On the Oddity of Extra Dimensions

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Abstract

This talk describes work in progress on the properties of massless propagators in odd spacetime dimensions and their implications for Hawking radiation. Motivated by the fact that in odd-dimensional Minkowski spacetime an accelerating detector (Unruh-De Witt detector) coupled to bosons (fermions) register a Fermi-Dirac (Bose-Einstein) thermal spectrum because of the breakdown of geometrical optics, we describe an ongoing investigation into whether this apparent spin statistics flip can be found in any form in the Hawking radiation from a black hole and discuss the implications for searches for large extra dimensions at the LHC.

It is by now well understood that there are many reasons to study phenomena related to the presence of extra spacetime dimensions. The extra dimensions paradigm (which can be motivated within string theory) has inspired novel solutions/rephrasings of the hierarchy problem using large [1] and/or warped [2] extra dimensions and a low fundamental (higher-dimensional Planck) scale. These scenarios generically lead to distinctive collider signatures [3]; one of the most striking signatures is that forthcoming colliders such as the LHC can copiously produce black holes [4, 5], opening up the exciting possibility of observing Hawking radiation [6] in the laboratory.

If spacetime is D-dimensional, an important goal is to measure the number of extra dimensions. In this talk, we will focus on a possible probe of whether D is even or odd related to the properties of massless propagators in even and odd spacetime dimensions and their possible implications for the spectrum of Hawking radiation.

To begin, let us review the Green functions for massless scalar particles in D dimensions (the extension to spinors is straightforward). The classical retarded propagator $G_R^D(x,x')$ satisfies the usual equation $G_R^D(x,x') = \delta^{(D)}(x-x')$, with the boundary conditions $G_R^D(x,x') = 0$ for $(x-x')^2 < 0$, $x^0 - x'^0 < 0$.[2] In terms of scalar field commutators

$$G_R^D(x,x') = i\langle[\varphi(x), \varphi(x')]\rangle\theta(x^0 - x'^0). \tag{1}$$

In the spectral representation

$$G_R^D(x,x') = -\int \frac{dk^0}{2\pi}\frac{d^{D-1}k}{2\pi^{D-1}}\frac{e^{i(k^0 y_0 - \vec{k}\cdot\vec{y})}}{(k^0 - |\vec{k}| - i\epsilon)(k^0 + |\vec{k}| - i\epsilon)}, \tag{2}$$

in which $y \equiv x - x'$. When D is even, $G_R^D(x,x')$ is given by

$$G_R^{D\,\mathrm{even}}(x,x') = (-)^{\frac{D-4}{2}}\Omega'_{D-2}\sum_{p=0}^{\frac{D-4}{2}} b_p \partial_{y^0}^{D-4-2p}\partial_y^{2p} I_E \tag{3}$$

[1]Work in collaboration with D.J.H. Chung.

[2]Throughout this paper, analogous expressions hold for the advanced propagator $G_A^D(x,x')$, defined by $G_A^D(x,x') = \delta^{(D)}(x-x')$, $G_A^D(x,x') = 0$ for $(x-x')^2 < 0$, $x^0 - x'^0 > 0$.

$$b_p = \frac{(-)^p \left(\frac{D-4}{2}\right)!}{\left(\frac{D-4}{2} - p\right)!p!} \quad (4)$$

$$I_E = \frac{\pi}{(2\pi)^{D-1}} \frac{\delta(y^0 - y)}{y}. \quad (5)$$

Note that this propagator (as expected) has support only on the lightcone. However, this no longer holds if D is odd. More precisely,

$$G_R^{D\,\mathrm{odd}}(x, x') = (-)^{\frac{D-3}{2}} \Omega'_{D-2} \sum_{p=0}^{\frac{D-3}{2}} c_p \partial_{y^0}^{D-3-2p} \partial_y^{2p} I_O, \quad (6)$$

$$c_p = \frac{(-)^p \left(\frac{D-3}{2}\right)!}{\left(\frac{D-3}{2} - p\right)!p!} \quad (7)$$

$$I_O = \frac{\pi}{(2\pi)^{D-1}} \frac{\theta((y^0)^2 - y^2)}{\sqrt{(y^0)^2 - y^2}}. \quad (8)$$

Clearly, $G_R^{D\,\mathrm{odd}}(x, x')$ has support inside the lightcone, even though the propagator describes massless particles. Signal propagation hence is not sharp; sources of finite duration have an everlasting afterglow, indicating the well-known violation of Huygen's principle in odd spacetime dimensions. Compactification does not spoil this property; *i.e.* the support structure inside the lightcone in compact space is qualitatively different in even- and odd-dimensional spacetimes (this difference is most extreme in the decompactified limit).

