Instabilities in 5-dimensional supersymmetric orbifold models

Stefan Groot Nibbelink
University Bonn

collaboration with

Hans-Peter Nilles, Marek Olechowski

based on:

hep-th/0205012
Overview

Supersymmetric theories on an orbifold

- the supersymmetry in 5D with boundaries
- the Fayet-Iliopoulos tadpoles

Localization of charge bulk fields

- the zero mode shapes
- the fate of massive excitations

Anomalies in orbifold models

- five dimensional gauge invariance
- localization of chiral modes

Conclusions
Supersymmetry multiplets:

<table>
<thead>
<tr>
<th></th>
<th>bulk</th>
<th>vector $\mathbf{V}$ and hyper $\mathbb{H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>branes</td>
<td></td>
<td>chiral $\mathcal{C}<em>I = (\phi_I, \psi</em>{I L}, F_I)$</td>
</tr>
</tbody>
</table>

$\mathbf{V}$:

<table>
<thead>
<tr>
<th>state</th>
<th>$A_\mu$</th>
<th>$A_5$</th>
<th>$\Phi$</th>
<th>$\lambda_+$</th>
<th>$\lambda_-$</th>
<th>$D_3$</th>
<th>$D_{1,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>parity</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

$\mathbb{H}$:

<table>
<thead>
<tr>
<th>state</th>
<th>$\phi_+$</th>
<th>$\phi_-$</th>
<th>$\psi_{+L}$</th>
<th>$\psi_{-L}$</th>
<th>$f_+$</th>
<th>$f_-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>parity</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

One half of the supersymmetries are broken on the orbifold

$$\delta A_\mu = i\tilde{\eta}_+ \gamma_\mu \lambda_+, \quad \delta \lambda_+ = \frac{1}{4} F^{\mu\nu} \gamma_{\mu\nu} \eta_+ - \frac{1}{2} i \tilde{D}_3 \eta_+,$$

But with: $\tilde{D}_3 = D_3 - \partial_y \Phi$! Mirabelli, Peskin’98
Fayet-Iliopoulos terms in 5D

For an abelian vector multiplet in 4D the Fayet, Iliopoulos'74 term: \( \mathcal{L}_{FI} = \xi D \) is supersymmetric and gauge invariant.

- At one-loop this FI-term is quadratically divergent if \( \sum D \phi \sim \sum Y \neq 0 \), Fischler, Nilles, Polchinski, Raby, Susskind'81

- But then a mixed gauge-gravitational anomaly arises!

In 5 dimensions there are tadpoles for \( D_3 \) Ghilencea, GN, Nilles'01, due to scalar loops Scrucca, Serone, Silvestrini, Zwirner'01, and for \( \Phi \) due to fermion loops GN, Nilles, Olechowski'02:

A calculation in modes gives

\[
\xi_{D_3} \sim g_5 q \sum_{n,n',n''} 2 \delta_{n,n'+n''} \int \frac{d^4p_4}{(2\pi)^4} \frac{\delta_{n',n''}}{p_4^2 - \frac{n''^2}{R^2}} D_n
\]

\[
\xi_{\Phi} \sim (-2)g_5 q \sum_{n,n',n''} 2 \delta_{n,n'+n''} \int \frac{d^4p_4}{(2\pi)^4} \frac{\delta_{n',n''}}{p_4^2 - \frac{n''^2}{R^2}} \left( -\frac{n''}{R} \right) \Phi_n
\]

The last part of \( \xi_{\Phi} \) can also be written as:

\[
\left( \frac{1}{2} \partial_y \Phi(y) \right)_n
\]
Fayet-Iliopoulos terms in 5D

The divergent part of the momentum integral is given by

$$
\int \frac{d^4p_4}{(2\pi)^4} \frac{1}{p_4^2 - n^2/R^2} \bigg|_{\text{div}} = \frac{1}{16\pi^2} \left( \Lambda^2 - \frac{n^2}{R^2} \ln \Lambda^2 \right).
$$

So that in terms of a coordinate space representation of the divergent tadpoles read

$$
\xi_{bulk} = \int dy g_5 \frac{\text{tr}(q)}{2} (-D_3 + \partial_y \Phi) \left( \frac{\Lambda^2}{16\pi^2} + \frac{\ln \Lambda^2}{16\pi^2} \frac{1}{4} \partial_y^2 \right) \sum_I \delta(y - IR)
$$

$$
\xi_{branes} = g_5 \frac{\Lambda^2}{16\pi^2} \int dy (-D_3 + \partial_y \Phi) \sum_{I=0,\pi} \text{tr}(q_I) \delta(y - IR)
$$

The total FI parameter is therefore given by

$$
\xi(y) = \sum \left( -\xi_I + \xi''_I \partial_y^2 \right) \delta(y - IR) \quad \text{with}
$$

$$
\xi_I = g_5 \frac{\Lambda^2}{16\pi^2} \left( \frac{1}{2} \text{tr}(q) + \text{tr}(q_I) \right), \quad \xi''_I = \frac{1}{4} g_5 \frac{\ln \Lambda^2}{16\pi^2} \frac{1}{2} \text{tr}(q)
$$
Supersymmetric background solutions

A supersymmetric background is given by
\[ \partial_y \Phi = g_5(\phi_+^\dagger q_\Phi^+ - \phi_-^\dagger q_\Phi^-) + \sum_{I=0,\pi} \delta(y - IR) g_5 \phi_I^\dagger q_I \phi_I + \xi(y), \]

\[ D_3 = \partial_y \Phi, \quad \xi(y) = \sum (-\xi_I + \xi_I'' \partial_y^2) \delta(y - IR) \]

