PHENOMENOLOGY OF NON-COMMUTATIVE FIELD THEORIES

Stephen Godfrey[a,b] and Michael A. Doncheski[c]

[a] *Department of Physics, Carleton University, Ottawa, ON K1S 5B6, Canada*

[b] *DESY, Deutsches Elektronen-Synchrotron, D22603 Hamburg, Germany*

[c]*Department of Physics, Pennsylvania State University, Mont Alto, PA 17237 USA*

Abstract

We study the effects of non-commutative QED (NCQED) in fermion pair production, $\gamma + \gamma \to f + \bar{f}$ and Compton scattering, $e + \gamma \to e + \gamma$. Non-commutative geometries give rise to 3- and 4-point photon vertices and to momentum dependent phase factors in QED vertices which will have observable effects in high energy collisions. We consider e^+e^- colliders with energies appropriate to the TeV Linear Collider proposals and the multi-TeV CLIC project operating in $\gamma\gamma$ and $e\gamma$ modes. Non-commutative scales roughly equal to the center of mass energy of the e^+e^- collider can be probed, with the exact value depending on the model parameters and experimental factors. The Compton process is sensitive to Λ_{NC} values roughly twice as large as those accessible to the pair production process.

1 Introduction

Non-commutative quantum field theories (NCQFT) [1] arise in string/M theory by describing the low-energy excitations of D-branes in a background of "EM-like" fields. Numerous ideas originating in string theory have stimulated particle physics phenomenology. We can regard NCQFT as yet another such example. NCQFT gives rise to an interesting set of operators not yet explored and therefore not ruled out. These give rise to a rich phenomenology with testable differences between QFT and NCQFT [2]. In this contribution we give a brief overview of NCQFT phenomenology with an emphasis on limits that can be extracted at high energy e^+e^- colliders [3, 4].

2 Non-commutative Field Theory

The central idea of NCQFT is that the conventional commuting coordinates are replaced with non-commuting space-time operators:

$$[\hat{X}_\mu, \hat{X}_\nu] = i\theta_{\mu\nu} \equiv \frac{i}{\Lambda_{NC}^2} C_{\mu\nu} \tag{1}$$

Here we adopt the Hewett-Petriello-Rizzo parametrization [4] where the overall scale, Λ_{NC}, characterizes the threshold where non-commutative (NC) effects become relevant and $C_{\mu\nu}$ is a real antisymmetric matrix whose dimensionless elements are presumably of order unity. One might expect the scale Λ_{NC} to be of order the Planck scale. However, given the possibility of large extra dimensions [5, 6] where gravity becomes strong at scales of order a TeV, it is possible that NC effects could set in at a TeV. We therefore consider the possibility that Λ_{NC} may lie not too far above the TeV scale.

We stress that the C matrix is not a tensor since its elements are identical in all reference frames resulting in the violation of Lorentz invariance. Thus, NCQFT violates Lorentz invariance at Λ_{NC}. The C-matrix can be parameterized, following the notation of [7], as

$$C_{\mu\nu} = \begin{pmatrix} 0 & C_{01} & C_{02} & C_{03} \\ -C_{01} & 0 & C_{12} & -C_{13} \\ -C_{02} & -C_{12} & 0 & C_{23} \\ -C_{03} & C_{13} & -C_{23} & 0 \end{pmatrix} \tag{2}$$

where $\sum_i |C_{0i}|^2 = 1$. Thus, the C_{0i} are related to space-time NC and are defined by the direction of the background "**E**-field". Furthermore, the C_{0i} can be parameterized as

$$\begin{aligned} C_{01} &= \sin\alpha\cos\beta \\ C_{02} &= \sin\alpha\sin\beta \\ C_{03} &= \cos\alpha. \end{aligned} \tag{3}$$

Likewise, the C_{ij} are related to the space-space non-commutativeness and are defined by the direction of the background "**B**-field". They can be parameterized as

$$\begin{aligned} C_{12} &= \cos\gamma \\ C_{13} &= \sin\gamma\sin\beta \\ C_{23} &= -\sin\gamma\cos\beta. \end{aligned} \tag{4}$$

In both cases β defines the origin of the ϕ axis which we set to $\beta = \pi/2$ so that we can parametrize C with the two angles α and γ, the angles of the background "**E**-field" and "**B**-field" relative to the z-axis. Since experiments are sensitive to the direction of the C-vectors an astronomical coordinate systems must be employed and events must be time stamped event by event.

