Unification into a product GUT: SU(5) × SU(5)

Graham D. Kribs

Department of Physics, University of Wisconsin, Madison, WI 53706

kribs@physics.wisc.edu

Abstract

A new grand unified model based on the product group SU(5) × SU(5) is presented. The product GUT is broken to $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{U}(1)_Y$, by an odd number of link fields acquiring suitable vacuum expectation values. The resulting SU(5) gauge anomalies are canceled by endpoint fields that are identified as the ordinary matter and Higgs fields of the MSSM. Doublet-triplet splitting is automatic, with the up-type and down-type Higgs fields charged under different SU(2)'s. The TeV scale effective theory has left-handed quarks charged under $\mathrm{SU}(2)_1$ and left-handed leptons charged under $\mathrm{SU}(2)_2$, a supersymmetrized version of the "ununified standard model". The μ parameter is generated once $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$ breaks to $\mathrm{SU}(2)_L$ near the electroweak scale. A robust test of this model is weak scale supersymmetry plus an additional SU(2) gauge symmetry appearing near a few TeV.

Introduction One of the most compelling reasons for physics beyond the Standard Model is to understand the origin of the quantum numbers of matter. SU(5) grand unification is perhaps the most elegant explanation of these quantum numbers since the SM matter fits precisely into (three generations of) the $10 + \overline{5}$ anomaly-free combination of SU(5) representations [1]. SU(5) is also the most minimal choice that embeds $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ with no additional gauge group structure.

Non-supersymmetric SU(5), however, is in strong disagreement with experiment. The prediction of $\sin^2\theta_W$ at the weak scale disagrees with experiment by many σ. Proton decay should have been observed at a rate easily detectable by now. Finally, non-supersymmetric GUTs suffer from the infamous hierarchy problem: the electroweak scale is not protected against quadratically divergent radiative corrections from GUT scale physics.

Supersymmetric SU(5) solves all of these problems. By including superpartners in the evolution equations, the prediction of $\sin^2\theta_W$ matches experiment to within about 2σ. Simultaneously, the unification scale is predicted to about 2×10^{16} GeV, alleviating the fast proton decay problem through dimension-6 operators. Finally, supersymmetry renders radiative corrections to the Higgs sector to be at most log divergent.

Supersymmetry is, however, not devoid of its own problems. One of the most troubling aspects of weak scale supersymmetry concerns the Higgs sector. To cancel $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge anomalies, and to couple to both up-type and down-type quarks, the Higgs sector must be made vector-like with both up-type and down-type Higgs fields with opposite quantum numbers. Hence, in the minimal supersymmetric standard model (MSSM) a supersymmetric mass for the Higgs fields $\mu H_u H_d$ is allowed. Avoiding current experimental bounds on Higgsinos requires $\mu \gtrsim 100$ GeV, while a non-fine-tuned solution to electroweak symmetry breaking suggests μ is not too far above the TeV scale. A viable low energy model with electroweak symmetry breaking therefore appears to force a relationship between the *a priori* unrelated supersymmetric mass μ and soft supersymmetry breaking masses that must also not be too far from the TeV scale. How this relationship arises is a puzzle in the MSSM, but becomes a very severe problem in supersymmetric SU(5).

Supersymmetry also suffers from additional proton decay problems. The most severe issue is at the renormalizable level, in which dimension-three and dimension-four operators that break lepton and/or baryon number, but this is typically solved with the addition of matter parity (that becomes R-parity on component fields). However, even with exact matter parity, proton decay can proceed through dangerous dimension-5 operators involving the color triplet Higgsino exchange. To avoid the current bound on the proton lifetime, the Higgsinos must be larger in mass by a factor of three or so over the GUT gauge boson masses.

