

# LEP Constraints on 5-Dimensional Extensions of the Standard Model

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## ABSTRACT

We study minimal 5-dimensional extensions of the Standard Model, in which all or only some of the  $SU(2)_L$  and  $U(1)_Y$  gauge fields and Higgs bosons propagate in the fifth compact dimension. In all the 5-dimensional settings, the fermions are assumed to be localized on a 3-brane. In addition, we present the consistent procedure for quantizing 5-dimensional models in the generalized  $R_\xi$  gauge. Bounds on the compactification scale between 4 and 6 TeV, depending on the model, are established by analyzing electroweak precision measurements and LEP2 cross sections.

## 1 Introduction

In the original formulations of string theory [1], the compactification radius  $R$  of the extra dimensions and the string mass  $M_s$  were considered to be set by the 4-dimensional Planck mass  $M_P = 1.9 \times 10^{16}$  TeV. However, recent studies have shown [2–6] that conceivable scenarios of stringy nature may exist for which  $R$  and  $M_s$  practically decouple from  $M_P$ . For example, in the model of Ref. [5],  $M_s$  may become as low as a few TeV. In this case,  $M_s$  constitutes the only fundamental scale in nature at which all forces including gravity unify. This low string-scale effective model could be embedded within e.g. type I string theories [4], where the Standard Model (SM) may be described as an intersection of higher-dimensional  $Dp$  branes [5–7].

As such intersections may be higher dimensional as well, in addition to gravitons the SM gauge fields could also propagate within a higher-dimensional subspace with compact dimensions of order  $\text{TeV}^{-1}$  for phenomenological reasons. Since such low string-scale constructions may result in different higher-dimensional extensions of the SM [7], the actual experimental limits on the compactification radius are, to some extent, model dependent. Nevertheless, most of the derived phenomenological limits in the literature were obtained by assuming that all the SM gauge fields propagate in a common higher-dimensional space [8–14].

Here, we wish to lift the above restriction and focus on the phenomenological consequences of models which minimally depart from the assumption of a universal higher-dimensional scenario [15]. Specifically, we will consider 5-dimensional extensions of the SM compactified on an  $S^1/Z_2$  orbifold, where the  $SU(2)_L$  and  $U(1)_Y$  gauge bosons may not both live in the same higher-dimensional space, the so-called bulk. In all our models, the SM fermions are localized on the 4-dimensional subspace, i.e. on a 3-brane or, as it is often called, brane. For each higher-dimensional model, we calculate the effects of the

fifth dimension on the electroweak observables and analyze their impact on constraining the compactification scale.

The organization of this brief report is as follows: in Section 2 we introduce the basic concepts of higher-dimensional theories by considering a simple 5-dimensional Abelian model. After briefly discussing how these concepts can be applied to the SM in Section 3, we turn our attention to the phenomenological aspects of the models of our interest in Section 4. Because of the limited space, technical details are omitted in this note. A complete discussion, along with detailed analytic results and references, is given in our paper in [15]. Section 5 summarizes our numerical results and presents our conclusions.

## 2 5-Dimensional Abelian Models

As a starting point, let us consider the Lagrangian of 5-dimensional Quantum Electrodynamics (5D-QED) given by

$$\mathcal{L}(x, y) = -\frac{1}{4}F_{MN}(x, y)F^{MN}(x, y) + \mathcal{L}_{\text{GF}}(x, y), \quad (2.1)$$

where  $F_{MN}$  denotes the 5-dimensional field strength tensor, and  $\mathcal{L}_{\text{GF}}(x, y)$  is the gauge-fixing term. Our notation is:  $M, N = 0, 1, 2, 3, 5$ ;  $\mu, \nu = 0, 1, 2, 3$ ;  $x = (x^0, \vec{x})$ ; and  $y = x^5$ .

In the absence of the gauge-fixing and ghost terms, the 5D-QED Lagrangian is invariant under U(1) gauge transformations. To compactify the theory on an  $S^1/Z_2$  orbifold, we demand for the fields to satisfy equalities like

$$\begin{aligned} A_\mu(x, y) &= A_\mu(x, y + 2\pi R), \\ A_\mu(x, y) &= A_\mu(x, -y). \end{aligned} \quad (2.2)$$

The field  $A_\mu(x, y)$  is taken to be even under  $Z_2$ , so as to embed conventional QED with a massless photon into our 5D-QED. Then, the reflection properties of the  $A_5(x, y)$  field with respect to  $y$  are dictated by gauge invariance, i.e.  $A_5(x, y) = -A_5(x, -y)$ .

Given (2.2), we can expand the fields in Fourier series, where the Fourier coefficients, denoted  $A_{(n)}^\mu(x)$ , are the so-called KK modes. Integrating out the  $y$  dimension we obtain the effective 4-dimensional Lagrangian including massless QED. The other terms describe two infinite towers of massive vector excitations  $A_{(n)}^\mu$  and (pseudo)-scalar modes  $A_{(n)}^5$  that mix with each other, for  $n \geq 1$ . The scalar modes  $A_{(n)}^5$  play the rôle of the would-be Goldstone modes in a non-linear realization of an Abelian Higgs model, in which the corresponding Higgs fields are taken to be infinitely massive.

