Radiative brane mass terms in orbifold gauge theories*

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Orbifolds play a prominent role in theories with extra dimensions due to their property to create chirality in the massless sector, an indispensable property in any phenomenologically relevant theory. Another interesting feature of orbifolds is their ability to break symmetries, in particular gauge and supersymmetry. While local symmetries remain intact in the bulk by an appropriate choice of parities for the transformation parameters, they are in general broken to smaller subgroups on the boundaries (the fixed points of the orbifold symmetry). As in any quantum field theory, in the effective action we must allow for all operators consistent with the symmetries. Allowed operators not present at tree level will be generated by radiative corrections [1–3].

The orbifold breaking of the bulk gauge symmetry proceeds by projecting out some fields, i.e. only a subset of the 4D gauge bosons A_μ and the 4D scalars A_i ($i = 5, \cdots, D$) will be non-vanishing at the boundaries. While these A_μ generate the unbroken gauge group $\mathcal{H}$, the A_i transform in some representation of $\mathcal{H}$. It is then necessary to determine how the symmetries restrict possible brane localized operators of those fields, especially possible mass terms for the scalars [3]. Would $\mathcal{H}$ be the only symmetry left on the brane, mass terms for A_i would be perfectly allowed leading to a quadratic sensitivity to the UV cut-off.

In this talk we demonstrate that the remnant symmetry on the brane is larger than the $\mathcal{H}$ gauge symmetry left over from the bulk. This provides a further restriction on the possible brane terms. We find that brane mass terms for scalars can only occur in $D \geq 6$ and only for $U(1)$ factors in $\mathcal{H}$ that were not already present in the bulk gauge group $\mathcal{G}$. These brane mass terms are radiatively generated by bulk fermions.

We will consider a gauge theory (gauge group $\mathcal{G}$) coupled to fermions in $D > 4$ dimensional space-time parametrized by coordinates $x^M = x^\mu, y^i$ where $\mu = 0, 1, 2, 3$ and $i = 5, \cdots, D$. The bulk Lagrangian is

$$\mathcal{L}_D = -\frac{1}{4} F^A_{MN} F^{AMN} + i\overline{\Psi}\gamma^M D_M \Psi, \tag{1}$$

where $F^A_{MN} = \partial_M A^A_N - \partial_N A^A_M - g\, f^{ABC} A^B_M A^C_N$ with the indices A, B, C running over the adjoint representation of $\mathcal{G}$ and f^{ABC} being the $\mathcal{G}$ structure constants. The local

*Talk heavily based on hep-ph/0210134, a collaboration with G. von Gersdorff and M. Quiros symmetry of (1) is the invariance under the (infinitesimal) gauge transformations

$$\delta_{\mathcal{G}} A_M^A = \frac{1}{g} D_M^{AB} \xi^B = \frac{1}{g} \partial_M \xi^A - f^{ABC} \xi^B A_M^C. \tag{2}$$

We now compactify the $p \equiv D - 4$ extra dimensions on the $T^p/\mathbb{Z}_2$ orbifold with all the radii of the torus equal to R [1] and with the $\mathbb{Z}_2$ action defined as $y^i \to -y^i$.

In the compactified theory the surviving gauge symmetry on the boundaries of the orbifold is a subgroup $\mathcal{H}$ of $\mathcal{G}$, according to the action of $\mathbb{Z}_2$ on the gauge fields

$$A(x^\mu, -y^i) = \mathcal{P}_{\mathcal{A}} A(x^\mu, y^i), \qquad \mathcal{P}_{\mathcal{A}} = \Lambda \otimes \mathcal{P}_1. \tag{3}$$

Here $\mathcal{P}_1$ acts on the vector indices and it is the diagonal matrix with eigenvalues $\alpha_\mu = +1, \alpha_i = -1$. Λ acts on the gauge indices and can also be taken diagonal. Its eigenvalues $\eta^A = \pm 1$ then define the breaking pattern. We split the bulk gauge index as $A = a, \hat{a}$ corresponding to the unbroken ($\eta^a = +1$) and the broken generators ($\eta^{\hat{a}} = -1$) respectively. The nonzero fields on the brane are the even fields, namely A_μ^a and $A_i^{\hat{a}}$, while $A_\mu^{\hat{a}}$ and A_i^a are odd and thus vanish on the brane. The orbifold consistency constraint on the structure constants comes essentially from the invariance of (1) and it provides the automorphism condition [4]

$$\eta^A \eta^B \eta^C = 1, \qquad \text{for } f^{ABC} \neq 0. \tag{4}$$

Finally, in the gauge sector, the Faddeev-Popov ghosts c transform as the μ-components of the gauge fields, and for them the parity action is $\mathcal{P}_c = \Lambda$.