NCQFT can be cast in the from of a conventional QFT using two approaches: the Seiberg-Witten mapping [8] and the Weyl-Moyal approach [9]. In the Moyal-Weyl approach only $U(N)$ Lie algebras are closed under Moyal brackets so that NC gauge theories are only based on $U(N)$ groups and covariant derivatives can only be constructed for fields with $Q = 0, \pm 1$. In contrast a NCSM has been developed using the Sieberg-Witten approach but it is non-renormalizable order by order.

It is difficult to construct the NCSM but NCQED is well defined in the Moyal-Weyl approach. It is the simplest extension of the standard model so that we will focus our discussion on NCQED but note that others have studied the NCSM. This approach provides a testing ground for the ideas of NCQFT.

To study the phenomenology of NCQED non-commuting coordinates are introduced and the Lagrangian is then cast in terms of conventional commuting quantum fields [10]. This gives rise to several modifications of the standard commuting QED. It introduces momentum dependent phase factors in the fermion-photon vertices and introduces new three and four-point photon vertices. These changes violate Lorentz invariance. The hallmark signal is azimuthal dependencies in the cross sections in $2 \to 2$ processes resulting from the existence of a preferred direction.

We note another approach to non-commuting phenomenology is to consider Lorentz violating operators such as

$$\theta^{\mu\nu}\bar{q}\sigma_{\mu\nu}q \tag{5}$$

to place limits on the scale noncommutativity [11, 12]. The $\theta^{\mu\nu}\bar{q}\sigma_{\mu\nu}q$ operator acts like a $\vec{\sigma} \cdot \vec{B}$ interaction with $\vec{B}$ fixed. The different sensitivities of Cs and Hg atomic clocks to external $\vec{\sigma} \cdot \vec{B}$ leads to the estimate of [11] $\Lambda_{NC} > 10^{17}$ GeV. The operator $\theta^{\mu\nu}\bar{N}\sigma_{\mu\nu}N$, where N are nucleon wavefunctions, leads to a shift in nuclear magnetic moments. Measurements of sidereal variation in hyperfine splittings in atoms gives $\Lambda_{NC} > 10^{15}$ GeV [12]. If one accepts these conclusions it is unlikely that colliders can probe NC phenomenology. The standard way out is the supposition that loop contributions might be canceled by other new physics. Some loop induced tests of non-commutative field theories are summarized in Table 1.

Table 1: Loop induced tests of NCSM.

Process	Reference
$b \rightarrow s\gamma$, sg	Iltan [13]
Lamb shift	Chaichian, Sheikh-Jabbari and Tureanu [14]
CP violation	Chang and Xing [15]
	Hinchliffe and Kersting [16]
$(g-2)_\mu$	Kersting [17]
Hyperfine Structure	Mocioiu, Pospelov and Roiban [12]
$Z \rightarrow \gamma\gamma$, gg	Mocioiu, Pospelov and Roiban [12]
	Behr *et al.* [18]
$\pi \rightarrow \gamma\gamma\gamma$	Grosse and Liao [19]
$Z \rightarrow \ell^+\ell^-$, $W \rightarrow \ell\nu$	Iltan [20]

3   Collider Tests of NCQED

Collider tests of non-commutative field theories have mainly concentrated on NCQED and by implication e^+e^- colliders. In this context a number of processes have been studied: Compton scattering, $e\gamma \rightarrow e\gamma$ [3, 21], pair production, $\gamma\gamma \rightarrow e^+e^-$ [3, 22], pair annihilation $e^+e^- \rightarrow \gamma\gamma$ [4], Moller scattering, $e^-e^- \rightarrow e^-e^-$ [4], Bhabha scattering, $e^-e^+ \rightarrow e^-e^+$ [4], two photon scattering, $\gamma\gamma \rightarrow \gamma\gamma$ [4], and Higgs production, $\gamma\gamma \rightarrow H^+H^-$ [24].

