invisible* in the $\theta \to 0$ limit, even though the volume of compactification is held fixed. This result is discussed further in Ref. [5].

5 Correspondence relations: Is one torus is merely the base of another?

Let us now consider general tori in one, two, and three dimensions. These tori are illustrated in Fig. 3. The one-dimensional torus, of course, is nothing but a circle and has no corresponding shape moduli. By contrast, the two- and three-dimensional tori are described not only by radii but also by the shift angles θ and α_{ij} which mix the periodicities associated with translations along the corresponding directions. Note that we shall now use the symbol ρ to indicate the radius of the one-torus; lowercase r_1, r_2 to indicate the radii of the two-torus, with θ serving as the shape angle; and uppercase R_1, R_2, R_3 to indicate the radii of the three-torus, with $\alpha_{12}, \alpha_{13}, \alpha_{23}$ denoting the corresponding shape angles.

Our goal is to study the extent to which various low-energy observers can determine the shapes of these tori by studying their associated KK spectra. Towards this end, let us assume that the "true" compactification geometry is given by the three-torus shown in Fig. 3(c). Furthermore, let us assume that there is a hierarchy of length scales such that $R_3 \ll R_2 \ll R_1$. For example, R_3 might be near the Planck scale, while R_1 might be at the inverse TeV scale (or even the millimeter scale) and R_2 might be at some intermediate scale. Of course, to a high-energy observer with access to energies $E_{\rm max} \gg \mathcal{O}(R_3^{-1})$, the KK spectrum will reveal the presence of all three dimensions of the torus. Such an observer can then determine all three radii R_i and shape angles α_{ij} through a detailed spectral analysis of the KK states. However, for an observer with access to only intermediate energies $\mathcal{O}(R_2^{-1}) \ll E_{\rm max} \ll \mathcal{O}(R_3^{-1})$, the third dimension will be inaccessible; the compactification manifold would then appear to be a two-torus, as illustrated in Fig. 3(b). Finally, for the low-energy observer with access to energies $\mathcal{O}(R_1^{-1}) \ll E_{\rm max} \ll \mathcal{O}(R_2^{-1})$, only one dimension worth of KK states will be accessible. Such an observer would then conclude that the compactification manifold is merely a circle, as illustrated in Fig. 3(a).

This change in the effective dimensionality of the compactification space is obvious, and is not our focus in this talk. However, given the hierarchy $R_3 \ll R_2 \ll R_1$, it is natural to expect that the intermediate-energy observer would experience a two-torus whose parameters (r_1, r_2, θ) are related to the underlying parameters (R_i, α_{ij}) of the three-torus via the relations

$$r_1 = R_1 \ , \qquad r_2 = R_2 \ , \qquad \theta = \alpha_{12} \ . \tag{5.1}$$

After all, at energies much below R_3^{-1}, we expect all remnants of the third dimension to vanish, so that the two-torus experienced by the intermediate-energy observer is merely the *base* of the original three-torus in Fig. 3(c). Likewise, it is natural to expect that the lowest-energy observer would perceive a circle with radius $\rho = r_1$, which by Eq. (5.1) implies

$$\rho \ = \ r_1 \ = \ R_1 \ . \tag{5.2}$$

Once again, this would be the naïve expectation, given that we have only sufficient energy to probe the largest dimension of the original three-torus.

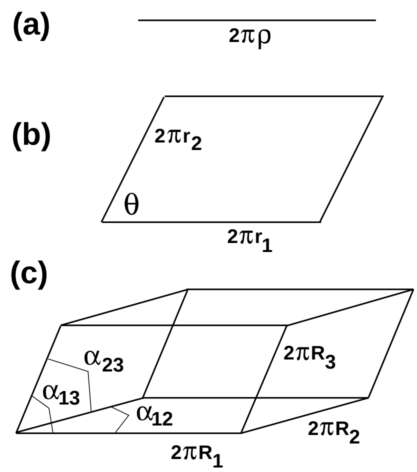

Figure 3: General one-, two-, and three-dimensional tori with arbitrary shape angles.

6 Shadowing

We shall now demonstrate that the above relations in Eqs. (5.1) and (5.2) are incorrect, even in the presence of a large hierarchy $R_3 \ll R_2 \ll R_1$, and must be replaced by relations which are far more non-trivial. Indeed, as we shall see, the relations in Eqs. (5.1) and (5.2) hold only when the shape moduli are ignored (*i.e.*, when all shape angles are taken to be $\pi/2$). In the presence of non-rectangular shape angles, by contrast, we shall see that these relations completely fail to describe the process by which small extra dimensions can be "integrated out" when passing to larger and larger length scales. We stress that this failure occurs *no matter how small* the smallest radii become. As we shall see, this failure arises because of a phenomenon called "shadowing" [6] which can have dramatic phenomenological consequences at low energies.

