The Scalar Sector of the Randall-Sundrum Model

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Outline

- Is the vacuum unique?
- Understanding the parameter space
- Basics of the couplings
- Phenomenology
- Conclusions

Presuming the new physics scale to be close to the TeV scale, there can be a rich new phenomenology in which Higgs and radion physics intermingle, especially if the $\xi R \widehat{H}^{\dagger} \widehat{H}$ mixing term is present in \mathcal{L} . Previous work:

• $\boldsymbol{\xi} = 0$:

1. Bae:2000pk S. B. Bae, P. Ko, H. S. Lee and J. Lee, Phys. Lett. B 487, 299 (2000) [arXiv:hep-ph/0002224].

- 2. Davoudiasl:1999jd H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. Lett. 84, 2080 (2000) [arXiv:hep-ph/9909255].
- 3. Cheung:2000rw K. Cheung, Phys. Rev. D 63, 056007 (2001) [arXiv:hep-ph/0009232].
- 4. Davoudiasl:2000wi H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. D 63, 075004 (2001) [arXiv:hep-ph/0006041].
- 5. Park:2000xp S. C. Park, H. S. Song and J. Song, Phys. Rev. D 63, 077701 (2001) [arXiv:hep-ph/0009245].

• $\xi \neq 0$:

- 1. wellsmix G. Giudice, R. Rattazzi, J. Wells, Nucl. Phys. B595 (2001), 250, hep-ph/0002178.
- 2. csakimix C. Csaki, M.L. Graesser, G.D. Kribs, Phys. Rev. D63 (2001), 065002-1, hep-th/0008151.
- 3. Han:2001xs T. Han, G. D. Kribs and B. McElrath, Phys. Rev. D 64, 076003 (2001) [arXiv:hep-ph/0104074].
- 4. Chaichian:2001rq M. Chaichian, A. Datta, K. Huitu and Z. h. Yu, Phys. Lett. B 524, 161 (2002) [arXiv:hep-ph/0110035].
- 5. Hewett:2002nk J. L. Hewett and T. G. Rizzo, hep-ph/0202155.
- 6. Csaki:1999mp C. Csaki, M. Graesser, L. Randall and J. Terning, Phys. Rev. D 62, 045015 (2000) [arXiv:hep-ph/9911406].

Some possibly very dramatic changes in phenomenology.

- We consider the usual two-brane (one visible, one hidden) RS 5D warped space scenario.
- The model is defined by the 5D action:

$$egin{aligned} S &=& -\int d^4x\,dy\sqrt{-\widehat{g}}\left(rac{R}{2\widehat{\kappa}^2}+\Lambda
ight) \ &+\int d^4x\,\sqrt{-g_{hid}}(\mathcal{L}_{hid}-V_{hid})+\int d^4x\,\sqrt{-g_{vis}}(\mathcal{L}_{vis}-V_{vis}), \end{aligned}$$

where $\hat{g}^{\hat{\mu}\hat{\nu}}(\hat{\mu},\hat{\nu}=0,1,2,3,y)$ is the bulk metric and $g_{hid}^{\mu\nu}(x) \equiv \hat{g}^{\mu\nu}(x,y=0)$ and $g_{vis}^{\mu\nu}(x) \equiv \hat{g}^{\mu\nu}(x,y=1/2)$ $(\mu,\nu=0,1,2,3)$ are the induced metrics on the branes.

• If $\Lambda/m_0 = -V_{hid} = V_{vis} = -6m_0/\hat{\kappa}^2$ and if periodic boundary conditions identifying (x, y) with (x, -y) are imposed, then the 5D Einstein equations

 $ds^{2} = e^{-2\sigma(y)} \eta_{\mu\nu} dx^{\mu} dx^{\nu} - b_{0}^{2} dy^{2}, \qquad (2)$

where $\sigma(y) \sim m_0 b_0 |y|.$

 \Rightarrow

- Fluctations of $g_{\mu\nu}$ relative to $\eta_{\mu\nu}$ are the KK excitations $h^n_{\mu\nu}$.
- Fluctations of b(x) relative to b_0 define the radion field.
- In addition, we place a Higgs doublet \widehat{H} on the visible brane.