There are solutions to the doublet-triplet splitting problem in field theory [2], string theory [3], and recently extra physical [4] and deconstructed [6] (see also [7]) dimensions. Quite recently, Witten [8] (see also [9]) has argued that by separating the Higgs chiral superfields in "theory space" one can naturally solve the doublet-triplet splitting problem. His idea relied a high energy theory with $\mathrm{SU}(5)' \times \mathrm{SU}(5)''$ times an additional discrete symmetry F. The $\mathrm{SU}(5)' \times \mathrm{SU}(5)''$ breaks to the SM gauge group, while F times a discrete subgroup of the $\mathrm{U}(1)''_Y$ (subgroup of $\mathrm{SU}(5)''$) breaks to a diagonal discrete subgroup F'. Fields originally charged under $\mathrm{SU}(5)''$ will have their SM component fields distinguished by this discrete symmetry. In particular, F' can be used to ensure the Higgs triplet acquires a GUT scale mass while the Higgs doublets remain massless. Of course at low energies the Higgs doublets must acquire an electroweak scale mass. This can happen if the F' discrete symmetry is spontaneously broken by the vev of singlet field carrying a discrete F' charge, leading to the operator

$$\frac{S^m H_u H_d}{M_{\mathrm{Pl}}^{m-1}} \quad \rightarrow \quad \mu \sim \frac{\langle S \rangle^m}{M_{\mathrm{Pl}}^{m-1}} \,. \tag{1}$$

Here, $m < n$ is determined by the discrete Z_n charge of S. There are two requirements of this source of supersymmetric Higgs mass: First, one must ensure that the discrete symmetry breaking does not reintroduce a proton decay problem, since dimension-4 baryon/lepton number violation, as well as dimension-5 proton decay was forbidden using this discrete symmetry. Second, the quantity $\langle S \rangle^m / M_{\mathrm{Pl}}^{m-1}$ must be arranged to coincide with the electroweak scale.

The Model In these proceedings I discuss a new approach to the doublet-triplet splitting problem and the dimension-5 proton decay problem using a product GUT $\mathrm{SU}(5) \times \mathrm{SU}(5)$ broken to $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{U}(1)_Y$ [10]. (For earlier papers that considered this product GUT, see [11].) Several fields are charged under both gauge groups, called link fields, while others are charged under just one or the other SU(5). Here I will build the model piece by piece.

The GUT breaking sector There are several possibilities for the choice of fields to break SU(5) $\times$ SU(5) down to 3-2-2-1. The $\mathrm{SU}(3)_c$ and $\mathrm{U}(1)_Y$ are the diagonal subgroups of the SU(3) $\times$ U(1)'s in the two SU(5)'s, while each SU(2) subgroup of SU(5) survives the GUT breaking. Perhaps the simplest set of fields that gives this breaking pattern is a pair of bifundamental link fields $Q_1(\overline{\mathbf{5}},\mathbf{5})$ and $Q_2(\mathbf{5},\overline{\mathbf{5}})$ in which each acquires a vev along the "color" direction

$$\langle Q_1 \rangle = \langle Q_2 \rangle = \begin{pmatrix} V & & & & \\ & V & & & \\ & & V & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} \tag{2}$$

where V is of order the usual GUT scale, few $\times 10^{16}$ GeV. A pair of bifundamentals is necessary to ensure that the product SU(5) $\times$ SU(5) gauge symmetry is fully broken down to 3-2-2-1. This can be seen by decomposing the bifundamentals under the surviving subgroups ($\mathrm{SU}(3)_c$,$\mathrm{SU}(2)_1$,$\mathrm{SU}(2)_2$,$\mathrm{U}(1)_Y$)

$$Q_1 = \begin{pmatrix} (\mathbf{8},\mathbf{1},\mathbf{1},0)+(\mathbf{1},\mathbf{1},\mathbf{1},0) & (\mathbf{3},\mathbf{1},\mathbf{2},-5/6) \\ (\overline{\mathbf{3}},\mathbf{2},\mathbf{1},5/6) & (\mathbf{1},\mathbf{2},\mathbf{2},0) \end{pmatrix} \tag{3}$$

$$Q_2 = \begin{pmatrix} (\mathbf{8},\mathbf{1},\mathbf{1},0)+(\mathbf{1},\mathbf{1},\mathbf{1},0) & (\mathbf{3},\mathbf{2},\mathbf{1},-5/6) \\ (\overline{\mathbf{3}},\mathbf{1},\mathbf{2},5/6) & (\mathbf{1},\mathbf{2},\mathbf{2},0) \end{pmatrix} \tag{4}$$

where the off-diagonal components get eaten by the X's and Y's of the two SU(5)'s. Notice that each bifundamental contributes a bidoublet under $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$.

