Localization and anomalies on orbifolds

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Abstract

In this talk we discus some properties of supersymmetric theories on orbifolds in five dimensions. The structure of FI–tadpoles may lead to (strong) localization of charged bulk scalars. Orbifold theories may suffer from various kinds of anomalies. The parity anomaly may render the construction of the orbifold theory ill–defined. The gauge anomaly on the orbifold are localized at the fixed points, which can sometimes be canceled by a Chern–Simons term.

1 Introduction and summary

Recently there has been a large interest in field theories with extra dimensions. When these extra dimensions have boundaries, one has to consider field theories with fields living both in the bulk and on these boundaries. As those theories are presumably some sort of low–energy description of string or M–theory, they often contain some remnant of supersymmetry. At the same time one would hope that they allow for phenomenology which resembles physics of the (supersymmetric) standard model.

A central question in the discussion of these field-theoretic orbifolds concerns the stability with respect to ultraviolet effects as, for example, quadratic divergences of scalar mass terms. One way to insure stability in this respect would be the consideration of supersymmetry [1]. In the presence of $U(1)$ gauge groups, however, supersymmtery is not enough as there might appear quadratically divergent Fayet-Iliopoulos (FI) terms even within the supersymmetric context. To obtain a stable theory such FI-terms have to be cancelled by a specific choice of the $U(1)$ charges of scalar fields. Higher order corrections are absent due to a nonrenormalization theorem[2]. The stability question of

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field theoretic orbifolds has been discussed extensively in the literature; see ref. [3] for a review and references.

This discussion is quite relevant for phenomenological considerations as the standard model contains the $U(1)$ gauge group of hypercharge. Quadratically divergent FI-tadpoles will appear if the sum of the hypercharges of the scalar fields does not vanish. A theory with a single Higgs multiplet (as in the standard model) will thus generically suffer from an ultraviolet instability. The simplest way to avoid this problem is the introduction of a second Higgs multiplet with opposite hypercharge (as e.g. in the minimal supersymmetric standard model).

In higher dimensional theories with bulk and brane fields the above discussion will become even more complex as localized FI-tadpoles (at some boundary or fixed point) might appear. In the present talk we shall elaborate on these complications and discuss the physical consequences of localized anomalies and FI-tadpoles. For simplicity we consider supersymmetric gauge theory in five dimensions compactified on $S^1/\mathbb{Z}_2$, coupled to hyper multiplets in the bulk and chiral multiplets on the boundaries. Before discussing these aspects in more detail, let us summarize the results that are reviewed in this talk:

The shape of the FI–terms over the fifth dimension leads to intriguing physical effects: it can cause the localization of the zero modes of the bulk hyper multiplets to the branes. Another important issue in field theory is the structure of anomalies. For orbifolds the anomaly structure can be quite rich: the gauge anomalies tend to be localized at the branes, and their cancellation may involve some contributions of a five dimensional Chern–Simons term. Another type of anomaly, the so–called parity anomaly, may lead to difficulties in defining the orbifold theory in the first place.

2 The setup of the 5D supersymmetric orbifold

We consider a five dimensional bulk on $S^1/\mathbb{Z}_2$ with one vector multiplet V and a set of hyper multiplets $H = \{H^b, b = 1, \ldots, n\}$. The vector multiplet $V = (A_M, \lambda, \Phi, \vec{D})$ contains a five dimensional vector field A_M, a Dirac gaugino λ, a real scalar Φ, and a triplet of auxiliary fields $\vec{D}$ in the off–shell formulation. The hyper multiplets $H = (\phi_+, \phi_-, \psi_{+L}, \psi_{-L}, f_+, f_-)$ consist of two complex scalars ϕ_+, ϕ_-, called the hyperons, two sets of chiral spinors ψ_{+L}, ψ_{-L}, the so–called hyperinos, and complex auxiliary scalars f_+, f_-. They are charged under the $U(1)$ gauge field, with charge operator q. The orbifolding leads to the following parity assignments