It is straightforward to determine the correct correspondence relations by comparing the KK spectra in each case. In the case of the circle in Fig. 3(a), the KK spectrum is given by $\mathcal{M}^2 = n_1^2/\rho^2$ where $n_1 \in \mathbb{Z}$. By contrast, for the general two-torus shown in Fig. 3(b), we have already seen in Eq. (2.3) that the KK spectrum is given by

$$\mathcal{M}^2 \;=\; \frac{1}{\sin^2\theta}\left[\sum_{i=1}^{2}\frac{n_i^2}{r_i^2} - \sum_{i\neq j}\frac{n_i n_j}{r_i r_j}\cos\theta\right] \tag{6.1}$$

where $n_i \in \mathbb{Z}$. Finally, for the general three-torus shown in Fig. 3(c), the KK spectrum is given by

$$\mathcal{M}^2 \;=\; \frac{1}{K}\left[\sum_{i=1}^{3}\frac{n_i^2}{R_i^2}s_{jk}^2 - \sum_{i\neq j}\frac{n_i n_j}{R_i R_j}\left(c_{ij} - c_{ik}c_{jk}\right)\right] \tag{6.2}$$

where $k \neq i, j$, where $c_{ij} \equiv \cos\alpha_{ij}$ and $s_{ij} \equiv \sin\alpha_{ij}$, and where K [the dimensionless squared volume of the parallelepiped in Fig. 3(c)] is given by

$$K \equiv 1 - \sum_{i<j} c_{ij}^2 + 2\prod_{i<j} c_{ij} = \sum_{i<j} s_{ij}^2 + 2\left(\prod_{i<j} c_{ij} - 1\right) . \tag{6.3}$$

Given these results, we can now determine the appropriate correspondence relations. In deriving these relations, we shall assume a hierarchy $R_3 \ll R_2 \ll R_1$ so that we can successively integrate out small extra dimensions when passing to larger length scales. Our procedure will be to disregard all KK states whose masses exceed the appropriate reference energy (either high, intermediate, or low) and are therefore inaccessible to the corresponding observer.

The observer at highest energy clearly sees three dimensions worth of KK states, and deduces the "true" geometry of the compactified space by comparing the measured KK masses with Eq. (6.2). However, the observer at intermediate energy cannot perceive excitations in the R_3 direction, since functionally $R_3 \to 0$ for this observer. His attention is therefore restricted to states with $n_3 = 0$, and he attempts a spectral analysis of the remaining states. This leads to the identifications

$$\begin{aligned}
\frac{1}{\sin^2\theta}\frac{1}{r_i^2} &= \frac{s_{jk}^2}{KR_i^2} \qquad (i = 1, 2)\\
\frac{\cos\theta}{\sin^2\theta}\frac{1}{r_1 r_2} &= \frac{c_{12} - c_{13}c_{23}}{KR_1R_2} .
\end{aligned} \tag{6.4}$$

This observer therefore deduces that the compactified space is a two-torus parametrized by (r_1, r_2, θ) given by

$$\begin{cases} r_i & = s_{i3} R_i \\ \cos\theta & = (c_{12} - c_{13}c_{23})/s_{13}s_{23} \ . \end{cases} \tag{6.5}$$

Note that both the radii r_i and the shape angle θ are affected, leading to apparent values for (r_1, r_2, θ) which are not present in the original three-torus.

The lowest-energy observer, by contrast, misses the n_2 excitations as well. Upon comparing with the circle KK spectrum $\mathcal{M}^2 = n_1^2/\rho^2$, he therefore concludes that the compactified space is a circle of radius

$$\rho \ = \ (\sin\theta) r_1 \ = \ \frac{\sqrt{K}}{s_{23}} \, R_1 \ . \tag{6.6}$$

Once again, this radius does not correspond to any periodicity in the original three-torus.

Mathematically, these results reflect the geometric "shadows" that successive smaller extra dimensions cast onto the larger extra dimensions when they are integrated out. As such, they indicate that the low-energy observer can see only those "projections" of the compactification space which are perpendicular to the extinguished dimensions. But given the assumed large hierarchy of length scales, the physical implications of this shadowing effect are rather striking. A small extra spacetime dimension — even one no larger than the Planck length! — is able to cast a huge shadow over all other length scales and their associated dimensions, completely distorting our low-energy perception and interpretation of the compactification geometry. Indeed, the physics which we would normally associate with the Planck scale (such as the angles that parametrize the shape of the Planck-sized extra dimensions relative to the larger extra dimensions) fail to decouple at low energies.