Including the ξ mixing term

• We begin with

$$S_{\boldsymbol{\xi}} = \boldsymbol{\xi} \int d^4x \sqrt{g_{\mathrm{vis}}} R(g_{\mathrm{vis}}) \widehat{H}^{\dagger} \widehat{H} \,,$$
 (3)

where $R(g_{vis})$ is the Ricci scalar for the metric induced on the visible brane.

• A crucial parameter is the ratio

$$\gamma \equiv v_0 / \Lambda_\phi \,. \tag{4}$$

where Λ_{ϕ} is vacuum expectation value of the radion field.

• After writing out the full quadratic structure of the Lagrangian, including $\xi \neq 0$ mixing, we obtain a form in which the h_0 and ϕ_0 fields for $\xi = 0$ are mixed and have complicated kinetic energy normalization.

We must diagonalize the kinetic energy and rescale to get canonical

normalization.

$$h_{0} = \left(\cos\theta - \frac{6\xi\gamma}{Z}\sin\theta\right)h + \left(\sin\theta + \frac{6\xi\gamma}{Z}\cos\theta\right)\phi$$

$$\equiv dh + c\phi$$
(5)

$$\phi_{0} = -\cos\theta\frac{\phi}{Z} + \sin\theta\frac{h}{Z} \equiv a\phi + bh.$$
(6)

• The mixing angle θ is given by

$$\tan 2\theta \equiv 12\gamma \xi Z \frac{m_{h_0}^2}{m_{\phi_0}^2 - m_{h_0}^2 (Z^2 - 36\xi^2 \gamma^2)}.$$
(7)

• In the above equations

$$Z^{2} \equiv 1 + 6\xi \gamma^{2} (1 - 6\xi) \,. \tag{8}$$

 $Z^2 > 0$ is required to avoid tachyonic situation.

This can be reexpressed as the requirement:

$$\frac{1}{12} \left(1 - \sqrt{1 + \frac{4}{\gamma^2}} \right) \le \xi \le \frac{1}{12} \left(1 + \sqrt{1 + \frac{4}{\gamma^2}} \right)$$
(9)

• The corresponding mass-squared eigenvalues are

$$m_{\pm}^{2} = \frac{1}{2Z^{2}} \left(m_{\phi_{0}}^{2} + \beta m_{h_{0}}^{2} \pm \left\{ [m_{\phi_{0}}^{2} + \beta m_{h_{0}}^{2}]^{2} - 4Z^{2}m_{\phi_{0}}^{2}m_{h_{0}}^{2} \right\}^{1/2} \right),$$
(10)
with $\beta \equiv 1 + 6\xi\gamma^{2}$ and $\operatorname{Max}[m_{h}, m_{\phi}] = m_{\pm}.$

- The process of inversion is very critical to the phenomenology and somewhat delicate.
- One finds:

$$[\beta m_{h_0}^2, m_{\phi_0}^2] = \frac{Z^2}{2} \left[m_+^2 + m_-^2 \pm \left\{ (m_+^2 + m_-^2)^2 - \frac{4\beta m_+^2 m_-^2}{Z^2} \right\}^{1/2} \right].$$
(11)

• For the quantity inside the square root appearing in Eq. (11) to be positive, we require that:

$$\frac{m_{+}^{2}}{m_{-}^{2}} > 1 + \frac{2\beta}{Z^{2}} \left(1 - \frac{Z^{2}}{\beta}\right) + \frac{2\beta}{Z^{2}} \left[1 - \frac{Z^{2}}{\beta}\right]^{1/2}, \qquad (12)$$

where $1-Z^2/eta=36\xi^2\gamma^2/eta>0.$

- I.e. since we will identify m_+ with either m_h or m_{ϕ} , the physical states h and ϕ cannot be too close to being degenerate in mass, depending on the precise values of ξ and γ ; extreme degeneracy is allowed only for small ξ and/or γ .
- A two-fold ambiguity remains in solving for $\beta m_{h_0}^2$ and $m_{\phi_0}^2$, corresponding to which we take to be the larger.

We resolve this ambiguity by requiring that $m_{h_0}^2 \to m_h^2$ in the $\xi \to 0$ limit. This means that for $\beta m_{h_0}^2$ we take the + (-) sign in Eq. (11) for $m_h > m_{\phi}$ ($m_h < m_{\phi}$), *i.e.* for $m_h = m_+$ ($m_h = m_-$), respectively.

