Flavor Physics and Warped Extra Dimensions

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Abstract

We consider a five dimensional model with warped geometry where the standard model fermions and gauge bosons correspond to bulk fields. Fermion masses and CKM mixings can be explained in a geometrical picture, without hierarchical Yukawa couplings. We discuss various flavor violating processes induced by (excited) gauge boson exchange and non-renormalizable operators. Some of them, such as muon-electron conversion, are in the reach of next generation experiments.

1 Introduction

The huge discrepancy between the Planck scale, $M_{\rm Pl} \sim 10^{19}$ GeV, and the scale of electroweak symmetry breaking, $M_W \sim 10^2$ GeV, is one of the most interesting challenges in modern physics. Recently, it was realized that a small but warped extra dimension provides an elegant solution to this gauge hierarchy problem [1]. The fifth dimension is an S_1/Z_2 orbifold with an AdS$_5$ geometry, bordered by two 3-branes with opposite tensions and separated by distance R. The AdS warp factor $\Omega = e^{-\pi kR}$ generates an exponential hierarchy between the effective mass scales on the two branes. If the brane separation is $kR \simeq 11$, the scale on the negative tension brane is of TeV-size, while the scale on the other brane is of order $M_{\rm Pl}$. The AdS curvature k and the 5D Planck mass M_5 are both assumed to be of order $M_{\rm Pl}$. At the TeV-brane gravity is weak because the zero mode corresponding to the 4D graviton is localized at the positive tension brane (Planck-brane).

We take the SM gauge bosons and fermions as bulk fields. The Higgs field is localized at the TeV-brane, otherwise the gauge hierarchy problem would reappear [2, 3]. Comparison with electroweak data, in particular with the weak mixing angle and gauge boson masses, requires the KK excitations of SM particles to be heavier than about 10 TeV [3–5]. If the fermions were confined to the TeV-brane, the KK scale would be even more constrained.

Models with localized gravity open up attractive possibilities for flavor physics. If the SM fermions reside in the 5-dimensional bulk, the hierarchy of quark and lepton masses can be interpreted in a geometrical way [6, 7]. Different flavors are localized at different

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positions in the extra dimension or, more precisely, have different wave functions. The fermion masses are in direct proportion to the overlap of their wave functions with the Higgs field [8]. Also the CKM mixing can be explained along these lines. Moreover, bulk fermions reduce the impact of non-renormalizable operators which, for instance, induce flavor violation and rapid proton decay, since closer to the Planck-brane the effective cut-off scale of the model is enhanced [6, 7]. Small Majorana neutrino masses can then arise from dimension five interactions without introducing new degrees of freedom [9]. The atmospheric and solar neutrino anomalies can be satisfactorily resolved. Alternatively, Dirac neutrino masses can generated by a coupling to right-handed neutrinos in the bulk [6, 11].

In this talk we review how fermion masses and mixings can be related to a "geography" of fermion locations in the extra dimension. Flavor violation by (excited) gauge boson exchange is a natural consequence of this approach. Contributions from non-renormalizable operators turn out to be safely suppressed. We discuss various flavor violating processes, especially focusing on the lepton sector. Some of them, such as muon-electron conversion, are in the reach of next generation experiments.

2 Fermion masses and CKM mixings

By the Kaluza-Klein (KK) procedure the 5D fields are decomposed into an infinite tower of 4D fields. The wave functions encode information on where the KK states are localized in the extra dimension. Together with the KK masses they are obtained by solving the 5D equations of motion. In five dimensions fermions are vector-like, and we can associate with them a 5D Dirac mass term, parameterized by $m_\Psi = c \cdot k\ \mathrm{sgn}(y)$, where y is the 5th coordinate. Depending on the Z_2 orbifold transformation property of the fermion, $\Psi(-y)_\pm = \pm\gamma_5\Psi(y)_\pm$, the left-handed (right-handed) zero mode, $f_0 \sim e^{(2-c)k|y|}$, of the KK decomposition is projected out by the boundary conditions [6, 7, 10]. The KK excited states come in left- and right-handed pairs, which are degenerate in mass. Note that the 5D Dirac mass parameter regulates whether the zero mode is localized towards the Planck-brane ($c > 1/2$) or the TeV-brane ($c < 1/2$).

