Effective Lagrangians in Pseudo-Supersymmetry

Matthias Klein[*]

SLAC, Stanford University, Stanford, CA 94309, USA

ABSTRACT

I discuss effective field theories of brane-world models where different sectors break different halves of the extended bulk supersymmetry. It is shown how to consistently couple $\mathcal{N} = 2$ supersymmetric matter to $\mathcal{N} = 1$ superfields that lack $\mathcal{N} = 2$ partners but transform in a non-linear representation of the $\mathcal{N} = 2$ algebra. In particular, I explain how to couple an $\mathcal{N} = 2$ vector to $\mathcal{N} = 1$ chiral fields such that the second supersymmetry is non-linearly realized. This method is then used to study systems where different sectors break different halves of supersymmetry, which appear naturally in models of intersecting branes.

1 Introduction

D-brane models provide an interesting supersymmetry breaking mechanism that I would like to study in an effective field theory approach [1]. Parallel D-branes of the same dimensionality break half of the supersymmetry that is present in the bulk. In the effective field theory on the D-brane world-volume, this is seen from the fact that the fields only fill multiplets of the smaller supersymmetry algebra. Part of the supersymmetry is explicitly broken on the D-branes since the world-volume fields lack the corresponding superpartners. Supersymmetry can be completely broken by adding anti-D-branes (e.g., [2, 3, 4, 5]), which preserve the other half of the bulk supersymmetry. Thus, supersymmetry is broken in a non-local way in such models. Each sector taken separately preserves part of the supersymmetry. A very similar situation arises in models containing stacks of D-branes at angles that intersect each other (e.g., [6, 7]). There is an extended supersymmetry on each stack of D-branes, but only a fraction of this supersymmetry is preserved at each intersection. Supersymmetry is completely broken in models where different intersections break different fractions of supersymmetry.

To determine the couplings of the bulk fields to the boundary fields, it is important to note that the part of supersymmetry that is broken on the D-branes is still non-linearly realized. Although a rigorous proof for this statement is still missing, there is much evidence in favor of it, including the following facts: (i) The $\mathcal{N} = 4$ vector multiplet of a

single D3-brane in flat ten-dimensional space contains just the right number of fermions to provide the goldstinos required for the non-linear realization of the bulk $\mathcal{N} = 8$ supersymmetry. The counting still works if the bulk supersymmetry is reduced to $\mathcal{N} = 4$ or $\mathcal{N} = 2$ by an orbifold projection. (ii) Consider an $\mathcal{N} = 1$ supersymmetric $U(1)$ theory. The requirement that the gaugino be the goldstino for a second non-linearly realized supersymmetry uniquely determines the full non-linear action. It turns out [8] that this action agrees with the supersymmetric generalization of the Born-Infeld action, which is known to describe the world-volume theory of D-branes. (iii) Consistent gravitino couplings are very constrained and it is hard to imagine how the bulk gravitinos corresponding to the broken supersymmetries can satisfy these constraints without non-linear supersymmetry [9].

In this talk, I want to discuss a four-dimensional toy model, where an $\mathcal{N} = 2$ $U(1)$ bulk vector $(A_m, \lambda^{(1)}, \lambda^{(2)}, \phi)$ couples to two boundary sectors that preserve different halves of the $\mathcal{N} = 2$ bulk supersymmetry [1]. On the first boundary, there is an $\mathcal{N} = 1$ chiral multiplet $(\phi^{(1)}, \psi^{(1)})$ which carries charge q_1 under the bulk vector and an $\mathcal{N} = 1$ goldstino multiplet $(\lambda_g, \tilde{A}_m)$. On the second boundary, there is an $\mathcal{N} = 1'$ chiral multiplet $(\phi^{(2)}, \psi^{(2)})$ which carries charge q_2 under the bulk vector and an $\mathcal{N} = 1'$ goldstino multiplet $(\lambda'_g, \tilde{A}'_m)$. The boundary multiplets transform linearly under one supersymmetry and non-linearly under the other supersymmetry. The bulk vector multiplet transforms linearly under both supersymmetries.