There are restrictions on the fermion representations as well. In even dimensions the bulk fermion representation has to be chosen anomaly free. Furthermore, for any number of extra dimensions, the resulting four dimensional massless fermion spectrum must also be anomaly free. In addition, there are orbifold consistency conditions analogous to (4). The $\mathbb{Z}_2$ action on the fermions is

$$\Psi(x^\mu, -y^i) = \mathcal{P}_\Psi \Psi(x^\mu, y^i), \qquad \mathcal{P}_\Psi = \lambda \otimes \mathcal{P}_{\frac{1}{2}} \tag{5}$$

where λ is a matrix acting on the representation indices. The constraint comes from the requirement that the coupling $i A_M^A \overline{\Psi} \gamma^M T^A \Psi$ is $\mathbb{Z}_2$ invariant. One obtains [3] for any number of dimensions [2]

$$[\lambda, T^a] = 0 \qquad \{\lambda, T^{\hat{a}}\} = 0. \tag{6}$$

$\mathcal{P}_{\frac{1}{2}}$ is the orbifold action on the spinor indices and will be given explicitly later on.

The non-vanishing fields on the branes are of the general form

$$\prod_{i=5}^{D} \partial_i^{n_i} \, \Phi|_{\text{brane}} \equiv \partial^n \Phi \tag{7}$$

where $n \equiv \sum_i n_i$ is even (odd) for even (odd) fields. Similarly, the gauge parameters ξ^a are even fields and $\xi^{\hat{a}}$ are odd. They couple to the branes according to (7).

[1]From now on we will work in units where $R \equiv 1$. Restoring the R dependence as well as introducing different radii R^i for different dimensions should be straightforward.

[2]Note that conditions (6) determine λ up to a sign.

The effective four dimensional Lagrangian can be written as

$$\mathcal{L}_4^{eff} = \int d^p y \big[\mathcal{L}_D + \mathcal{L}_4^{brane} \prod_i \{\delta(y^i) + \delta(y^i - \pi)\}\big] \tag{8}$$

where $\mathcal{L}_D$ is given by (1) and $\mathcal{L}_4^{brane}$ should be the most general Lagrangian consistent with the symmetries. The latter can be nothing but the original bulk symmetry (2) modded out by the orbifold action and subsequently evaluated at the location of the brane. Let us call the transformation resulting from this operation δ_ξ. Applying this rule to (2) acting on the massless even fields, one obtains the transformations

$$\delta_\xi(A_\mu^a) = \frac{1}{g}\partial_\mu \xi^a - f^{abc}\xi^b A_\mu^c, \tag{9}$$

$$\delta_\xi(A_i^{\hat{a}}) = \frac{1}{g}\partial_i \xi^{\hat{a}} - f^{\hat{a}b\hat{c}}\xi^b A_i^{\hat{c}}. \tag{10}$$

In the above equations and in what follows, all fields should be interpreted as coupled to the brane in (8) according to (7).

The brane symmetry is however much larger than the transformations (9) and (10). In fact, there is an infinite number of non-zero independent fields on the brane, i.e. $\partial^{2k}\{A_\mu^a, A_i^{\hat{a}}\}$ and $\partial^{2k+1}\{A_\mu^{\hat{a}}, A_i^a\}$, and an infinite number of corresponding transformation parameters $\{\partial^{2k}\xi^a\}$ and $\{\partial^{2k+1}\xi^{\hat{a}}\}$ induced by the bulk. Using (2), one can derive the transformation of any non-zero brane field. We show explicitly only the first two at the next level:

$$\delta_\xi(\partial_j A_i^a) = \frac{1}{g}\partial_j(\partial_i \xi^a) - f^{a\hat{b}\hat{c}}(\partial_j \xi^{\hat{b}})A_i^{\hat{c}} - f^{abc}\xi^b(\partial_j A_i^c), \tag{11}$$

$$\delta_\xi(\partial_i A_\mu^{\hat{a}}) = \frac{1}{g}\partial_\mu(\partial_i \xi^{\hat{a}}) - f^{\hat{a}\hat{b}c}(\partial_i \xi^{\hat{b}})A_\mu^c - f^{\hat{a}b\hat{c}}\xi^b(\partial_i A_\mu^{\hat{c}}). \tag{12}$$