Masses for the fermionic zero modes, which are associated with the SM quarks and leptons, are generated by the Higgs mechanism. The induced fermion masses

$$M_{ij} = \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} \lambda_{ij}^{(5)} e^{-4\sigma} H(y) f_{0L}^{(i)}(y) f_{0R}^{(j)}(y) \tag{1}$$

crucially depend on the overlap between the Higgs and fermion wave functions in the extra dimension, and naturally become small for fermions residing close to the Planck-brane. Here $\lambda_{ij}^{(5)}$ are the 5D Yukawa couplings, H is the Higgs profile localized at the TeV-brane, and $f_0^{(j)}$ are the zero modes of the relevant quark and lepton fields. In fig. 1 we sketch this five dimensional "geography". Assuming non-hierarchical 5D Yukawa couplings of the order of the 5D weak gauge coupling $g_2^{(5)}$, the localized Higgs field

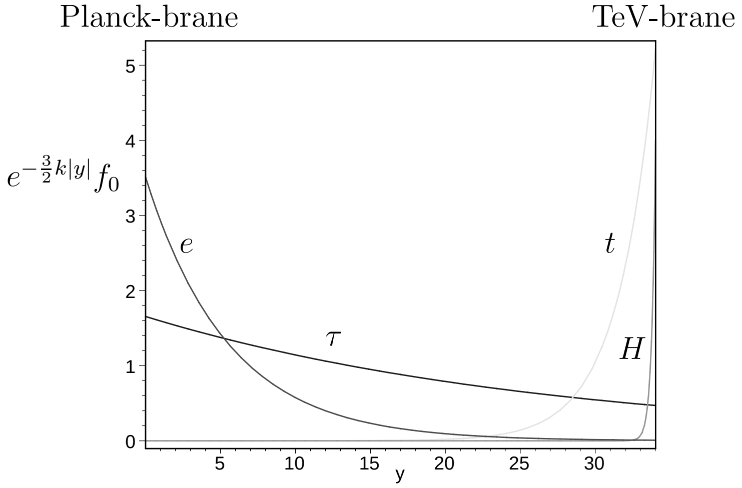

Figure 1: Localization of the electron, tau and top quark zero modes in the fifth dimension together with the Higgs profile (y is given in units of k).

induces a product-like pattern for the mass matrices

$$M \sim \begin{pmatrix} a_1b_1 & a_1b_2 & a_1b_3 \\ a_2b_1 & a_2b_2 & a_2b_3 \\ a_3b_1 & a_3b_2 & a_3b_3 \end{pmatrix} \tag{2}$$

where a_i and b_i depend exponentially on the associated c parameters. If the mass matrix is diagonalized by $U_L M U_R^\dagger$, the left- and right-handed mixings are typically $U_{L,ij} \sim a_i/a_j$ and $U_{R,ij} \sim b_i/b_j$, respectively. Only fermions which have similar positions (c parameters) have large mixings. The mass matrix (2) predicts the approximate relation $U_{13} \sim U_{12}U_{23}$ between the mixing angles, which in case of the observed CKM matrix is satisfied up to a factor of about two [12].

Building up the fermion mass matrices from eq. (2) requires the specification of c parameters and Yukawa couplings. Thus there are considerably more independent parameters in the model than there are observable fermion masses and mixings. In the following we assume that the pattern of fermion masses is, at the first place, determined by the fermion locations. We are looking for a set of c parameters, where typical Yukawa couplings $\lambda_{ij}^{(5)} \sim g_2^{(5)}$ reproduce the observed fermion properties. More precisely, we are generating random sets of Yukawa couplings and require the averaged fermion masses and mixings to fit the experimental data. Taking $1/\sqrt{2} < |\lambda_{ij}^{(5)}/g_2^{(5)}| < \sqrt{2}$ and random phases from 0 to 2π, we find the "optimal" locations

$$\begin{aligned} &c_{Q1} = 0.65, && c_{D1} = 0.65, && c_{U1} = 0.67, \\ &c_{Q2} = 0.59, && c_{D2} = 0.61, && c_{U2} = 0.53, \\ &c_{Q3} = 0.32, && c_{D3} = 0.61, && c_{U3} = -0.60. \end{aligned} \tag{3}$$