We will concentrate on the two processes $e\gamma \rightarrow e\gamma$ and $\gamma\gamma \rightarrow e^+e^-$ [3] as they provide good examples of the behavior we expect in $2 \rightarrow 2$ processes in general. We mention other processes [4] where the analysis provides additional ingredients beyond what was done in these analysis. We consider $\sqrt{s} = 0.5$, 0.8, 1.0, 1.5, 3, 5, and 8 TeV appropriate to the TESLA/LC/CLIC proposals. We assume an integrated luminosity of $L = 500$ fb^{-1}. To take into account finite detector acceptance we assume angular acceptance of $10^o \leq \theta \leq 170^o$ and $p_T > 10$ GeV.

3.1   Pair Production: $\gamma\gamma \rightarrow e^+e^-$

The Feynman diagrams for $\gamma\gamma \rightarrow e^+e^-$ are shown in Fig. 1. There are two important differences between the non-commuting version of this process and SM QED. The first is that there are momentum dependent phases in the fermion-photon vertices and the
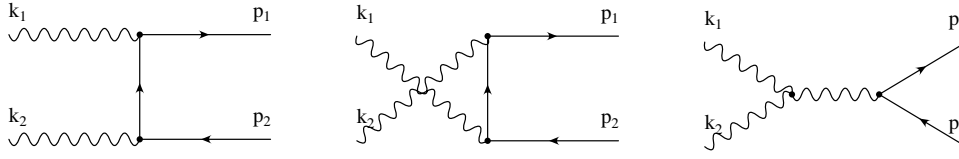
Figure 1: The Feynman diagrams contributing to the process $\gamma\gamma \to e^+e^-$.

second is the introduction of the third diagram which includes a tri-linear photon vertex. The resulting cross section is given by [3]:

$$\frac{d\sigma(\gamma\gamma \to f\bar{f})}{d\cos\theta\, d\phi} = \frac{\alpha^2}{2s}\left\{\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - 4\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}\sin^2\left(\frac{k_1 \cdot \theta \cdot k_2}{2}\right)\right\} \tag{6}$$

where k_1 and k_2 are the momenta of the incoming photons. The first two terms in the cross section are the standard model result. The third term is the NCQED contribution which includes contributions from the fermion-photon phase factors as well as the diagram with the tri-linear photon vertex. The bilinear product in the phase factor is given by:

$$\frac{1}{2}k_1 \cdot \theta \cdot k_2 = \frac{\hat{s}}{4\Lambda_{NC}^2}C_{03} = \frac{\hat{s}}{4\Lambda_{NC}^2}\cos\alpha. \tag{7}$$

This phase factor breaks Lorentz invariance. There is no ϕ dependence in this cross section and as $\Lambda_{NC} \to \infty$ the phase angle goes to 0 and the SM is recovered.

Fig. 2 shows the cross section for $\gamma\gamma \to e^+e^-$ vs. Λ_{NC} for QED and NCQED with $\alpha = 0$ and $\pi/4$, for a $\sqrt{s} = 0.5$ TeV e^+e^- collider operating in $\gamma\gamma$ mode [23]. The event rate is high with statistics that can exclude NCQED to a fairly high value of Λ_{NC}. Note that the QED (solid) curve is actually a central QED value with $\pm 1\sigma$ bands (assuming 500 fb^{-1} of integrated luminosity).

To quantify the sensitivity to NCQED, we calculate the χ^2 for the deviations between NCQED and the SM for a range of parameter values. We start by calculating statistical errors based on an integrated luminosity of 500 fb^{-1}. We consider two possibilities for systematic errors. In the first case we do not include systematic errors while in the second case we obtain limits by combining a 2% systematic error combined in quadrature with the statistical errors; $\delta = \sqrt{\delta_{stat}^2 + \delta_{sys}^2}$. The 2% systematic error is a very conservative estimate of systematic errors, for example the TESLA TDR calls for only a 1% systematic error. We calculate the χ^2 for the following obervables; total cross sections, $\cos\theta$ and ϕ angular distributions, binning the angular distributions into 20 bins in $\cos\theta$ and ϕ.