• Given this choice, we complete the inversion by writing out the kinetic energy of Eq. (??) using the substitutions of Eqs. (5) and (6) and demanding

that the coefficients of $-\frac{1}{2}h^2$ and $-\frac{1}{2}\phi^2$ agree with the given input values for m_h^2 and m_{ϕ}^2 .

It is easy to show that these requirements are equivalent and imply

$$\sin 2\theta = \frac{12\gamma\xi m_{h_0}^2}{Z\left(m_{\phi}^2 - m_{h}^2\right)}.$$
 (13)

Note that the sign of $\sin 2\theta$ depends upon whether $m_h^2 > m_{\phi}^2$ or vice versa. It is convenient to rewrite the result for $\tan 2\theta$ of Eq. (7)

$$\tan 2\theta = \frac{12\gamma \xi m_{h_0}^2}{Z\left(m_{\phi}^2 + m_h^2 - 2m_{h_0}^2\right)}.$$
 (14)

In combination, Eqs. (13) and (14) are used to determine $\cos 2\theta$. Together, $\sin 2\theta$ and $\cos 2\theta$ give a unique solution for θ .

Using this inversion, for given ξ , γ , m_h and m_{ϕ} we compute

• Z^2 from Eq. (8),

- $m_{h_0}^2$ and $m_{\phi_0}^2$ from Eq. (11),
- and then θ from Eq. (7).
- With this input, we can then obtain a, b, c, d as defined in Eqs. (5) and (6).
- Net result

4 independent parameters to completely fix the mass diagonalization of the scalar sector when $\xi \neq 0$. These are:

$$\boldsymbol{\xi}, \quad \boldsymbol{\Lambda}_{\boldsymbol{\phi}}, \quad \boldsymbol{m}_{\boldsymbol{h}}, \quad \boldsymbol{m}_{\boldsymbol{\phi}}, \tag{15}$$

where we recall that $\gamma \equiv v_0/\Lambda_\phi$ with $v_0 = 246~{
m GeV}.$

Two additional parameters will be required to completely fix the phenomenology of the scalar sector, including all possible decays. These are

$$\widehat{\Lambda}_W, \quad m_1,$$
 (16)

where $\widehat{\Lambda}_W$ will determine KK-graviton couplings to the h and ϕ and m_1 is the mass of the first KK graviton excitation.

We recall the earlier formulae:

$$\widehat{\Lambda}_{W} \equiv \frac{2\sqrt{b_{0}}}{\epsilon\chi^{n}(1/2)} \simeq \sqrt{2}M_{Pl}\Omega_{0},$$

$$m_{n} = m_{0}x_{n}\Omega_{0},$$

$$\Lambda_{\phi} = \sqrt{6}M_{Pl}\Omega_{0} = \sqrt{3}\widehat{\Lambda}_{W},$$
(17)

where $\Omega_0 M_{Pl} = e^{-m_0 b_0/2} M_{Pl}$ should be of order a TeV to solve the hierarchy problem. In Eq. (17), the x_n are the zeroes of the Bessel function J_1 ($x_1 \sim 3.8$, $x_2 \sim 7.0$). A useful relation following from the above equations is:

$$m_1 = x_1 \frac{m_0}{M_{Pl}} \frac{\Lambda_\phi}{\sqrt{6}} \,. \tag{18}$$

 m_0/M_{Pl} is related to the curvature of the brane and should be a relatively small number for consistency of the RS scenario.

• Sample parameters that are safe from precision EW data and Runl Tevatron constraints are $\Lambda_{\phi} = 5 \,\, {
m TeV}$ ($\Rightarrow \, \widehat{\Lambda}_W \sim 3 \,\, {
m TeV}$) and $m_0/M_{Pl} = 0.1$.

We will also consider a marginal scenario with $\Lambda_{\phi} = 1$ TeV.

• For m_h and m_{ϕ} we will consider a range of possibilities, but with some prejudice towards $m_{\phi} < m_h$. There are theoretical arguments in favor of this.

A light radion ϕ eigenstate presents a particularly rich phenomenology.