With exception of V_{ub} which is too large by a factor of two, all quark masses and mixings are on average within their allowed ranges. As expected from their similar locations, the right-handed rotations of the down quarks turn out to be large. Note that (3) is different from the locations we used in ref. [7] to maximally suppress proton decay. There is a degeneracy in the solution (3) since the fermion masses do not change if the all left- and right-handed quarks are shifted oppositely by the same amount. The quarks can be localized closer towards the Planck-brane by $\delta c = \ln(l)/(2\pi R)$ if the 5D Yukawa couplings are increased by a common factor l.

To determine the lepton locations we have to take into account neutrino masses and mixings. We assume that dimension five interactions induce small Majorana neutrino masses [9]. Large neutrino mixings require the neutrinos and thus the SU(2) lepton doublets to have similar positions c_{Li}. To suppress the matrix element U_{e3} it is favorable to separate the electron doublet somewhat from the muon and tau doublets

$$\begin{aligned} c_{L1} &= 0.63, \quad & c_{L2} &= 0.58, \quad & c_{L3} &= 0.58, \\ c_{E1} &= 0.75, \quad & c_{E2} &= 0.62, \quad & c_{E3} &= 0.50. \end{aligned} \tag{4}$$

The right-handed positions c_{Ei} we fixed by requiring that with random Yukawa couplings the average charged lepton masses fit their observed values. The mixings of the left-handed charged leptons are similar to the neutrino mixings, while the right-handed mixings are small.

The parameter sets (3) and (4) demonstrate that bulk fermions in a warped geometry can nicely fit with order unity parameters not only the huge fermion mass hierarchy but also the fermion mixings. Note that in our model a non-trivial wave function for fermions is automatically induced by the non-factorizable geometry.

3 Flavor violation

There are various sources of flavor violation in the warped model [5–7, 13–16]. The low cut-off scale dramatically amplifies the impact of non-renormalizable operators at the weak scale. With bulk fields localized towards the Planck-brane the corresponding suppression scales can be significantly enhanced [6, 7]. However, there are limits because the SM fermions need to have sufficient overlap with the Higgs field at the TeV-brane to acquire their observed masses. In the case of proton decay, typical dimension-six operators still have to be suppressed by small couplings of order 10^{-8} to be compatible with observations [7]. We consider the following generic four-fermion operators which are relevant for flavor violation as well as for proton decay

$$\int d^4x \int dy \sqrt{-g} \frac{1}{M_5^3} \bar{\Psi}_i \Psi_j \bar{\Psi}_k \Psi_l \equiv \int d^4x \frac{1}{M_4^2} \bar{\Psi}_i^{(0)} \Psi_j^{(0)} \bar{\Psi}_k^{(0)} \Psi_l^{(0)}. \tag{5}$$

The effective 4D suppression scales M_4 associated with these operators depend on where the relevant fermion states are localized in the extra dimension. Let us focus on some examples. Constraints on $K - \bar{K}$ mixing require the dimension-six operator $(ds)^2$ to be suppressed by $M_4 > 1 \cdot 10^6$ GeV. Using the fermion positions of eq. (3) we find $M_4((ds)^2) = 9 \cdot 10^7$ GeV, safely above the experimental bound. The lepton flavor violating decay $\mu \to eee$ is induced the operator μeee at a rate $\Gamma \sim m_\mu^5/M_4^4$. ¿From eq. (4) we obtain $M_4(\mu eee) = 2 \cdot 10^7$ GeV, considerably larger than the experimental bound

$M_4 > 3 \cdot 10^6$ GeV. Other possible dimension-six operators are suppressed in a similar way.