Of course, no observer at any energy scale can use this shadowing phenomenon in order to deduce the existence of an extra spacetime dimension beyond his own energy scale. Nevertheless, the observer's *interpretation* of that portion of the compactification geometry accessible to him is completely distorted, leading him to deduce geometric radii and shape angles that have no basis in reality. Since the existence of an even smaller extra dimension beyond those already perceived can never be ruled out, this shadowing effect implies that *one can never know the "true" compactification geometry*. Even when light KK states are detected and successful fits to the KK mass formulae are obtained via spectral analyses, the presence of further additional dimensions with appropriate shape moduli can always reveal the previous successes to have been illusory.

We are not claiming that no "true" compactification geometry can ever exist. Indeed, if one takes the predictions of string theory seriously, then there is ultimately a true, maximum number of compactified dimensions, with associated radii and shape moduli. However, as an *experimental* question, one can never be satisfied concerning the true number of extra dimensions. Thus, our result implies that one can correspondingly never be certain of the nature of whatever compactification geometry is ultimately discovered. In this sense, the concept of a "true" compactification geometry does not exist.

7 Examples of shadowing

We begin with a simple example: suppose $R_1^{-1} = 1$ TeV, $R_2^{-1} = 10^{11}$ GeV, $R_3^{-1} = 10^{19}$ GeV, with $\alpha_{12} = 90°$ and $\alpha_{13} = \alpha_{23} = 60°$. How is this torus perceived at lower energies? Using the above results, we see that at intermediate energies this appears to be a two-torus with $r_1^{-1} \approx 1.155$ TeV, $r_2^{-1} \approx 1.155 \times 10^{11}$ GeV, and $\theta \approx 71°$. Finally, at lowest energies we observe a single one-dimensional circle with radius $\rho^{-1} \approx 1.225$ TeV. Note that the distortions are not large in this example. However, this distortion for the size of the largest extra dimension persists over *eight orders of magnitude* in this example, and it does not disappear even as the smallest dimension(s) are taken to zero size!

Let us now consider more dramatic examples of this "shadowing" phenomenon. Suppose the original three-torus has $R_1 = R_2$ and $R_3^{-1} = 10^{19}$ GeV, with $\alpha_{12} = 90°$, $\alpha_{13} = 60° + \epsilon$, and $\alpha_{23} = 30° + \epsilon$, where $\epsilon \ll 1$. In this configuration, the third periodicity associated with the Planck-sized extra dimension is nearly in the plane formed by the periodicities of the two larger extra dimensions. After integrating out the Planck-sized extra dimension, we apparently observe a two-torus with $r_1/r_2 = \sqrt{3}$ and $\theta \approx \epsilon$. Thus the "squashing" of the original Planck-sized extra dimension relative to the large dimensions is perceived by a low-energy observer as a squashing of the two large dimensions with respect to each other! Note, moreover, that this also reproduces the preconditions (algebraic irrational ratio of radii with collapsing angle) that were required for the "invisibility" mechanism discussed earlier in Sect. 4.

Now let us suppose that the original three-torus has $R_1 = R_2$ and $R_3^{-1} = 10^{19}$ GeV, with $\alpha_{12} = 90°$ and $\alpha_{13} \ll \alpha_{23}$. We then find that the effective two-torus at lower energies apparently has $r_1 \ll r_2$. Thus, an apparent physical hierarchy in "large" radii has been *generated* by the effects of a Planck-scale extra dimension!

Finally, let us consider a case with $R_1 = R_2$ and $R_3^{-1} = 10^{19}$ GeV, where $\alpha_{12} = 60°$ and $\alpha_{13} = \alpha_{23} = 45°$. We then find that the resulting two-torus at low energies appears to be rectangular: $\theta = 90°$. Thus, even *trivial* shape moduli can be low-energy illusions produced by shadowing.

8 The "renormalization" of compactification geometry

What are we to make of these results? Clearly, as we go to higher and higher energies and discover additional spacetime dimensions, our description of the compactification manifold changes. Our perception of quantities such as the radii and shape moduli associated with the largest (and experimentally accessible) extra dimensions continually evolves as a function of the energy with which we probe this manifold — even though the largest extra dimensions are already detected and their geometric properties are already presumed known. Thus, the apparent compactification geometry is not fixed at all, but rather undergoes *renormalization* much like other "constants" of nature.

This may seem to be a rather unusual way to interpret these results. However, compactifications on these different tori constitute a series of different effective theories. As we cross the energy thresholds associated with the different extra dimensions, our effective theory changes. In this sense, then, our correspondence relations are "matching conditions" or "threshold effects" which describe the same physics at different energy scales.