V :

state	A_μ	A_5	Φ	$\lambda_{\pm L}$	$\lambda_{\pm R}$	D_3	$D_{1,2}$
parity	$+$	$-$	$-$	$\pm$	$\pm$	$+$	$-$

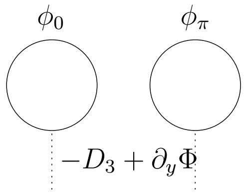
H :

state	$\phi_\pm$	$\psi_{\pm L}$	$\psi_{\pm R}$	$f_\pm$
parity	$\pm$	$\pm$	$\pm$	$\pm$

From the five dimensional supersymmetry transformations, one can obtain the unbroken four dimensional supersymmetry transformation with Majorana parameter η_+, which exists at the orbifold fixed points. This implies [4] that the four dimensional vector multiplet on the branes is given by $V| = (A_\mu, \lambda_+, \tilde{D}_3)$ with "modified" auxiliary field $\tilde{D}_3 = D_3 - \partial_y \Phi$. In addition we allow for an arbitrary number of chiral multiplets $C_0 = (\phi_0, \psi_{0L}, \tilde{f}_0)$ and $C_\pi = (\phi_\pi, \psi_{\pi L}, \tilde{f}_\pi)$ on the branes $y = 0$ and $y = \pi R$.

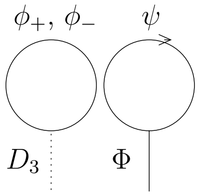
3 Fayet–Iliopoulos tadpoles and localization of bulk fields

Now we turn to the first central topic of this talk: the structure of the one loop induced FI–tadpoles. As $\tilde{D}_3$ not D_3 is the relevant auxiliary field for the vector multiplet at the branes, the one-loop FI-terms due to brane chiral multiplets, C_0 or C_π, are proportional to $\tilde{D}_3$



$$\xi_{branes}(y) = g_5 \frac{\Lambda^2}{16\pi^2} \sum_{I=0,\pi} \delta(y - IR) \mathrm{tr}(q_I). \tag{1}$$

The situation for tadpoles due to the bulk fields is more subtle. In ref. [5] it was shown that hyper multiplets may lead to a quadratically divergent zero mode FI–term, and in ref. [6] it was argued that the counter term for D_3 is located at the branes. This localization on the branes implies that there exists tadpole for the $\partial_y \Phi$ as well. In ref. [7, 8] it was shown that the bulk hyperinos ψ give rise to a tadpole for this operator. The bulk induced tadpoles involve a double derivative acting on the brane delta functions:



$$\xi_{bulk}(y) = g_5 \frac{\mathrm{tr}(q)}{2} \Big(\frac{\Lambda^2}{16\pi^2} + \frac{\ln \Lambda^2}{16\pi^2} \frac{\partial_y^2}{4} \Big) \Big[\delta(y) + \delta(y - \pi R) \Big]. \tag{2}$$

The derivative $\partial_y \Phi$ arises because of the Kaluza–Klein mass coupling of the hyperinos to Φ, which is dictated by five dimensional supersymmetry. Combining both the brane and bulk contributions to the FI–terms gives

$$\xi(y) = \sum_{I=0,\pi} (\xi_I + \xi_I'' \partial_y^2) \delta(y - IR), \tag{3}$$

$$\xi_I = g_5 \frac{\Lambda^2}{16\pi^2} \Big(\frac{1}{2} \mathrm{tr}(q) + \mathrm{tr}(q_I) \Big), \quad \xi_I'' = \frac{1}{4} g_5 \frac{\ln \Lambda^2}{16\pi^2} \frac{1}{2} \mathrm{tr}(q). \tag{4}$$

The second main issue of this talk is the question how these FI–terms can cause an instability that finally leads to localization of bulk hyper multiplet fields. For this we first investigate the background for Φ in the presence of the FI–tadpoles. Its (BPS) field equation, that respects four dimensional supersymmetry, is given by

$$D_3 = \partial_y \Phi = g_5 (\phi_+^\dagger q \phi_+ - \phi_-^\dagger q \phi_-) + \sum_{I=0,\pi} \delta(y - \pi R) g_5 \phi_I^\dagger q_I \phi_I + \xi(y). \tag{5}$$