$$
\begin{aligned}
\delta\phi^{(1)} &= \sqrt{2}\,\xi^{(1)}\psi^{(1)} - 2i\kappa\,\lambda_g\sigma^m\bar{\xi}^{(2)}\partial_m\phi^{(1)} \\
\delta A_m &= -i\lambda^{(1)}\sigma_m\bar{\xi}^{(1)} - i\lambda^{(2)}\sigma_m\bar{\xi}^{(2)} \;+\; \text{h.c.} \\
\delta\phi^{(2)} &= \sqrt{2}\,\xi^{(2)}\psi^{(2)} - 2i\kappa\,\lambda'_g\sigma^m\bar{\xi}^{(1)}\partial_m\phi^{(2)}
\end{aligned}
\tag{1.1}
$$

To determine the Lagrangian describing this toy model, I will first develop a method to couple $\mathcal{N} = 1$ multiplets to non-supersymmetric matter and then generalize it to couplings of $\mathcal{N} = 2$ multiplets to $\mathcal{N} = 1$ matter. Using the results of [8] on partially broken $\mathcal{N} = 2$ supersymmetry, the pseudo-supersymmetry Lagrangian is then easily obtained. A supergravity approach to pseudo-supersymmetry is pursued in [10].

2   Non-linear supersymmetry

Let us start by briefly reviewing the formalism of non-linearly realized supersymmetry. A supersymmetry transformation acts as a shift on the superspace coordinates:

$$
\begin{aligned}
x^m &\rightarrow x'^m = x^m - i(\xi\sigma^m\bar{\theta} - \theta\sigma^m\bar{\xi}) \\
\theta &\rightarrow \theta' = \theta + \xi \\
\bar{\theta} &\rightarrow \bar{\theta}' = \bar{\theta} + \bar{\xi}
\end{aligned}
\tag{2.1}
$$

An important observation is that the goldstino λ_g of broken supersymmetry can be viewed as a hypersurface in superspace defined through

$$
\theta = -\kappa\,\lambda_g(x),
\tag{2.2}
$$

where κ is a constant of mass dimension -2 related to the supersymmetry breaking scale. This is very similar to the Goldstone bosons of a spontaneously broken global internal symmetry being hypersurfaces in the parameter space of the global symmetry group.

The requirement that the hypersurface (2.2) be invariant under supersymmetry transformations, $\theta'(x) = \theta(x')$, implies the standard non-linear transformation law [11]

$$\delta_\xi \lambda_g \;=\; \frac{1}{\kappa}\xi - i\kappa\,(\lambda_g \sigma^m \bar{\xi} - \xi \sigma^m \bar{\lambda}_g)\partial_m \lambda_g \tag{2.3}$$

for the goldstino. This forms a non-linear realization of the supersymmetry algebra [11]

$$[\delta_\eta, \delta_\xi] \;=\; -2i\,(\eta\sigma^m \bar{\xi} - \xi\sigma^m \bar{\eta})\,\partial_m. \tag{2.4}$$

A matter field $f(x)$ is well-defined on the hypersurface (2.2) if, under a supersymmetry transformation, one has $f'(x) = f(x')$. This implies

$$\delta_\xi f \;=\; -i\kappa\,(\lambda_g \sigma^m \bar{\xi} - \xi\sigma^m \bar{\lambda}_g)\partial_m f, \tag{2.5}$$

which is the standard non-linear transformation for a matter field in the goldstino background.

To construct an invariant action, one introduces covariant derivatives [12]

$$\begin{aligned} D_m &\;=\; \left(\omega^{-1}\right)_m{}^n \partial_n, \\ \omega_m{}^n &\;=\; \delta_m{}^n - i\kappa^2(\partial_m \lambda_g \sigma^n \bar{\lambda}_g - \lambda_g \sigma^n \partial_m \bar{\lambda}_g), \end{aligned} \tag{2.6}$$

and notes that $\det(\omega)$ transforms as a density under the non-linear supersymmetry [11]. It is then straightforward to verify that

$$S = \int d^4x \, \det(\omega) \left(-\frac{1}{2\kappa^2} - D_m f D^m f - V(f) \right) \tag{2.7}$$

is invariant under (2.3), (2.5).