The Higgs sector Following previous ideas that separate the up-type from down-type Higgs multiplets in "theory space", here the up-type Higgs arises from a $(\mathbf{5},\mathbf{1})$ while the down-type Higgs arises from a $(\mathbf{1},\overline{\mathbf{5}})$. No Higgs triplet or doublet mass between these fields is allowed by the SU(5) $\times$ SU(5) gauge symmetries of the theory. A triplet mass is, however, generated after the product GUT is broken to 3-2-2-1. This is easy to see from the superpotential

$$W = (\mathbf{5},\mathbf{1})_H Q_1(\overline{\mathbf{5}},\mathbf{5})(\mathbf{1},\overline{\mathbf{5}})_H \quad \longrightarrow \quad V t_u t_d \tag{5}$$

where t_u is a $(\mathbf{3},\mathbf{1},\mathbf{1},-1/3)$ and t_d is a $(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},1/3)$. That is, the triplets acquire a GUT scale mass while the doublets are forbidden from pairing up due to the $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$ gauge symmetry.

The μ term Clearly to generate a weak scale μ term, we must break the product $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$ gauge symmetry down to $\mathrm{SU}(2)_L$ near the weak scale. This can be done using a bidoublet $(\mathbf{2},\mathbf{2})$ under $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$ that acquires a vev near the weak scale. Perhaps the simplest mechanism to induce a bidoublet vev is through the Coleman-Weinberg mechanism. In particular, the bidoublet scalar (mass)^2 will run negative near the weak scale if it is coupled to other fields with positive (mass)^2 through order one Yukawa couplings, analogous to the up-type Higgs (mass)^2 in the MSSM.

Here, we clearly see the role of preserving an extra SU(2) gauge symmetry to the weak scale: Doublet-triplet splitting is automatic, and the explanation for the size of the μ term is determined by SUSY breaking.

Higgs mass terms? With only a vector-like pair of bifundamentals under $SU(5)_1 \times SU(5)_2$, additional fields clearly must be added to cancel the gauge anomalies induced by just the Higgs fields. Adding an additional pair $(\overline{\mathbf{5}}, \mathbf{1})$ plus $(\mathbf{1}, \mathbf{5})$ brings us back to a situation like in the MSSM with (now two) vector-like Higgs sectors. I propose an alternative path: Forbid vector-like matter on the endpoints. This retains the spirit of ensuring that there is no vector-like sector in the model. (We shall see later, however, that a vector-like Froggatt-Nielsen sector will be needed to generate the b-Yukawa.)

Non-vector-like link fields: Add a third bifundamental $Q_3(\overline{\mathbf{5}}, \mathbf{5})$ to the model. The endpoints are now anomalous, but the anomalies can be canceled with suitable "matter". The most economical arrangement is to charge all three generations of the $\mathbf{10}_m$'s of matter under $SU(5)_1$ and similarly the $\overline{\mathbf{5}}_m$'s of matter under $SU(5)_2$. Three generations plus one Higgs fundamental (or anti-fundamental) is almost sufficient to cancel each SU(5) gauge anomaly, but one additional "spectator" set of multiplets, $S + S'$, must also be added. These spectators could be a fourth generation $(\mathbf{10}, \mathbf{1}) + (\mathbf{1}, \overline{\mathbf{5}})$ or a fundamental + anti-fundamental $[(\mathbf{5}, \mathbf{1}) + (\mathbf{1}, \overline{\mathbf{5}})]$, etc. Notice that the two SU(2)'s forbid the doublet components of a $(\mathbf{5}, \mathbf{1}) + (\mathbf{1}, \overline{\mathbf{5}})$ from pairing up through the link field vev, just like in the Higgs sector. This means that a fourth generation has the advantage that gauge coupling unification is not affected (to one-loop), while a fundamental + antifundamental must be forbidden to pair up (e.g., through an additional discrete symmetry).