Can lightcone support be used to detect the oddity of D? In particle physics, tree-level scattering amplitudes are blind to this property (momentum space propagators have qualitatively similar features). Certainly physics which depends crucially on configuration space properties might probe this feature. One known example is the Unruh effect: a uniformly accelerated detector (Unruh-De Witt detector) in Minkowski vacuum perceives a thermal bath of particles [7]. In even spacetime dimensions, the accelerated detector coupled to bosons (fermions) detects the usual Bose-Einstein (Fermi-Dirac) thermal spectrum. However, it was discovered in the 1980's that in odd spacetime dimensions there is a spin-statistics flip: the Unruh-De Witt detector coupled to bosons (fermions) detects Fermi-Dirac (Bose-Einstein) statistics [8]! The statistics flip can be traced back to the support structure of the classical propagators, which was outlined in a beautiful paper by Ooguri [9].

To see this more clearly, consider an Unruh-De Witt monopole detector in D dimensions with the uniformly accelerated trajectory (in Rindler coordinates with acceleration α^{-1}) $x_i \ldots, x_{D-2} = 0$,$x_{D-1} = \sqrt{t^2 + \alpha^2}$,$t = \alpha \sinh(\tau/\alpha)$, where τ is the proper time of the detector. The quantity of interest for the particle detection transition rate is the detector response function

$$F(\omega) = \int_{-\infty}^{\infty} d\Delta\tau e^{-i\omega\Delta\tau} G_+(x(\tau), x(\tau')), \quad (9)$$

where $\Delta\tau = \tau - \tau'$. $F(\omega)$ is the Fourier transform of the Wightman function

$$G_+(x(\tau)x(\tau')) = \langle 0_M|\varphi(x(\tau))\varphi(x(\tau'))|0_M\rangle. \tag{10}$$

In $D = 4$,

$$G_+(\Delta\tau) = -\frac{1}{16\pi^2\alpha^2}\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha} - \frac{i\epsilon}{\alpha})} = G_+^R, \tag{11}$$

and one can show by contour integration that $F(\omega)$ is proportional to the Planck spectrum, $F(\omega) \propto 1/(e^{2\pi\alpha\omega} - 1)$. However, for $D = 5$

$$G_+(\Delta\tau)\frac{i}{64\pi^2\alpha^3}\frac{1}{\sinh^3(\frac{\Delta\tau}{2\alpha} - \frac{i\epsilon}{\alpha})} = G_+^R, \tag{12}$$

and the analogous contour integration yields $F(\omega) \propto 1/(e^{2\pi\alpha\omega} + 1)$, *i.e.*, Fermi-Dirac statistics. (The crucial feature is the $\sinh^3$ rather than $\sinh^2$ dependence.) In general dimension $D = N$, one finds $F(\omega) \propto 1/(1 + (-)^{N-1}e^{2\pi\alpha\omega})$ [8].

The statistics flip in odd D can be traced back to the lightcone support of $G_R^D(x, x')$ [9]. The argument is as follows. Consider the thermal Green functions

$$\begin{aligned}
G_+^\beta &= \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H}\varphi(x)\varphi(x')\right] && (13)\\
G_-^\beta &= \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H}\varphi(x')\varphi(x)\right] && (14)\\
G_C^\beta &= \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H}[\varphi(x), \varphi(x')]\right] && (15)\\
G_{AC}^\beta &= \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H}\{\varphi(x), \varphi(x')\}\right], && (16)
\end{aligned}$$

in which $Z \equiv \mathrm{Tr}e^{-\beta H}$, and define their Fourier transforms

$$F_X^\beta(\omega) = \int_{-\infty}^{\infty} dt e^{-i\omega t} G_X^\beta(t). \tag{17}$$

The KMS condition $G_\mp^\beta(t + i\beta; \vec{x}, x') = G_\pm^\beta(t, \vec{x}, x')$ implies that

$$F_+^\beta = \frac{-F_C^\beta}{e^{\beta\omega} - 1} = \frac{F_{AC}^\beta}{e^{\beta\omega} + 1}. \tag{18}$$

In addition, Sewell's theorem dictates that G_+^R for the Rindler system (constantly accelerating observer coordinatization of a patch of Minkowski space) coordinates behaves like G_+^β with $\beta \equiv 2\pi\alpha$, such that the detector response function F is given by

$$F(\omega; \alpha, D) = \frac{-F_C^R(\omega; \alpha, D)}{e^{\beta\omega} - 1} = \frac{F_{AC}^R(\omega; \alpha, D)}{e^{\beta\omega} + 1}. \tag{19}$$