If the SM fermions are located at different positions, KK gauge bosons couple non-universally to the fermion flavors. The same is true for the zero modes of Z and W bosons since their wave functions are y-dependent as well [3, 4]. The 4D gauge couplings are obtained from an integration over the extra dimension

$$g = \frac{g^{(5)}}{(2\pi R)^{3/2}} \int_{-\pi R}^{\pi R} e^{\sigma} f_0(y)^2 f_A(y)\, dy. \tag{6}$$

Here f_A denotes a generic gauge boson wave function. Going from the interaction to the mass eigenstates, flavor non-diagonal couplings are generated as

$$G = U^{\dagger} g U, \tag{7}$$

where g is a diagonal matrix in flavor space. This type of flavor violation is therefore especially important for large fermion mixing. The effect is completely analogous to what happens in models with family non-universal Z' bosons, so we can simply adopt the formalism described, for instance, in ref. [17]. Separating the fermions in the extra dimension increases the non-universality of the gauge couplings while suppressing the fermion mixing. Note that also the right-handed fermion rotations become physically relevant.

In the numerical evaluations we use again the fermion locations of eqs. (3) and (4) and average over random sets of Yukawa couplings. We find a typical value of $\mathrm{Br}(\mu \to eee) \approx 1 \cdot 10^{-16}$, safely below the experimental bound $1 \cdot 10^{-12}$ [12]. The branching ratio of $\mu \to e\gamma$ is found to be even six orders of magnitude below the present experimental sensitivity $1.2 \cdot 10^{-11}$. In the case of muon-electron conversion in muonic atoms we find $\mathrm{Br}(\mu N \to eN) \approx 1 \cdot 10^{-16}$ while the current sensitivity is $6.1 \cdot 10^{-13}$. The MECO Collaboration plans to improve this bound by four orders of magnitude and could therefore reach the predicted rate. Flavor violation in the quark sector is also safely suppressed. The $K^0 - \bar{K}^0$ mass splitting, for instance, induces an upper bound $\mathrm{Re}(G_{12}^2) < 1 \cdot 10^{-8}$ for the coupling $G_{12} Z d s$ [17] while we obtain $\mathrm{Re}(G_{12}^2) \approx 1 \cdot 10^{-12}$. Moreover, we find $\mathrm{Im}(G_{12}^2) \approx 3 \cdot 10^{-13}$ where CP violation in the Kaon system leads to the bound $\mathrm{Im}(G_{12}^2) < 8 \cdot 10^{-11}$. Why are the rates for flavor violation so small in the warped model whereas in the case of a flat extra dimension they can push the KK scale up to 5000 TeV [19]? The reason is that in a warped geometry the gauge boson wave functions are almost constant near the Planck-brane [3, 4, 6], where the light fermions have to reside in order to explain their small masses. Therefore the induced deviations from universality are tiny from the very beginning.

In the scenario of Dirac neutrino masses [10, 11] the rate of $\mu \to e\gamma$ transitions is considerably enhanced by the presence of heavy sterile neutrino states. If the SM neutrinos are confined to the TeV-brane, its large branching ratio pushes the KK scale up to 25 TeV and thus imposes the most stringent constraint on the model [16]. However, the rate for $\mu \to e\gamma$ is very sensitive to the mixing between light and heavy neutrino states. With bulk neutrinos the mixing with heavy states is considerably reduced. In the case of the large angle MSW solution we obtain $\mathrm{Br}(\mu \to e\gamma) \approx 10^{-15}$ [11]. While this value is still well below the experimental sensitivity, it is two orders of magnitude larger than the contribution from gauge boson exchange and comes close to the reach of the MEG experiment [18].

4 Conclusions

We have shown that bulk quarks and leptons in a warped background can naturally accommodate the fermion mass hierarchies and mixings in geometrical way, without relying on hierarchical Yukawa couplings. Flavor violation by (excited) gauge boson exchange is an immediate consequence of this approach, while contributions from non-renormalizable operators are automatically suppressed. Some processes, such as muon-electron conversion, are in the reach of next generation experiments and can provide valuable hints to the higher dimensional theory.

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