Matter There are several interesting features resulting from the above distribution of matter among the two SU(5)'s. First, the gauge anomaly resulting from one $(\mathbf{10}, \mathbf{1})_m$ or one $(\mathbf{1}, \overline{\mathbf{5}})_m$ is no longer canceling generation-by-generation. Anomaly cancellation is instead resulting from a combination of an odd number of link fields and the product $SU(5) \times SU(5)$ gauge structure. This has the advantage that the number of generations is not arbitrary in the model.

Second, the dimension-5 operator for proton decay resulting after the Higgs triplets are integrated out,

$$\frac{\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\overline{\mathbf{5}}}{M_{\mathrm{t}}}\,, \tag{6}$$

is forbidden by gauge invariance. It can be regenerated at dimension-6

$$\frac{(\mathbf{10}, \mathbf{1})_m\,(\mathbf{10}, \mathbf{1})_m\,(\mathbf{10}, \mathbf{1})_m\,Q_1(\mathbf{1}, \overline{\mathbf{5}})_m}{M_{\mathrm{t}} M_{\mathrm{Pl}}} \tag{7}$$

leading to one of the two usual operators

$$\frac{V}{M_{\mathrm{Pl}}} \frac{e\,u\,u\,d}{M_{\mathrm{t}}}\,, \tag{8}$$

but with an additional $V/M_{\mathrm{Pl}} \sim 10^{-2}$ suppression. This is easily sufficient to cure the triplet-induced dimension-5 proton decay problem.

Finally, Yukawa couplings are quite distinct in this model. The top Yukawa is allowed

$$\lambda_t (\mathbf{10}, \mathbf{1})_m (\mathbf{10}, \mathbf{1})_m (\mathbf{5}, \mathbf{1})_H \tag{9}$$

but the b and τ Yukawas are forbidden by gauge invariance. They are regenerated at dimension ≥ 5, but with only bifundamental link fields acquiring the vev in eq (2) with bidoublets acquiring a weak scale vev, they are highly suppressed. One possibility is to add a vector-like pair of link fields $Q_4(\overline{\mathbf{10}},\mathbf{10})+Q_5(\mathbf{10},\overline{\mathbf{10}})$, in antisymmetric tensor reps that acquire a vev along the hypercharge direction. New operators could be written such as

$$\frac{(\mathbf{10},\mathbf{1})_m Q_4(\mathbf{1},\overline{\mathbf{5}})_m(\mathbf{1},\overline{\mathbf{5}})_H}{M_{\rm Pl}} \tag{10}$$

which leads to a τ Yukawa that is naturally suppressed relative to the top Yukawa

$$\frac{V}{M_{\rm Pl}} e\, L\, H_d\,. \tag{11}$$

by the amount $V/M_{\rm Pl} \sim 10^{-2}$. The b-Yukawa is more subtle. The basic problem is that a quark doublet from $\mathrm{SU}(5)_1$ is forbidden from pairing up with the down-type Higgs doublet from $\mathrm{SU}(2)_2$ by the product $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$ gauge symmetry. One solution is to add a Froggatt-Nielsen sector to $\mathrm{SU}(5)_2$, and when combined with the additional link fields $Q_{4,5}$ one can obtain a b-Yukawa suppressed by $V/M_{\rm Pl}$ so long as the mass of the vector-like Froggatt-Nielsen sector is of order μ.