It is convenient to separate the above transformations into two different classes:

$$\delta_\xi = \delta_{\mathcal{H}} + \delta_{\mathcal{K}} \quad \text{with} \quad \delta_{\mathcal{H}} = \{\xi^a\}, \quad \delta_{\mathcal{K}} = \{\partial^{2k}\xi^a, \partial^{2k+1}\xi^{\hat{a}}\}. \tag{13}$$

This is a natural separation because $\delta_{\mathcal{H}}$ is the surviving gauge transformation on the brane reflecting its $\mathcal{H}$ gauge invariance. One can see immediately by inspection of Eqs. (9)−(12) that A_μ^a are the gauge bosons of $\mathcal{H}$ while all other fields transform homogeneously in either the adjoint of $\mathcal{H}$, $(T^a)_{bc} = if^{abc}$, or in the representation spanned by $(T^a)_{\hat{b}\hat{c}} = if^{a\hat{b}\hat{c}}$ [3]. The rest of the transformations is a set of local (but not gauge) transformations which we named $\delta_{\mathcal{K}}$.

Once the symmetries under which the brane action should be invariant are known, one can start constructing the allowed terms by these symmetries. A useful guiding principle in this task is the gauge symmetry $\mathcal{H}$. We know that it is a necessary condition that the building blocks should be $\mathcal{H}-$covariant combinations of the fields since this (and only this) can ensure that the square of these covariant objects are $\delta_{\mathcal{H}}-$invariant. Given a set

[3] As a simple example consider the breaking $SU(3) \to SU(2) \otimes U(1)$. The adjoint of $SU(3)$, $f^{ABC} = \mathbf{8}$ then splits into the $SU(2)$ representations $f^{abc} = \mathbf{3} \oplus \mathbf{1}$ ($\mathcal{H}$ is not simple and hence its adjoint is reducible) and $f^{a\hat{b}\hat{c}} = \mathbf{2} \oplus \mathbf{2}$.

of $\mathcal{H}-$covariant objects, invariance under $\delta_{\mathcal{K}}$ is a sufficient condition for their square to be invariant under both $\delta_{\mathcal{H}}$ and $\delta_{\mathcal{K}}$ and therefore to be an allowed terms in the effective action. The reason for which we required $\mathcal{K}-$invariance is because there is no notion of $\mathcal{K}-$covariance, since $\mathcal{K}$ is not a gauge symmetry. Thus, even though at this point we have not proved that $\mathcal{K}-$invariance is not only a sufficient but also a necessary condition, we will enforce it.

A simple and very important example is the field $A_i^{\hat{a}}$. By looking at (10) one can see that this field is indeed $\delta_{\mathcal{H}}-$covariant but not $\delta_{\mathcal{K}}-$invariant. A naive interpretation would then be that an explicit brane mass term as $(A_i^{\hat{a}} M_{\hat{a}\hat{b}} A_j^{\hat{b}})$ is forbidden in the four dimensional effective action. However, as we will see below, under particular circumstances such a term can be part of a $\delta_{\mathcal{H}}-$ and $\delta_{\mathcal{K}}-$invariant term in the Lagrangian in which case such a term can be generated radiatively.

The terms which are at the same time $\mathcal{H}-$covariant and $\mathcal{K}-$invariant are easily found from the transformation properties:

$$\delta_{\mathcal{H}} F^a_{\mu\nu} = -f^{abc}\xi^b F^c_{\mu\nu}, \qquad \delta_{\mathcal{K}} F^a_{\mu\nu} = 0 \tag{14}$$

$$\delta_{\mathcal{H}} F^{\hat{a}}_{i\mu} = -f^{\hat{a}b\hat{c}}\xi^b F^{\hat{c}}_{i\mu}, \qquad \delta_{\mathcal{K}} F^{\hat{a}}_{i\mu} = 0 \tag{15}$$

$$\delta_{\mathcal{H}} F^a_{ij} = -f^{abc}\xi^b F^c_{ij}, \qquad \delta_{\mathcal{K}} F^a_{ij} = 0. \tag{16}$$