- Not \( \langle D_3 \rangle \) but \( \langle D_3 - \partial_y \Phi \rangle \) is decided on susy,
- Integrability of equation for \( \langle \Phi \rangle \) with \( U(1) \) unbroken \( \langle \phi_a \rangle = 0 \):

\[ 0 = \int_0^{\pi R} dy \partial_y \langle \Phi \rangle = \xi_0 + \xi_\pi \Rightarrow \text{tr}(q) + \text{tr}(q_0) + \text{tr}(q_\pi) = 0 \]

Localization due to (tree-level \( \delta \)-like) FI-terms has been studied before Kaplan,Tait'01, Arkani-Hamed,Gregoire,Wacker'01.

We take the tadpole induced FI-terms as starting point:

\[ \partial_y \phi_{0+} - g_5 q_b \langle \Phi \rangle \phi_{0+} = 0 \]

for the zero mode \( \phi_{0+} \) with \( \langle \Phi \rangle \) the solution of the equation above with no spontaneous gauge symmetry \( \langle \phi_a \rangle = 0 \).
Shape of the zero mode

The solution of the zero mode with charge $q_b$ equation reads

$$\phi_{0+} \propto \exp\left\{ g_5 q_b \int_0^y dy \langle \Phi \rangle \right\} ; \quad \int_0^y dy \langle \Phi \rangle = \frac{\xi_0}{2} y + \xi_0'' \sum_I \delta(y - IR)$$

using that $U(1)$ is unbroken.

Pictorially the solutions take the form:

The spectrum of a bulk scalars ($\phi_+, \phi_-$) with charge $q_b$:

*before*

<table>
<thead>
<tr>
<th>$m_n R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

*after*

<table>
<thead>
<tr>
<th>$m_n R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

$$\left\{ \frac{1}{2} g_5 q_b \xi_0 |R \right\}$$
In a 5D theory there are no local anomalies

$$\delta_A \Gamma(A) = 2\pi i \int_{S^5} d\Omega_4^1(A; \Lambda) = 0,$$

using the Wess-Zumino consistency conditions, unless the space has boundaries

$$\delta_A \Gamma(A) = \pi i \int_{\mathcal{M}_5} (\delta(y) + \delta(y - \pi R)) \Omega_4^1(A; \Lambda) dy,$$

since we have the same orbifold projection on both branes Horava,Witten’96. A similar conclusion is reached using a perturbative calculation of the anomaly. Arkani-Hamed,Cohen,Georgi’01 Scrucca,Serone,Silvestrini,Zwirner’01, Pilo,Riotto’02, Barbieri, et al.’02.

Local 5D anomaly cancelation can be achieved by adding an appropriate Chern-Simons term

$$\Gamma_{CS} = \pi i \int \Omega_5(A),$$

which transforms as

$$\delta_A \Gamma_{CS}(A) = \pi i \int (-\delta(y) + \delta(y - \pi R)) \Omega_4^1(A; \Lambda) dy.$$
Localization and anomalies

Consider a simple orbifold model with one hyper multiplet in the bulk, and one chiral multiplet with the opposite charge on the 0-brane.

Localization of the zero mode cannot change the shape of the bulk anomaly: the anomaly is independent on the complete set of modes used. \cite{Arkani-Hamed,Cohen,Georgi01}

The bulk anomaly splits up into the zero mode and the massive modes:

Thus zero-mode anomalies and “massive” anomalies (with CS variations) cancel separately.
The parity anomaly on $S^1$

How is the orbifold model $\mathcal{M}^4 \times S^1/\mathbb{Z}_2$ constructed?

- mod out the discrete $\mathbb{Z}_2$ symmetry: $y \to -y$ on $S^1$

$$\psi(-y) = i\gamma^5 \psi(y), \quad A_\mu(-y) = A_\mu(y), \quad A_5(-y) = -A_5(y)$$

Parity anomaly: what if the fermion determinant

$$e^{-\Gamma(A)} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S}, \quad S = -\int dx \bar{\psi} \mathcal{D}(A) \psi.$$ 

is not parity invariant?

Add the parity anomaly counter term Alvarez-Gaume, Della Pietra, Moore’85

$$\Gamma_{PAC}(A) = -\pi i \int_{S^4 \times S^1} \Omega_5(A).$$

But this breaks gauge invariance GN, Nilles, Olechowski’02 if:

The gauge group contains either

- an $SU(n)$ factor, with $n \geq 3$,

- or a $U(1)$ factor (and a simple Lie group factor).

The fermionic content consists of

- an odd number fundamental representations,

- charges that add up to an odd number.
Conclusions

5 dimensional **Fayet-Iliopoulos terms** were calculated

- tadpole $D_3$ (scalars) and tadpole $\partial_y \Phi$ (fermions)
- localized on branes as $\delta$ and $\delta''$ contributions

The profile of $\Phi$ leads to **localization of bulk zero modes**

- the $\delta''$-FI tadpole often gives **delta-like** localization
- 3 types of localization (for $\Lambda \to \infty$): at a brane, near a brane, and on both branes.

**Anomalies of orbifold models** were investigated

- the $S^1$ **parity anomaly** can make orbifold **ill-defined**, 
- **localization** changes the **appearance** of anomalies.