Finally, to obtain the ω dependence of $F(\omega; \alpha, D)$, it is necessary to determine whether or not $F_{C,AC}^R(\omega; \alpha, D)$ are finite or infinite polynomials in ω. There is a mathematical theorem which states if a given function $f(t)$ has support only at $t = 0$, it can be expressed as a finite sum of derivatives of δ-functions $f(t) = \sum_n^{N_c<\infty} \gamma_n \delta^{(n)}(t)$, and its Fourier transform is $\int_{-\infty}^{\infty} dte^{-i\omega t} = \sum_n^{N_c<\infty} \rho_n \omega^n$. For even D, the previous discussion of the classical propagators tells us that F_C^R is a finite polynomial in ω, and F registers Bose-Einstein statistics. However, this is *not* true for odd D, because the commutators have support inside the lightcone. In this case it is actually the *anticommutator* which has support only on the lightcone, such that F_{AC}^R is the finite polynomial in ω and F registers Fermi-Dirac statistics!

Unfortunately, the Unruh effect is not of practical applicability for the laboratory setting. However, it is related in a nontrivial way to Hawking radiation. More precisely, if a detector were to observe Hawking radiation at a constant radius outside a black hole, the detector is undergoing uniform acceleration. In the rest frame of the detector, for physical wavelengths λ arbitrarily small compared to the radius r of any sphere enveloping the black hole, the physics should be identical to that of a Rindler system and at least for those wavelengths, one expects to recover the statistics flip observed for Unruh detectors in odd dimensions. Since the Unruh effect is only appreciable for λ less than inverse acceleration $1/a$, and since the acceleration of a constant radius detector diverges near the black hole horizon, the full statistics flip should be recovered in the limit that the detector is arbitrarily close to the black hole surface.

Hence, *if* black holes are detected at the LHC and *if* this statistics flip were to occur for their Hawking radiation it would provide an experimental probe into the oddity of the extra dimensions. However, it is necessary to mention at the outset that phenomenologically the effect is likely to be too small to observe even if present because it would be the bulk (graviton) states which would demonstrate this feature, while black holes radiate mainly on the brane [10] (the phase space is also suppressed for higher spins).

The appearance of Hawking radiation is associated with the formation of a spacetime horizon during the formation of the black hole. Hence, it is first necessary to identify the definition of the vacuum states before and after the formation of the black hole. For the spherically symmetric black hole the vacuum state before the black hole creation is the Kruskal vacuum and the vacuum state after the black hole creation is the Schwarzschild vacuum. To calculate the Hawking radiation, two complete bases of eigenmodes (for the creation and annihilation operators) covering the same spacetime are then obtained by solving the mode equation with each of the boundary conditions. The modes can then be related using the Bogoliubov transformation method.

In practice, instead of dealing with a time dependent system of collapse, it is simpler to replace the system with an eternal black hole and use the natural boundary conditions afforded by the Cauchy surface in the future infinity $\mathcal{I}^+ \cup H^+$ (in Schwarzschild coordinates) and the boundary condition on the past horizon H^- (in Kruskal). Denoting the modes with Schwarzschild (Kruskal) boundary conditions as φ_S^i (φ_K^j), where the superscript denotes the mode quantum number, we then need to solve for the Bogoliubov transformation coefficient β^{ij}. It is obtained using $\varphi_S^i = \sum_j [\alpha^{ij}\varphi_K^j + \beta^{ij}\varphi_K^{*j}]$, which yields the spectrum as $n_i = \sum_j |\beta^{ij}|^2$.

Consider the Kruskal coordinates in 5D, which are written as

$$ds^2 = -(1 + \frac{r}{r_H})^2 e^{-2r/r_H} d\bar{u}d\bar{v} + r^2 d\Omega^2, \tag{20}$$

with $\bar{u} = -r_H e^{-u/r_H}$, $\bar{v} = r_H e^{v/r_H}$, and where u and v are the usual tortoise light-cone coordinates. Since the relationship between the Kruskal coordinates $(\bar{u}, \bar{v})$ and the Schwarzschild coordinates (u, v) does not depend on D, the same functional result for the thermal spectrum is obtained in 5D as in 4D, indicating *no* apparent statistics flip.

However, the boundary condition on H^- matching the real black hole collapse is contingent upon the validity of the geometrical optics approximation, which breaks down for odd D due to the violation of Huygen's principle in odd spacetime dimensions. The validity of this calculation is thus currently under investigation. Furthermore, if one uses an Unruh detector far from the black hole and near the black hole, it should be possible at least to interpolate smoothly between one spectrum statistics and the other. Calculations along these lines are currently in progress.

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