The Couplings

The $f\overline{f}$ and VV couplings

The VV couplings

• The h_0 has standard ZZ coupling while the ϕ_0 has ZZ coupling deriving from the interaction $-\frac{\phi_0}{\Lambda_\phi}T^{\mu}_{\mu}$ using the covariant derivative portions of $T^{\mu}_{\mu}(h_0)$. The result for the $\eta_{\mu\nu}$ portion of the ZZ couplings is:

$$g_{ZZh} = \frac{g m_Z}{c_W} \left(d + \gamma b \right) , \quad g_{ZZ\phi} = \frac{g m_Z}{c_W} \left(c + \gamma a \right) . \tag{19}$$

g and c_W denote the SU(2) gauge coupling and $\cos \theta_W$, respectively. The WW couplings are obtained by replacing gm_Z/c_W by gm_W .

• Additional contributions to the ZZh and $ZZ\phi$ couplings come from $-\frac{\phi_0}{\Lambda_{\phi}}T^{\mu}_{\mu}$ for the gauge fixing portions of $T_{\mu\nu}$. These terms vanish when contracted with on-shell W or Z polarizations, which is the physical situation we are interested in. In addition, these extra couplings vanish in the unitary gauge.

• The $f\overline{f}$ couplings

- The h_0 has standard fermionic couplings.
- The fermionic couplings of the ϕ_0 derive from $-\frac{\phi_0}{\Lambda_\phi}T^{\mu}_{\mu}$ using the Yukawa interaction contributions to T^{μ}_{μ} .
- One obtains results in close analogy to the VV couplings just considered:

$$g_{f\bar{f}h} = -\frac{g m_f}{2 m_W} (d + \gamma b), \quad g_{f\bar{f}\phi} = -\frac{g m_f}{2 m_W} (c + \gamma a).$$
 (20)

• Note same factors for WW and $f\bar{f}$ couplings.



The gg and $\gamma\gamma$ couplings

• There are the standard loop contributions, except rescaled by $f\overline{f}/VV$ strength factor.

For c_{γ} , the \sum_{i} comprises all charged fermions (including quarks, with $N_{c}^{i} = 3$ and $e_{i} = 2/3$ or -1/3, and leptons, with $e_{i} = -1$ and $N_{c}^{i} = 1$) and the W boson (with $e_{i} = 1$ and $N_{c}^{i} = 1$).

For c_g , the \sum_i is over all colored fermions (assumed to have $N_c^i = 3$). The auxiliary functions are:

$$F_{1/2}(\tau) = -2\tau [1 + (1 - \tau)f(\tau)], \qquad (21)$$

$$F_1(\tau) = 2 + 3\tau + 3\tau(2 - \tau)f(\tau), \qquad (22)$$

for spin-1/2 and spin-1 loop particles, respectively, with

$$f(\tau) = -\frac{1}{4} \ln \left[-\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} \right]^2$$
(23)

 $\tau \equiv 4m^2/M^2$, where *m* is the mass of the internal loop particle and *M* is the mass of the scalar state, *h* or ϕ .

- Must include the anomalous contributions, which are expressed in terms of the SU(3)×SU(2)×U(1) β function coefficients $b_3 = 7$, $b_2 = 19/6$ and $b_Y = -41/6$.
- For the h, $g_{fV} = d + \gamma b$ and $g_r = \gamma b$. For the ϕ , $g_{fV} = c + \gamma a$ and $g_r = \gamma a$.



 $Zh\phi$ tree level couplings are absent.

The cubic interactions

1. First, we have

$$\mathcal{L} \ni -V(H_0) = -\lambda (H_0^{\dagger} H_0 - \frac{1}{2} v_0^2)^2 = -\lambda (v_0^2 h_0^2 + v_0 h_0^3 + \frac{1}{4} h_0^4),$$
 (24)

after substituting $H_0 = \frac{1}{\sqrt{2}}(v_0 + h_0)$. Expressing λ in terms of m_{h_0} as in Eq. (??), the h_0^3 term of Eq. (24) becomes

$$\mathcal{L} \ni -\frac{m_{h_0}^2}{2v_0} h_0^3.$$
 (25)

2. The interaction of ϕ_0 with $T^{\mu}_{\mu}(h_0)$:

$$-\frac{\phi_0}{\Lambda_\phi}T^{\mu}_{\mu}(h_0) = -\frac{\phi_0}{\Lambda_\phi} \left(-\partial^{\rho}h_0\partial_{\rho}h_0 + 4\lambda v_0^2 h_0^2\right) \,. \tag{26}$$