3   Coupling $\mathcal{N} = 1$ to $\mathcal{N} = 0$

Consider an $\mathcal{N} = 1$ superfield

$$\Phi(x, \theta, \bar{\theta}) \;=\; \phi(x) + \theta\psi(x) + \ldots \tag{3.1}$$

on which supersymmetry is linearly realized. To be able to couple the lowest component ϕ of this superfield to a sector where supersymmetry is non-linearly realized, we need to find a composite field $\hat{\phi}$ that transforms according to (2.5) and reduces to ϕ in the limit $\kappa \to 0$. It is easy to see that the desired composite field is given by [12]

$$\hat{\phi}(x) \equiv \Phi(x, -\kappa\lambda_g(x), -\kappa\bar{\lambda}_g(x)). \tag{3.2}$$

Indeed, using the invariance of the hypersurface (2.2) under linear supersymmetry transformations, one finds

$$\delta_\xi \hat{\phi} \;=\; \hat{\phi}(x + \delta x) - \hat{\phi}(x) \;=\; -i\kappa (\lambda_g \sigma^m \bar{\xi} - \xi \sigma^m \bar{\lambda}_g) \partial_m \hat{\phi}. \tag{3.3}$$

As an illustrative example, consider a chiral multiplet Φ. One has

$$\hat{\phi} = \phi - \kappa\, \lambda_g \psi + \mathcal{O}(\kappa^2). \tag{3.4}$$

The supersymmetric completion of the dilaton-like coupling $(\phi^\dagger + \phi) F_{mn} F^{mn}$ is thus given by

$$\int d^4 x \left((\phi + \phi^\dagger - \kappa\, \lambda_g \psi - \kappa\, \bar{\lambda}_g \bar{\psi}) F_{mn} F^{mn} + \mathcal{O}(\kappa^2) \right). \tag{3.5}$$

4 Partially broken $\mathcal{N} = 2$

Let us see how the above formalism of non-linearly realized supersymmetry generalizes to partially broken $\mathcal{N} = 2$ supersymmetry. The goldstino is now the lowest component of a superfield $\Lambda_g(x, \theta, \bar{\theta})$ with respect to the unbroken supersymmetry. I concentrate on the case, where the superpartner of the goldstino is a $U(1)$ gauge boson:

$$\Lambda_{g\,\alpha} \;=\; -\frac{i}{2} \left(\lambda_{g\,\alpha} + (\sigma^{mn}\theta)_\alpha F_{mn} + \ldots \right) \;=\; \frac{1}{2} W_\alpha + \mathcal{O}(\kappa^2). \tag{4.1}$$

Under the second supersymmetry, Λ_g transforms as

$$\delta^{(2)}\Lambda_g = \frac{1}{\kappa}\xi^{(2)} - i\kappa\,(\Lambda_g \sigma^m \bar{\xi}^{(2)} - \xi^{(2)} \sigma^m \bar{\Lambda}_g)\partial_m \Lambda_g. \tag{4.2}$$

The full non-linear invariant action for the goldstino superfield has been determined by the authors of [8]. They find

$$S \;=\; \int d^4 x \left[\frac{1}{4} \int d^2\theta\, W^2 + \frac{1}{4} \int d^2\bar{\theta}\, \bar{W}^2 + \frac{\kappa^2}{8} \int d^2\theta d^2\bar{\theta}\, W^2 \bar{W}^2 + \mathcal{O}(\kappa^4) \right]. \tag{4.3}$$

The bosonic terms of (4.3) coincide with the Born-Infeld action,

$$S_{\text{bos}} \;=\; \frac{1}{\kappa^2} \int d^4 x \left(1 - \sqrt{-\det(\eta_{mn} + \kappa\, F_{mn})} \right). \tag{4.4}$$

There is a chiral version of the non-linear supersymmetry transformation which acts on chiral superfields $\tilde{\Phi}$ as

$$\delta^{(2)}\tilde{\Phi} = -2i\kappa\, \tilde{\Lambda}_g \sigma^m \bar{\xi}^{(2)} \partial_m \tilde{\Phi}, \tag{4.5}$$

where $\tilde{\Lambda}_g(x, \theta, \bar{\theta}) = \Lambda(x^m - i\kappa^2\, \Lambda_g \sigma^m \bar{\Lambda}_g, \theta, \bar{\theta})$.