$$\chi_{\mathcal{O}}^2(\Lambda) = \sum_i\left(\frac{\mathcal{O}_i(\Lambda) - \mathcal{O}_i^{QED}}{\delta\mathcal{O}_i}\right)^2 \tag{8}$$

where $\mathcal{O}$ represents the observable under consideration and the sum is over the bins of the angular distributions. $\chi^2 = 4$ represents a 95% C.L. deviation from QED, which we'll define as the sensitivity limit. The $\cos\theta$ distribution consistently gives the highest exclusion limits on Λ_{NC}, regardless of $\sqrt{s}$ and α (as long as $\alpha \neq \pi/2$, where, again, no limits are possible). We obtain limits of $\Lambda_{NC} > (0.5 - 1)\sqrt{s}$.
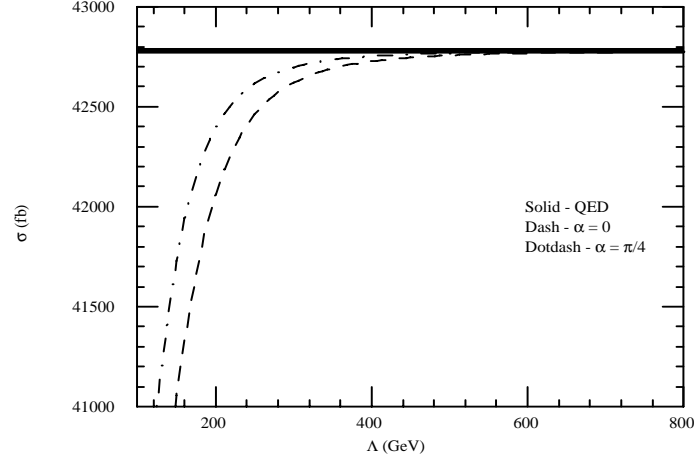
Figure 2: σ vs. Λ_{NC} for the pair production process, $\sqrt{s} = 500$ GeV. The solid line corresponds to the SM cross section $\pm$ 1 standard deviation (statistical) error based on integrated luminosity of $L = 500 fb^{-1}$.
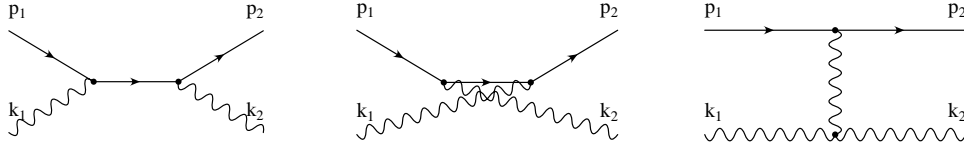

Figure 3: The Feynman diagrams contributing to the process $e\gamma \to e\gamma$.

3.2   Compton scattering

The Feyman diagrams contributing to Compton scattering are shown in Fig. 3. The differential cross section is given by [3]:

$$\frac{d\sigma(e^-\gamma \to e^-\gamma)}{d\cos\theta \, d\phi} = \frac{\alpha^2}{2s}\left\{ -\frac{\hat{u}}{\hat{s}} - \frac{\hat{s}}{\hat{u}} + 4\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\sin^2\left(\frac{k_1 \cdot \theta \cdot k_2}{2}\right)\right\}. \tag{9}$$

The first two terms in the expression are the standard QED contribution, while the last term is due to NCQED effects. As before, the NC phase factor only appears in this new term.