Gauge coupling unification With matter distributed among the two SU(5)'s, and an extra SU(2) gauge symmetry, gauge coupling unification is obviously not manifest. Whether or not gauge coupling unification is successful depends on detailing precisely what matter survives to the weak scale. Consider a situation in which n_b bidoublets $(\mathbf{2},\mathbf{2})$ survive to the weak scale, but nothing else. (This is unrealistic, but this calculation is a useful warmup to understand what generally happens in this model.) The $\mathrm{SU}(3)_c$ and $\mathrm{U}(1)_Y$ gauge couplings are, to one-loop,

$$\frac{1}{\alpha_c(M_Z)} = \frac{1}{\alpha_{\mathrm{SU}(5)_1}} + \frac{1}{\alpha_{\mathrm{SU}(5)_2}} + \frac{b_c^{\rm MSSM}}{2\pi}\ln\frac{V}{M_Z} \tag{12}$$

$$\frac{1}{\alpha_Y(M_Z)} = \frac{1}{\alpha_{\mathrm{SU}(5)_1}} + \frac{1}{\alpha_{\mathrm{SU}(5)_2}} + \frac{b_Y^{\rm MSSM}}{2\pi}\ln\frac{V}{M_Z} \tag{13}$$

while the SU(2)'s are

$$\frac{1}{\alpha_1(M_Z)} = \frac{1}{\alpha_{\mathrm{SU}(5)_1}} + \frac{b_1}{2\pi}\ln\frac{V}{\mu} \tag{14}$$

$$\frac{1}{\alpha_2(M_Z)} = \frac{1}{\alpha_{\mathrm{SU}(5)_2}} + \frac{b_2}{2\pi}\ln\frac{V}{\mu}\,. \tag{15}$$

Here $b_{c,Y}^{\rm MSSM}$ are the usual MSSM beta function coefficients, while $b_{1,2}$ are for the two SU(2)'s. The diagonal electroweak group is

$$\frac{1}{\alpha_L(M_Z)} = \frac{1}{\alpha_1(\mu)} + \frac{1}{\alpha_2(\mu)} + \frac{b_L^{\rm MSSM}}{2\pi}\ln\frac{\mu}{M_Z} \tag{16}$$

which can be rewritten as

$$\frac{1}{\alpha_L(M_Z)} = \frac{1}{\alpha_{\mathrm{SU}(5)_1}} + \frac{1}{\alpha_{\mathrm{SU}(5)_2}} + \frac{b_1+b_2}{2\pi}\ln\frac{V}{\mu} + \frac{b_L^{\rm MSSM}}{2\pi}\ln\frac{\mu}{M_Z}\,. \tag{17}$$

With n_b bidoublets it is straightforward to calculate $b_1 + b_2 = b_L^{\mathrm{MSSM}} - 6 + 2n_b$ where $-6 = -3C_2$ arises from the "extra" set of gauge bosons and the bidoublets contribute $2n_b$. If the number of bidoublets $n_b = 3$, these two contributions cancel out, and we are left with *precisely* the same one-loop gauge coupling unification as in the MSSM. Notice that the unification is to

$$\frac{1}{\alpha_{\mathrm{GUT}}} = \frac{1}{\alpha_{\mathrm{SU(5)}_1}} + \frac{1}{\alpha_{\mathrm{SU(5)}_2}} \,, \tag{18}$$

i.e., does not depend on either high energy SU(5) gauge coupling individually, and thus there is no loss in predictivity over minimal SU(5) with one gauge coupling. In this model, there are three bifundamentals that yield three bidoublets, and one "merely" needs to ensure that these bidoublets survive to weak scales with nothing else (in incomplete SU(5) reps). This is, however, not an easy task for reasons I won't go into here [10].