Note the different structure of $F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$ and $F^a_{ij} \equiv \partial_i A^a_j - \partial_j A^a_i - g f^{a\hat{b}\hat{c}} A^{\hat{b}}_i A^{\hat{c}}_j$ in the nonlinear terms. Further terms could be constructed from covariant derivatives of these operators. At the renormalizable level the following terms can appear in the Lagrangian:

$$\mathcal{L}_4^{brane} = -\frac{1}{4}\mathcal{Z}_{ab} F^a_{\mu\nu} F^{b\,\mu\nu} - \frac{1}{4}\mathcal{Z}^{ij}_{\hat{a}\hat{c}} F^{\hat{a}}_{i\mu} F^{\hat{c}\,\mu}_j - \frac{1}{4}\mathcal{Z}^{ijkl}_{ab} F^a_{ij} F^b_{kl} + \mathcal{Z}^{ij}_\alpha F^\alpha_{ij} + \mathcal{Z}^{klij}_\alpha D^{\alpha A}_k D^{AB}_l F^B_{ij}. \tag{17}$$

where the $\mathcal{Z}$ tensors in extra-dimensional indices must be proportional to either the torus metric g^{ij} or to possible invariant tensors under the symmetry group of the torus. We differentiate in the last two terms of (17) possible $U(1)$ factors of $\mathcal{H}$ from the remaining semi-simple part and denote these $U(1)$ generators by T^α. In fact Eq. (16) implies that the field strength of a $U(1)$ gauge field is invariant by itself allowing for the term [4]

$$F^\alpha_{ij} = \partial_{[i} A^\alpha_{j]} - g f^{\alpha\hat{b}\hat{c}} A^{\hat{b}}_i A^{\hat{c}}_j. \tag{18}$$

that can give rise to a quadratic renormalization. In a similar way, the term $D^{\alpha A}_k D^{AB}_l F^B_{ij}$ is invariant allowing for the last term in (17). It is dimension four and gives rise to a logarithmic renormalization, as we will see.

One might think that the term $\mathrm{tr}(\lambda_R T^a_R) F^a_{ij}$, where λ_R satisfies Eqs. (6) and the index R denotes some arbitrary irreducible representation, would give a further invariant linear in F_{ij}. However, for T^a_R belonging to a simple factor of $\mathcal{H}$, λ_R must act as the identity in this subspace by Eqs. (6) and Schur's Lemma, so the trace vanishes. Only $U(1)$ factors will thus contribute to the trace and we do not get any new invariant. We conclude that the terms F^α_{ij} are the most general linear terms.

[4] Notice that unbroken $U(1)$ factors in $\mathcal{G}$ do not give rise in (18) to bilinear terms in even fields.

We will be concerned mainly with the appearance of scalar mass terms in $\mathcal{L}_4^{brane}$. For a general unbroken gauge group $\mathcal{H}$ the most general renormalizable Lagrangian allowed by the symmetries of the theory contains the terms in (17). The first term in (17) corresponds to kinetic terms for the four dimensional gauge bosons, the second one corresponds to kinetic terms for the even scalars (plus some interactions), while the third term contains brane mass terms for the odd scalars. One consequence of the appearance of brane mass terms in this particular way is that their renormalization is expected to be governed by the (wave function) renormalization of F^2, which does not contain quadratic divergences. They are expected to pick up only logarithmically divergent renormalization effects. Brane mass terms for even scalars can appear in $\mathcal{L}_4^{brane}$ in the case where there are $U(1)$ group factors in $\mathcal{H}$ corresponding to unbroken generators T^α. Under this circumstance we have seen that the operator (18) is allowed by all symmetries on the brane and we expect that both a tadpole for the derivative of odd fields, $\partial_i A_j^\alpha$, and a mass term for the even fields, $f^{\alpha\hat{b}\hat{c}} A_i^{\hat{b}} A_j^{\hat{c}}$, will be generated on the brane by bulk radiative corrections. Moreover, since these operators have dimension two, we expect that their respective renormalizations will lead to quadratic divergences, making the theory ultraviolet sensitive.