Indeed, such relations merely reflect the requirement that the physical KK spectrum remain invariant under change in our effective field theories with different cutoffs. However, this is nothing but renormalization. Indeed, we can imagine extrapolating our calculations to incorporate a continuing series of hierarchies corresponding to a continuing series of extra dimensions. The corresponding series of matching conditions would then constitute a renormalization group "flow."

We stress that this is a purely classical effect. It is triggered entirely by non-trivial shape moduli, and thus is a property of geometry itself. However, as an *experimental* question, there can always be smaller which have not yet been discovered. We see, then, that there is no such thing as "true" compactification geometry.

9 KK states, winding states, and the string scale

Finally, if string theory is the ultimate theory, then spacetime compactification should produce not only KK states, but also winding states. Ordinarily, the masses of KK states and winding states play a reciprocal role: if the lightest KK states are lighter than the string scale, then the corresponding winding states are necessarily heavier than the string scale. Similarly, the reverse situation in which the lightest KK states are heavier than the string scale ordinarily results in winding states which are lighter than the string scale (and is equivalent to the previous configuration as a result of T-duality). The expectation, then, is that *either* the KK states *or* the winding states must have masses at or below the string scale. Thus, it would seem to be impossible to cross the string scale without seeing at least *some* states (either KK or winding) associated with the spacetime compactification.

However, this expectation also fails when non-trivial shape moduli are involved [7]. Specifically, it can be shown [7] that it is possible for the string scale to be simultaneously *lighter* than *all* the KK states as well as *all* the winding states. Thus, in such theories, it is possible to cross the string scale without seeing a single resonance associated with the spacetime compactification! Needless to say, this can therefore give rise to a low-energy phenomenology which is markedly different from that of the usual KK effective field theories. Further details can be found in Ref. [7].

10 Conclusions and open questions

We have seen that non-trivial shape moduli have the potential to produce some surprising and counter-intuitive results. In Sect. 4, for example, we have shown that non-trivial shape moduli have the potential to make certain large, flat extra dimensions essentially invisible. This feature can therefore be exploited to alleviate (or perhaps even eliminate) many of the experimental bounds (such as those from table-top Cavendish experiments, colliders, and astrophysical phenomena) which constrain such higher-dimensional theories. Likewise, we have seen that non-trivial shape moduli lead to a shadowing phenomenon whereby small, Planck-sized extra dimensions can profoundly alter our perception of larger, TeV-sized (or even millimeter-sized) extra dimensions. These results provide important lessons for interpreting the results of potential KK discoveries.

Needless to say, these results concerning shape moduli leave many questions unanswered. What sets the values of these shape moduli [5]? What are the effects of non-trivial shape moduli in more general toroidal compactifications with background torsion fields [7], or in more complicated non-toroidal geometries? How do modular symmetries affect these results [6], and how might these results be interpreted within the context of string theory when non-trivial vacuum energies and GSO projections are present [7]? How can the degrees of freedom associated with these non-trivial shape moduli be exploited in order to produce phenomenologically interesting theories, and how do they affect current collider (and Cavendish) bounds on extra dimensions? How might theories with non-trivial shape moduli be deconstructed? If shape moduli can have such dramatic effects on KK masses, what are their effects on couplings and scattering amplitudes? How do quantum effects contribute to the renormalization of compactification geometry? And if compactification geometry is renormalized, to what extent is spacetime geometry knowable at all? Indeed, what are the analogues of the renormalization-group invariants? Finally, can we use the phenomenon of Planck-scale "shadowing" in order to develop a new approach for attacking the cosmological constant problem? Clearly, these and other questions await further study.

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References

[1] I. Antoniadis, Phys. Lett. B **246** (1990) 377; I. Antoniadis, K. Benakli and M. Quiros, Phys. Lett. B **331** (1994) 313.

[2] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429** (1998) 263; I. Antoniadis *et al.*, Phys. Lett. B **436** (1998) 257.

[3] K.R. Dienes, E. Dudas, and T. Gherghetta, Phys. Lett. B **436** (1998) 55; Nucl. Phys. B **537** (1999) 47; arXiv:hep-ph/9807522.

[4] E. Witten, Nucl. Phys. B **471** (1996) 135; J.D. Lykken, Phys. Rev. D **54** (1996) 3693; G. Shiu and S.-H.H. Tye, Phys. Rev. D **58** (1998) 106007;

[5] K. R. Dienes, Phys. Rev. Lett. **88** (2002) 011601.

[6] K. R. Dienes and A. Mafi, Phys. Rev. Lett. **88** (2002) 111602.

[7] K. R. Dienes and A. Mafi, Phys. Rev. Lett. **89** (2002) 171602.