TeV phenomenology A prediction of this model is that an additional SU(2) gauge symmetry survives to near the weak scale. Since our $\mathrm{SU(2)}_L$ results from a diagonal breaking of $\mathrm{SU(2)}_1 \times \mathrm{SU(2)}_2$, matter is coupled to the extra heavy $W^{\pm}, W^3$ gauge bosons with order TeV masses. In fact, quark doublets are charged under $\mathrm{SU(2)}_1$ while lepton doublets are charged under $\mathrm{SU(2)}_2$, analogous to an older non-supersymmetric idea called the "ununified model" [12]. Precision electroweak data strongly constraints any new gauge symmetries appearing near the TeV scale. The EW precision constraints on the ununified model were calculated some time ago [13] and recently updated [10] leading to the bound

$$M_{W_H} > 2.7 \text{ TeV} \quad \text{to } 95\% \text{ CL} \,. \tag{19}$$

Future colliders may well be able to probe gauge boson masses at and above this level, leading to an excellent test of the model— probing the TeV region one would discover an additional SU(2) gauge symmetry in addition to superpartners!

Summary I have presented a new model of grand unification based on the product group $\mathrm{SU(5)}_1 \times \mathrm{SU(5)}_2$ breaking to $\mathrm{SU(3)}_c \times \mathrm{SU(2)}_1 \times \mathrm{SU(2)}_2 \times \mathrm{U(1)}_Y$ at the GUT scale, and then the SU(2)'s breaking to $\mathrm{SU(2)}_L$ at the TeV scale. Doublet-triplet splitting is automatic, and the size of μ is tied to supersymmetry breaking. Dimension-5 proton decay is adequately suppressed. Gauge coupling unification may be preserved. New physics is expected near the weak scale with a new SU(2) gauge symmetry with a structure analogous to a supersymmetrized version of the ununified model.

Acknowledgments This work was supported in part by the U.S. Department of Energy under contract DE-FG02-95-ER40896.

References

[1] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[2] For a review, see L. Randall and C. Csáki, arXiv:hep-ph/9508208.

[3] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. B **258**, 46 (1985); E. Witten, Nucl. Phys. B **258**, 75 (1985).

[4] See for example Y. Kawamura, Prog. Theor. Phys. **105**, 999 (2001) [arXiv:hep-ph/0012125]; G. Altarelli and F. Feruglio, Phys. Lett. B **511**, 257 (2001) [arXiv:hep-ph/0102301];

[5] L. J. Hall and Y. Nomura, Phys. Rev. D **64**, 055003 (2001) [arXiv:hep-ph/0103125]; A. Hebecker and J. March-Russell, Nucl. Phys. B **613**, 3 (2001) [arXiv:hep-ph/0106166].

[6] C. Csáki, J. Erlich, C. Grojean, and G. D. Kribs, Phys. Rev. D **65**, 015003 (2002) [arXiv:hep-ph/0106044]; C. Csáki, G. D. Kribs, and J. Terning, Phys. Rev. D **65**, 015004 (2002) [arXiv:hep-ph/0107266]; H. C. Cheng, K. T. Matchev, and J. Wang, Phys. Lett. B **521**, 308 (2001) [arXiv:hep-ph/0107268].

[7] N. Weiner, arXiv:hep-ph/0106097.

[8] E. Witten, arXiv:hep-ph/0201018.

[9] M. Dine, Y. Nir, and Y. Shadmi, arXiv:hep-ph/0206268.

[10] G. D. Kribs, in preparation.

[11] R. Barbieri, G. R. Dvali, and A. Strumia, Phys. Lett. B **333**, 79 (1994) [arXiv:hep-ph/9404278]; R. N. Mohapatra, Phys. Lett. B **379**, 115 (1996) [arXiv:hep-ph/9601203]; S. M. Barr, Phys. Rev. D **55**, 6775 (1997) [arXiv:hep-ph/9607359].

[12] H. Georgi, E. E. Jenkins, and E. H. Simmons, Phys. Rev. Lett. **62**, 2789 (1989) [Erratum-ibid. **63**, 1540 (1989)]; Nucl. Phys. B **331**, 541 (1990).

[13] R. S. Chivukula, E. H. Simmons, and J. Terning, Phys. Lett. B **346**, 284 (1995) [arXiv:hep-ph/9412309].