Here, p_1 and k_1 are the momenta of the initial state electron and photon, respectively, while p_2 and k_2 are the momenta of the final state electron and photon, respectively. $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables $\hat{s} = (p_1 + k_1)^2$, $\hat{t} = (p_1 - p_2)^2$ and $\hat{u} = (p_1 - k_2)^2$. Choosing $k_1 = x\frac{\sqrt{s}}{2}(1,0,0,-1)$ and $k_2 = k(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, results in the following expression for the phase factor:

$$\begin{aligned}
\frac{1}{2}k_1 \cdot \theta \cdot k_2 &= \frac{xk\sqrt{s}}{4\Lambda_{NC}^2}[(C_{01} - C_{13})\sin\theta\cos\phi + (C_{02} + C_{23})\sin\theta\sin\phi + C_{03}(1 + \cos\theta)] \\
&= \frac{xk\sqrt{s}}{4\Lambda_{NC}^2}[-\sin\gamma\sin\theta\cos\phi + \sin\alpha\sin\theta\sin\phi + \cos\alpha(1 + \cos\theta)]. \tag{10}
\end{aligned}$$

where x is the momentum fraction of the incident photon, k is the magnitude of the 3-momentum of the final state photon, and θ and ϕ are the lab frame angles of the final state photon. Choosing $\beta = \pi/2$ leaves us with two free parameters in addition to Λ_{NC}. Compton scattering is sensitive to both space-space and space-time NC parts, probing γ in addition to α. It therefore complements $\gamma\gamma \to e^+e^-$.

Fig. 4 shows the cross section σ vs. Λ_{NC} for QED and NCQED with $\gamma = 0$ and $\pi/2$ and $\alpha = 0$, $\pi/4$ and $\pi/2$, for a $\sqrt{s} = 0.5\ TeV\ e^+e^-$ collider operating in $e\gamma$ mode. The event rate is high, so there are enough statistics to probe NCQED up to a fairly high value of Λ_{NC}. Again, the QED (solid) curve includes the central QED value and $\pm 1\sigma$ bands (assuming 500 fb^{-1} of integrated luminosity). Fig. 5 shows the angular distributions $d\sigma/d\phi$ for QED and NCQED with $\gamma = 0$ and $\pi/2$ and $\alpha = \pi/2$, and $\sqrt{s} = \Lambda_{NC} = 500\ GeV$. The error bars in Fig. 5 assume 500 fb^{-1} of integrated luminosity.
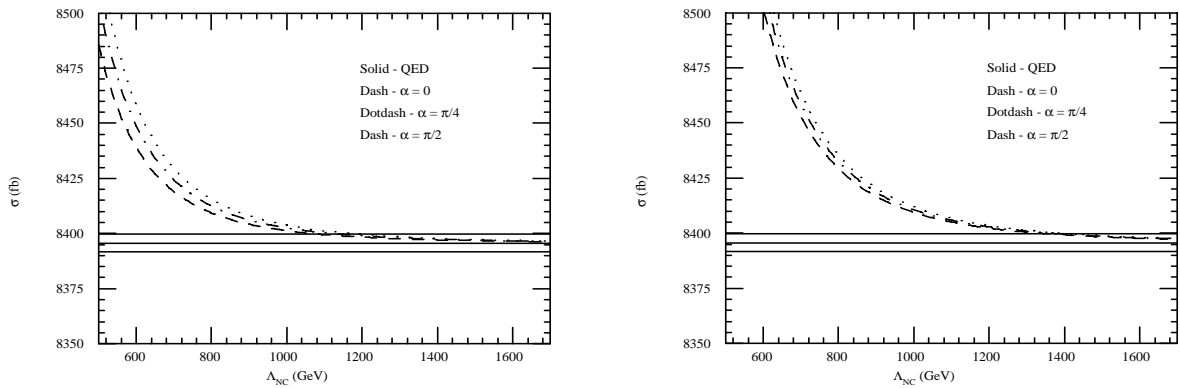

Figure 4: σ vs. Λ_{NC} for the Compton scattering process with $\sqrt{s} = 500$ GeV for (a) $\gamma = 0$ (b) $\gamma = \pi/2$. The horizontal band represents the SM cross section $\pm$ 1 standard deviation (statistical) error.

Note that there is no ϕ dependence for $\alpha = 0$ since for this case both $\mathbf{E}$ and $\mathbf{B}$ are parallel to the beam direction. In contrast, when $\alpha = \pi/2$, $\mathbf{E}$ is perpendicular to the beam direction which is reflected in the strong oscillatory behavior in the ϕ distribution. We find typical limits of $\Lambda_{NC} > (1-2)\sqrt{s}$.