3. The interaction of the KK-gravitons with $T^{\mu\nu}(h_0)$:

$$-\frac{\epsilon}{2}h_{\mu\nu}(x,y=1/2)T^{\mu\nu} \ni -\frac{1}{\widehat{\Lambda}_W}\sum_n h^n_{\mu\nu}\partial^\mu h_0\partial^\nu h_0, \qquad (27)$$

where we keep only the derivative contributions and we have dropped (using the gauge $h^{\mu n}_{\mu} = 0$) the $\eta^{\mu\nu}$ parts of $T^{\mu\nu}$.

4. The ξ -dependent tri-linear components of Eq. (??):

$$\begin{aligned} & 6\xi\Omega(x)\left(-\Box\Omega(x)+\epsilon h_{\mu\nu}(x,y=1/2)\partial^{\mu}\partial^{\nu}\Omega(x)\right)H_{0}^{\dagger}H_{0} \\ & \ni \left[-3\frac{\xi}{\Lambda_{\phi}}h_{0}^{2}\Box\phi_{0}-6\xi\frac{v_{0}}{\Lambda_{\phi}^{2}}h_{0}\phi_{0}\Box\phi_{0}\right. \\ & \left.-12\xi\frac{v_{0}}{\widehat{\Lambda}_{W}\Lambda_{\phi}}\sum_{n}h_{\mu\nu}^{n}\partial^{\mu}\phi_{0}\partial^{\nu}h_{0}-6\xi\frac{v_{0}^{2}}{\widehat{\Lambda}_{W}\Lambda_{\phi}^{2}}\sum_{n}h_{\mu\nu}^{n}\partial^{\mu}\phi_{0}\partial^{\nu}\phi_{0}\right] (28) \end{aligned}$$

where we have employed the expansion of $h_{\mu\nu}(x, y = 1/2)$ in terms of the $h^n_{\mu\nu}$, used the gauge conditions $\partial^{\mu}h^n_{\mu\nu} = 0$ and $h^{\mu\,n}_{\mu} = 0$, and also used the symmetry of $h_{\mu\nu}$.

Constraints from LEP/LEP2

- Choose $\Lambda_{\phi} = 5$ TeV. The $Z^2 > 0$ gives ξ constraint.
- LEP/LEP2 provides an upper limit on ZZs (s = h or ϕ) from which we can exclude regions in the (m_h, m_{ϕ}) plane for a given choice of R^2 .

Use upper limits on the ZZs coupling in both with and without b tagging, with computed branching ratios into b and non-b final states.

• Conclusion:

Small m_{ϕ} relative to m_h is entirely possible given current data so long as $m_h \gtrsim 115$ GeV. (The $ZZ\phi$ coupling does not blow up.)



Couplings

- First, consider the $f\overline{f}/VV$ couplings of h and ϕ relative to SM, taking $m_h = 120 \text{ GeV}$ and $\Lambda_{\phi} = 5 \text{ TeV}$.
- Next, the h^3 and ϕ^3 couplings relative to $h_{
 m SM}^3$ taking $m_{h_{
 m SM}}=m_h$ or m_ϕ , respectively.

Deviations shown should be readily explorable at an LC for the h^3 coupling, but the ϕ^3 coupling may be difficult to probe except where it gets near 1 (relative to SM comparison).

















Physics Implications

































More marginal case: $\Lambda_{\phi} = 1$ TeV







CONCLUSIONS

- The Higgs-radion sector will certainly be very revealing, and for some parameter choices may prove quite challenging to fully explore.
- In fact, at the LHC one can miss both the ϕ and the h for the most difficult parameter choices: $m_{\phi} = 200 \text{ GeV}$ and substantially negative ξ .
- \Rightarrow keep improving and working on every possible signature.
- The large deviations of h properties with respect to $h_{\rm SM}$ properties is not really surprising given the nearness of the $\Lambda_{\phi} = 1$ TeV scale to the Higgs mass scales being considered.
- It would be nice to rule out the very light ϕ possibility.
- The decays (such as $h \to \phi \phi$ and $h \to h^n \phi$) which are only present if $\xi \neq 0$ can have large branching ratios and would provide an incontrovertible signature for mixing.