There are supersymmetric vacua which do not spontaneously break the gauge symmetry (all charged scalars vanish in the vacuum) if the following integrability condition is satisfied

$$0 = \int_0^{\pi R} \mathrm{d}y \, \partial_y \langle \Phi \rangle = \frac{1}{2} \xi_0 + \frac{1}{2} \xi_\pi \quad \Rightarrow \quad \mathrm{tr}(q) + \mathrm{tr}(q_0) + \mathrm{tr}(q_\pi) = 0. \tag{6}$$

Note that this condition is identical to the requirement that the mixed $U(1)$ gauge gravitational anomaly is absent in the effective four dimensional theory. Therefore, from now we always assume that $\xi_\pi = -\xi_0$. Even with this requirement fulfilled, the shape of the background expectation value of $\langle \Phi \rangle$ is non–trivial. Its integral, for example, takes the form

$$\int_0^y \mathrm{d}y \, \langle \Phi \rangle (y) = \frac{1}{2}\xi_0 \Big(\pi R - |y - \pi R| \Big) + \xi_0'' \Big(\delta(y) + \delta(y - \pi R) \Big). \tag{7}$$

This affects the shape of the zero mode in a dramatic way. The shape of the zero mode ϕ^b_{0+} of an even bulk field with charge q_b is given by

$$\partial_y \phi^b_{0+} - g_5 q_b \langle \Phi \rangle \phi^b_{0+} = 0 \quad \Rightarrow \quad \phi^b_{0+}(y) = \exp \left\{ g_5 q_b \int_0^y \mathrm{d}y \, \langle \Phi \rangle \right\} \bar{\phi}^b_{0+}, \tag{8}$$

where the integral has been evaluated in (7). The remaining $N = 1$ four dimensional supersymmetry implies that the zero mode wave functions of the even bulk scalar ϕ^b_{0+} and the chiral fermion ψ^b_{0+L} with charges q_b are identical. The delta functions in that expression requires some form of regularization that takes the normalization $\int_0^{\pi R} \mathrm{d}y \, |\phi_{0+}(y)|^2 = 1$ into account. Skipping the computational details (which can be found in ref.[8]) we find that the shape of the zero mode crucially depends on the sign of the product of $\xi_0'' q_b$ (the special case $\xi_0'' = 0$ has be studied in refs.[9, 10]):

$$(\phi^b_{0+})^2(y) = \frac{2 e^{g_5 q_b \xi_0 \, y}}{1 + e^{g_5 q_b \xi_0 \, \pi R}} \left[\delta(y) + \delta(y - \pi R) \right], \qquad \xi_0'' q_b > 0; \tag{9}$$

$$(\phi^b_{0+})^2(y) = \frac{g_5 q_b \xi_0 \, e^{g_5 q_b \xi_0 \, y}}{e^{g_5 q_b \xi_0 \, \pi R} - 1} \begin{cases} 1 & 0 < y < \pi R, \\ 0 & y = 0, \pi R, \end{cases} \qquad \xi_0'' q_b < 0. \tag{10}$$

Hence, for $\xi_0'' q_b > 0$ the zero mode has the delta function support on the two fixed points, but the height at these two fixed points is not the same: while, for $\xi_0'' q_b < 0$ the zero mode vanishes at both branes identically, but has an exponential behavior on the open interval $]0, \pi R[$. In both cases, displayed in figures 1, the value of $|\xi_0''|$ does not appear anymore; it has been absorbed in the regularization of the delta functions when implementing the normalization of the modes. The shapes of the zero modes in the limit of the cut–off $\Lambda \to \infty$ are depicted in figure 2. When a zero mode becomes localized, it is natural to ask what happens to the other (massive) states in the KK–towers. Taking into account the non–trivial background for $\langle \Phi \rangle$ induced by the FI–terms, the KK–mass spectrum is given by

$$(m^b_n)^2 = \frac{1}{4}(g_5 q_b \xi_0)^2 + \frac{n^2}{R^2}, \quad n \in \mathbb{N}. \tag{11}$$

Clearly, in the limit of large cut-off Λ all non-zero mode states become extremely heavy, and should decouple from the theory.