Defining $\Phi(x, \theta, \bar{\theta}) = \tilde{\Phi}(x^m + i\kappa^2\, \Lambda_g \sigma^m \bar{\Lambda}_g, \theta, \bar{\theta})$, one finds that an invariant action is of the form

$$S \;=\; \int d^4 x \left[\int d^2\theta d^2\bar{\theta}\, \hat{E}\, \Phi^\dagger \Phi + \int d^2\theta\, E_L\, \mathcal{P}(\tilde{\Phi}) + \int d^2\bar{\theta}\, E_R\, \mathcal{P}(\tilde{\Phi}^\dagger) \right], \tag{4.6}$$

where

$$\hat{E} \;=\; 1 + \frac{\kappa^2}{8}\bar{D}^2\bar{\Lambda}_g^2 + \frac{\kappa^2}{8}D^2\Lambda_g^2 + \mathcal{O}(\kappa^4),$$

$$E_L \;=\; 1 + \frac{\kappa^2}{4}\bar{D}^2\bar{\Lambda}_g^2 + \mathcal{O}(\kappa^4), \qquad E_R \;=\; E_L^\dagger. \tag{4.7}$$

5   Coupling $\mathcal{N}=2$ to $\mathcal{N}=1$

The method to couple an $\mathcal{N}=2$ multiplet to $\mathcal{N}=1$ matter is a straightforward generalization of the method explained in section 3. In this talk, I will just give the result for the coupling of an $\mathcal{N}=2$ vector $(A_m, \lambda^{(1)}, \lambda^{(2)}, \phi)$ to $\mathcal{N}=1$ matter. The $\mathcal{N}=2$ vector can be split into an $\mathcal{N}=1$ vector $V = (A_m, \lambda^{(1)})$ and an $\mathcal{N}=1$ chiral multiplet $\Phi = (\phi, \lambda^{(2)})$.

Using

$$\delta^{(2)}V \;=\; -\frac{i}{\sqrt{2}}\,\theta\sigma^m\bar{\theta}(\xi^{(2)}\sigma_m\bar{D}\bar{\Phi}^\dagger + \bar{\xi}^{(2)}\bar{\sigma}^m D\Phi),$$

$$\delta^{(2)}\Phi \;=\; -i\sqrt{2}\,W\xi^{(2)}, \tag{5.1}$$

$$\delta^{(2)}W_\alpha \;=\; \frac{i}{\sqrt{2}}\,\bar{\xi}^{(2)}\bar{D}D_\alpha\Phi - \frac{i}{2\sqrt{2}}\xi_\alpha^{(2)}\bar{D}^2\Phi^\dagger,$$

one finds that

$$\hat{\Phi} \equiv \Phi + i\sqrt{2}\,\kappa\,\tilde{\Lambda}_g W - \frac{1}{4}\kappa^2\tilde{\Lambda}_g\tilde{\Lambda}_g\bar{D}^2\Phi^\dagger \tag{5.2}$$

transforms as $\delta^{(2)}\hat{\Phi} = -2i\kappa\,\tilde{\Lambda}_g\sigma^m\bar{\xi}^{(2)}\partial_m\hat{\Phi}$. This implies that the $\mathcal{N}=2$ supersymmetric generalization of the dilaton-like coupling $\Phi\,\mathcal{W}^\alpha\mathcal{W}_\alpha$, where $\mathcal{W}_\alpha$ is an $\mathcal{N}=1$ Maxwell superfield without $\mathcal{N}=2$ partner, is given by

$$S \;=\; \int d^4x\,d^2\theta\,E_L\,\hat{\Phi}\,\mathcal{W}^\alpha\mathcal{W}_\alpha \;+\; h.c. \tag{5.3}$$