3.3   Other Processes

In addition to pair creation and Compton scattering other processes have been studied. Similar results emerge through momentum dependent electron-photon vertices and in the case of pair annihilation, an additional diagram with the tri-linear photon vertex. Similar patterns of azimuthal dependence are found. Hewett, Petriollo and Rizzo studied Bhabba scattering, Moller scattering, and pair annihilation [4]. They found that the azimuthal dependences could be enhanced by introducing stiffer cuts on the angle of the final state particles. There is a tradeoff between reduced statistics with this cut and an enhanced effect. However, the sensitivity is more likely to be limited by systematic errors than by statistical errors so the stiffer cuts seem a worthwhile possibility to pursue.

In our analysis we chose a specific terrestial reference frame to perform our calculations. This is a reasonable approach in a preliminary survey of the sensitivity to the NC scale.
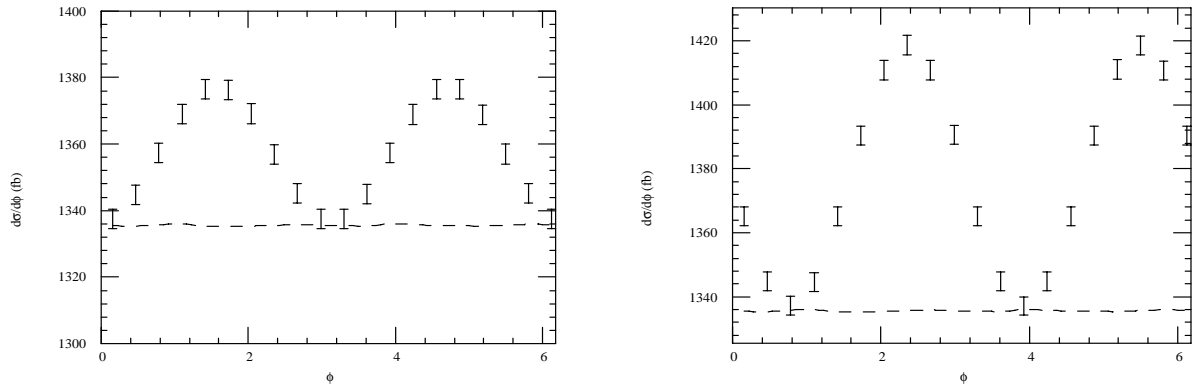
Figure 5: $d\sigma/d\phi$ for the Compton scattering process with $\sqrt{s} = 500$ GeV and for $\Lambda = 500$ GeV, $\alpha = \pi/2$ and (a) $\gamma = 0$ (b) $\gamma = \pi/2$. The dashed curve corresponds to the SM angular distribution and the points correspond to the NCQED angular distribution including 1 standard deviation (statistical) error.

However, as we have repeatedly pointed out, the background fields remain fixed with respect to some universal direction. To perform a proper analysis one would have to account for the motion of the experiment with respect to some celestial reference frame. This would need a detailed knowledge of not only the location of the experiment with respect to the earth but knowledge of when events were collected. Clearly this is beyond the scope of a simple theorist's estimate. However, along these lines, Grosse and Liao [24] did a preliminary analysis studying day-night asymmetries in $e^+e^- \to HH$ production.

4 Conclusions

To summarize, noncommutative field theory is a possibility whose phenomenology has received little attention. In this preliminary study we found that lepton pair production and Compton scattering at high energy linear colliders are excellent processes to study noncommutative QED. The hallmark signature is the appearance of azimuthally dependent cross sections in $2 \to 2$ processes.

The pair production process is only sensitive to space-time NC and is therefore insensitive to γ. As α increases towards $\pi/2$ the deviations from SM decrease towards zero, with $\alpha = \pi/2$ being identical to the SM. On the other hand, the Compton scattering process is sensitive to both space-space and space-time NC as parametrized by γ and α. On the whole, we found that the Compton scattering process is superior to lepton pair production in probing NCQED. Despite significantly smaller statistics, the large modification of angular distributions leads to higher exclusion limits, well in excess of the center of mass energy for all colliders considered.

Acknowledgments

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