4 Orbifold anomalies

The remainder of this talk is devoted to the subject of anomalies that can be associated to orbifold models. First we briefly mention the parity anomaly on the circle and then turn

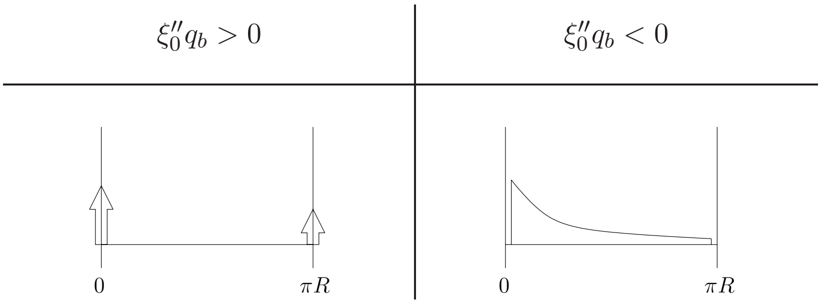


Figure 1: The two basic shapes (eqs. (9) and (10)) of the zero mode with charge q_b are displayed for a finite value of the cut-off Λ. Delta function localizations, denoted by the arrows.

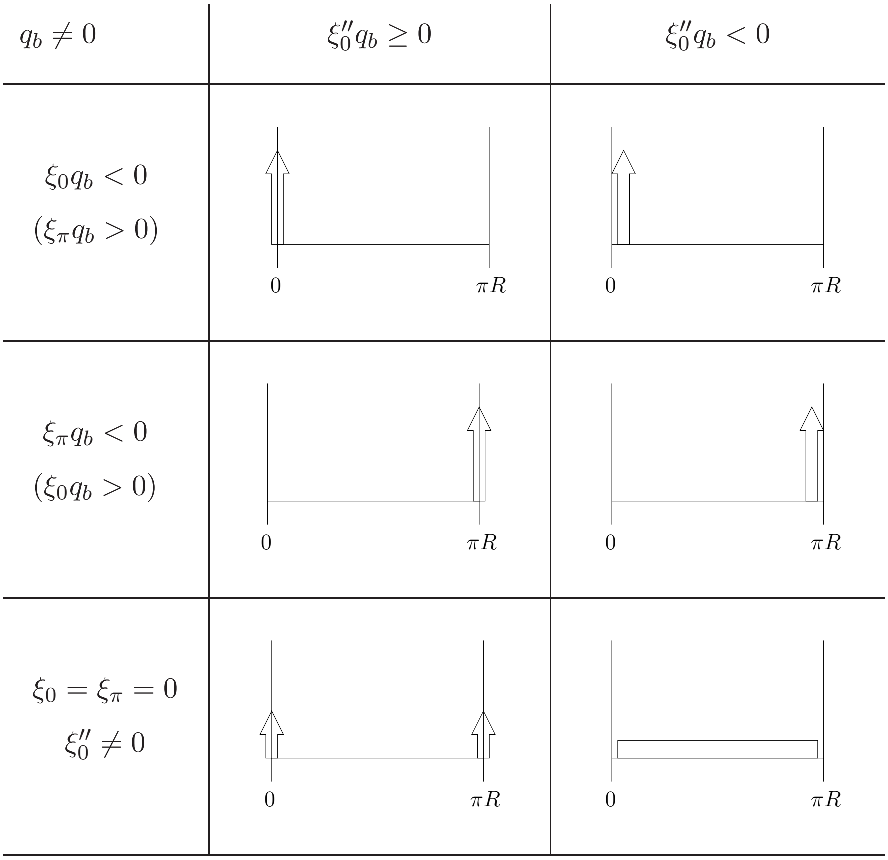


Figure 2: This table schematically displays the different shapes of a (bulk) zero mode with charge $q_b \neq 0$ when the cut–off Λ is taken to be very large.