Similarly, one finds that

$$\hat{V} = V + i\kappa\,\theta\sigma^m\bar{\theta}\left(\bar{\Lambda}_g\bar{\sigma}_m D\Phi + \Lambda_g\sigma_m\bar{D}\Phi^\dagger\right) + \mathcal{O}(\kappa^2). \tag{5.4}$$

transforms as $\delta^{(2)}\hat{V} = -i\kappa\left(\Lambda_g\sigma^m\bar{\xi}^{(2)} - \xi^{(2)}\sigma^m\bar{\Lambda}_g\right)\partial_m\hat{V}$. This implies that the $\mathcal{N}=2$ supersymmetric generalization of the gauge coupling term $\Phi_{\rm b}^\dagger\,e^V\,\Phi_{\rm b}$, where $\Phi_{\rm b}$ is a chiral superfield without $\mathcal{N}=2$ partner, is given by

$$S = \int d^4x\,d^2\theta\,d^2\bar{\theta}\,\hat{E}\,\Phi_{\rm b}^\dagger\,e^{\hat{V}}\,\Phi_{\rm b}. \tag{5.5}$$

boundary 1 $\mathcal{N}=1$ matter	bulk $\mathcal{N}=2$ vector	boundary 2 $\mathcal{N}=1'$ matter
$\tilde{\Phi}^{(1)} = (\phi^{(1)}, \psi^{(1)})$	$V = (A_m, \lambda^{(1)})$ $\Phi = (\phi, \lambda^{(2)})$ or $V' = (A_m, \lambda^{(2)})$ $\Phi' = (\phi, \lambda^{(1)})$	
		$\tilde{\Phi}^{(2)} = (\phi^{(2)}, \psi^{(2)})$
Λ_g		Λ'_g

Table 1: Field content of the pseudo-supersymmetry toy model. The $\mathcal{N}=2$ vector can be split either into two $\mathcal{N}=1$ multiplets or into two $\mathcal{N}=1'$ multiplets.

6   Pseudo-Supersymmetry

We are now in a position to write down the explicit Lagrangian for the pseudo-super-symmetry toy model described in the introduction. The field content of this model is summarized in table 1. Note that the $\mathcal{N}=2$ vector can either be split into an $\mathcal{N}=1$ vector V and an $\mathcal{N}=1$ chiral multiplet Φ or into an $\mathcal{N}=1'$ vector V' and an $\mathcal{N}=1'$ chiral multiplet Φ'. The chiral fields on the first boundary couple to V whereas the chiral fields on the second boundary couple to V'. The Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{bulk}} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)}, \tag{6.1}$$

with

$$\mathcal{L}^{(1)} = \int d^2\theta d^2\bar{\theta}\, \hat{E}^{(1)}\, \Phi^{(1)\dagger} e^{q_1 \hat{V}} \Phi^{(1)} + \int d^2\theta\, E_L^{(1)}\, \tilde{\Lambda}_g \tilde{\Lambda}_g + h.c.$$

$$\mathcal{L}_{\text{bulk}} = \int d^2\theta d^2\bar{\theta}\, \Phi^{\dagger} e^{V} \Phi + \frac{1}{4}\int d^2\theta\, WW + h.c. \tag{6.2}$$

$$\mathcal{L}^{(2)} = \int d^2\tilde{\theta} d^2\bar{\tilde{\theta}}\, \hat{E}^{(2)}\, \Phi^{(2)\dagger} e^{q_2 \hat{V}'} \Phi^{(2)} + \int d^2\tilde{\theta}\, E_L^{(2)}\, \tilde{\Lambda}'_g \tilde{\Lambda}'_g + h.c.$$

This supersymmetry breaking mechanism has several interesting consequences:

- There are no quadratic divergences at one-loop. Only Feynman diagrams involving fields from both boundaries can contribute to supersymmetry breaking. But such diagrams only arise at two-loop.

- The scalar masses squared arising at two loops are expected to be $\sim (g/4\pi)^4 M^2$, where g is the gauge coupling of the bulk vector and M is the cut-off scale of the effective field theory.

- A non-vanishing vacuum energy only arises at three-loop. (See [13] for a calculation of scalar masses and vacuum energy in a 5D version of the model described in this talk.)

- Both goldstinos stay massless (at tree-level) even when coupled to supergravity because the $U(1)$ gauge symmetries of their superpartners are unbroken.

- Gravitino masses arise only at three-loop.

Acknowledgments

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