We would like to confirm by explicit calculation that the allowed terms are indeed generated radiatively on the brane. In particular, mass terms for brane scalars (extra dimensional components of gauge bosons) are contained in the third term of (17) for the odd scalars A_i^a, and in (18) for the even scalars $A_i^{\hat{a}}$ when there are $U(1)$ group factors in $\mathcal{H}$. In all cases they arise from effective operators proportional to F_{ij}. An important special case is $D = 5$, i.e. a five dimensional gauge theory compactified on $S^1/\mathbb{Z}_2$. In this case the term F_{ij} does not exist and therefore we do not expect any type of brane mass terms to appear in $\mathcal{L}_4^{brane}$. This result has been confirmed by explicit one loop calculation in Ref. [3]. However for $D > 5$ F_{ij} does exist and we expect, from the previous symmetry arguments, the corresponding mass terms to be generated on the brane by radiative corrections. We will now compute these mass terms in a $D = 6$ model compactified on the orbifold $T^2/\mathbb{Z}_2$. The contribution of a chiral fermion $\Psi_\pm$ turns out to be

$$ig \ \mathrm{tr}\left(\lambda_R T_R^B\right) \epsilon_{ij} m^j \int \frac{d^4 q}{(2\pi)^4} \, \frac{1}{q^2 - \vec{m}^2/2}, \qquad m_5, m_6 \text{ even}, \tag{19}$$

where the external leg corresponds to the 4D scalar A_i^B (we have defined $\epsilon_{56} = -\epsilon_{65} = +1$). It leads to the brane terms (We have confirmed explicitly that the terms $f^{\alpha\hat{a}\hat{b}} A_i^{\hat{a}} A_j^{\hat{b}}$ in (18) receive the same renormalization $\mathcal{Z}_\alpha^{ij}$ at one loop as the tadpole)

$$\left(\mathcal{Z}_\alpha^{ij} F_{ij}^\alpha + \mathcal{Z}_\alpha^{klij} D_k^{\alpha A} D_l^{AB} F_{ij}^B\right) \left[\delta(y_5) + \delta(y_5 - \pi)\right] \left[\delta(y_6) + \delta(y_6 - \pi)\right], \tag{20}$$

where α runs over the different $U(1)$ factors of $\mathcal{H}$ and $\mathcal{Z}_\alpha^{ij}$ and $\mathcal{Z}_\alpha^{klij}$ are given by

$$\mathcal{Z}_\alpha^{ij} = \epsilon^{ij} \, \frac{g}{32\pi^2} \, \zeta^\alpha \, \Lambda^2, \quad \zeta^\alpha = \mathrm{tr}\left(\lambda_R T_R^\alpha\right), \tag{21}$$

$$\mathcal{Z}_\alpha^{klij} = \delta^{kl} \, \epsilon^{ij} \, \frac{g}{32\pi^2} \, \zeta^\alpha \, \log \frac{\Lambda}{\mu}, \tag{22}$$

where Λ is the ultraviolet and μ the infrared cut-off.

A further comment concerns the gauge contribution to the tadpole. Each graph is proportional to the trace

$$\mathrm{tr}\left(\lambda_{\mathrm{Adj}} T^{\alpha}_{\mathrm{Adj}}\right) = \eta^{A} \delta^{AB} f^{\alpha AB} = 0 \tag{23}$$

and thus vanishes by the asymmetry of the structure constants. Note that this is a generic feature of real representations.

We have also computed the one loop contribution to the terms $(F^a_{ij})^2$ in (17) and we found the logarithmic divergence we anticipated:

$$-\frac{1}{4} F^a_{ij} \mathcal{Z}^{ijkl}_{ab} F^b_{kl} \left[\delta(y_5) + \delta(y_5 - \pi)\right] \left[\delta(y_6) + \delta(y_6 - \pi)\right], \tag{24}$$

where

$$\mathcal{Z}^{ijkl}_{ab} = \delta^{ik} \delta^{jl} \frac{g^2}{2\pi^2} \left(C_2(\mathcal{H}_a) - \frac{1}{2} C_2(\mathcal{G}) \right) \delta_{ab} \, \log \frac{\Lambda}{\mu}. \tag{25}$$

Here, $C_2(\mathcal{H}_a)$ is by definition the Casimir of the group factor in $\mathcal{H}$ to which the generator T^a belongs (we define it to be zero for $U(1)$ factors). One expects a corresponding logarithmic contribution from the fermion sector.

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