our attention to the gauge anomalies on the orbifold. The construction of the orbifold field theory relies on the fact that

$$\psi(-y) = i\gamma^5\psi(y), \quad A_\mu(-y) = A_\mu(y), \quad A_5(-y) = -A_5(y), \tag{12}$$

is a symmetry of the theory on the circle which can be moded out, so as to obtain the orbifold theory. However, one has to be careful since this symmetry can be anomalous as was observed in ref. [11]. This anomaly can be canceled by adding the parity anomaly counter term $\Gamma_{PAC}(A) = -\pi i \int_{S^4 \times S^1} \Omega_5(A)$, with $\Omega_5(A)$ the Chern-Simons 5-form. (See for example ref.[12] for the definitions of forms $\Omega_{2n+2}, \Omega_{2n+1}$ and Ω^1_{2n}.) But then invariance under non–contractible gauge transformations may be lost. If this happens, it may not be possible to define a consistent quantum field theory on this orbifold. Here we do not go into detail but just state a rule of thumb: if the number of bulk fermions and the sum of their charges is even, no parity anomaly arises [8].

Next, we consider gauge anomalies on the orbifold $S^1/\mathbb{Z}_2$. Using a variant of the argumentation of Horava and Witten [13], we infer that the gauge anomaly of this five dimensional theory is localized at the fixed points

$$\delta_\Lambda \Gamma(A) = N\, \pi i \int_{\mathcal{M}_5} \left(\delta(y) + \delta(y - \pi R)\right) \Omega^1_{4|F}(A;\Lambda)\mathrm{d}y, \tag{13}$$

where $\Gamma(A)$ denotes the effective action with the fermions integrated out. Here the anomaly is normalized to the fundamental representation F to fix the normalization N of the anomaly uniquely. (Using a perturbative calculation a similar result was obtained in ref.[10]. See also refs.[14, 15] for a discussion on the orbifold $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2$.) When there are chiral fermions on the boundaries we may obtain additional anomaly contributions: the variation of their effective action $\Gamma_I(A)$ reads

$$\delta_\Lambda \Gamma_I(A) = N_I\, 2\pi i \int_{S^4} \Omega^1_{4|F}(A;\Lambda)\Big|_{IR}. \tag{14}$$

In addition we can allow for a five dimensional Chern-Simons action $\Gamma_{CS}(A)$:

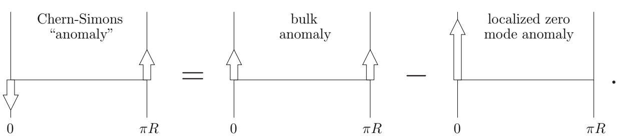
$$\begin{aligned} \delta_\Lambda \Gamma_{CS}(A) &= N_{CS}\, \delta_\Lambda \pi i \int_{\mathcal{M}_5} \Omega_{5|F}(A) \\ &= N_{CS}\, \pi i \int_{\mathcal{M}_5} \left(-\delta(y) + \delta(y - \pi R)\right) \Omega^1_{4|F}(A;\Lambda)\mathrm{d}y. \end{aligned} \tag{15}$$

To have a theory which does not have any anomaly we find the requirements

$$N_{CS} = N_0 - N_\pi, \qquad N + N_0 + N_\pi = 0. \tag{16}$$

Notice that the consistency requirement takes the form of a sum rule, and is determined by the fermionic zero mode spectrum of the bulk and branes only. Furthermore, a Chern-Simons term is required only if the anomalies at both branes are not equal to each other.

In the final part of this talk we combine the localization effects and the discussion on the gauge anomalies. One may define a "massive" anomaly as the anomaly of the bulk fields minus the bulk zero mode. Now if due to FI–terms the zero mode gets localized at a brane, the massive anomaly equals the variation of the Chern–Simons term. Pictorially this may be represented as



(17)

This shows that the localized bulk zero mode cancels the anomaly of the brane chiral fermions; while the heavy stuff from the four dimensional effective field theory point of view (the anomaly due to the massive KK–states and the Chern–Simons variation) cancel among themselves, leaving no trace in the zero mode four dimensional theory.

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