The above observation motivates us to seek for a higher-dimensional generalization of 't-Hooft's gauge-fixing condition. We choose the following generalized  $R_\xi$  gauge [15,16]:

$$\mathcal{L}_{\text{GF}}(x, y) = -\frac{1}{2\xi}(\partial^\mu A_\mu - \xi \partial_5 A_5)^2. \quad (2.3)$$

Upon integration over the extra dimension, all mixing terms disappear and the Lagrangian describes QED accompanied by a tower of massive gauge bosons  $A_{(n)}^\mu$  and the respective Goldstone modes  $A_{(n)}^5$ . The limit  $\xi \rightarrow \infty$  corresponds to the usual unitary gauge [17,

18]. Thus, for a simple model, we have seen how starting from a non-covariant higher-dimensional gauge-fixing condition, we can arrive at the known covariant 4-dimensional  $R_\xi$  gauge after compactification.

This quantization procedure can be successfully applied to theories that include Higgs and gauge bosons living in the bulk and/or on the brane [19]. A brane Higgs induces mixing terms between the Fourier modes. The KK mass eigenstates, found by diagonalizing the mass matrix, have slightly shifted masses and couplings to brane fermions compared to the Fourier modes.

### 3 5-Dimensional Extensions of the Standard Model

The ideas introduced in Section 2 can be generalized for non-Abelian theories. As a new feature, the self-interaction of gauge-bosons in non-Abelian theories leads to self-interactions of the KK modes which are restricted by selection rules reflecting the  $S^1/Z_2$  structure of the extra dimension.

For spontaneous symmetry-breaking theories, such as the Standard Model (SM), the existence of new compact dimensions opens up several possibilities in connection with the  $SU(2)_L \otimes U(1)_Y$  gauge structure. For example, the  $SU(2)_L$  and  $U(1)_Y$  gauge fields do not necessarily need to propagate both in the extra dimension. Such a realization may be encountered within specific stringy frameworks, where one of the gauge groups is effectively confined on the boundaries of the  $S^1/Z_2$  orbifold [7].

However, in the most frequently investigated scenario,  $SU(2)_L$  and  $U(1)_Y$  gauge fields live in the bulk of the extra dimension (bulk-bulk model). In this case, for generality, we will consider a 2-doublet Higgs model, where one Higgs field propagates in the fifth dimension, while the other one is localized. The phenomenology of this model is influenced by the vacuum expectation values  $v_1$  and  $v_2$ , or equivalently by  $\tan\beta = v_2/v_1$  and  $v^2 = v_1^2 + v_2^2$ .

An even more minimal 5-dimensional extension of electroweak physics constitutes a model in which only the  $SU(2)_L$ -sector feels the extra dimension while the  $U(1)_Y$  gauge field is localized at  $y = 0$  (bulk-brane model). In this case, the Higgs field being charged with respect to both gauge groups has to be localized at  $y = 0$  in order to preserve gauge invariance of the (classical) Lagrangian. For the same reason, a bulk Higgs is forbidden in the third possible model in which  $SU(2)_L$  is localized while  $U(1)_Y$  propagates in the fifth dimension (brane-bulk model).

In all these minimal 5-dimensional extensions of the SM we assume that the SM fermions are localized at the  $y = 0$  fixed point of the  $S^1/Z_2$  orbifold. All the KK modes of a bulk field couple to a brane fermion. Because the KK mass eigenmodes generally differ from the Fourier modes, their couplings to fermions have to be calculated for each model individually.

### 4 Effects on Electroweak Observables

In this section, we will concentrate on the phenomenology and present bounds on the compactification scale  $M = 1/R$  of minimal 5-dimensional extensions of the SM calculated

by analyzing a large number of high precision electroweak observables. We relate the SM prediction  $\mathcal{O}^{\text{SM}}$  [20,21] for an observable to the prediction  $\mathcal{O}^{\text{HDSM}}$  for the same observable obtained in the higher-dimensional SM under investigation through

$$\mathcal{O}^{\text{HDSM}} = \mathcal{O}^{\text{SM}} (1 + \Delta_{\mathcal{O}}^{\text{HDSM}}). \quad (4.1)$$

Here,  $\Delta_{\mathcal{O}}^{\text{HDSM}}$  is the tree-level modification of a given observable  $\mathcal{O}$  from its SM value due to the presence of one extra dimension. In order to enable a direct comparison of our predictions with the precision data [20,21], we include SM radiative corrections to  $\mathcal{O}^{\text{SM}}$ . However, we neglect SM- as well as KK-loop contributions to  $\Delta_{\mathcal{O}}^{\text{HDSM}}$  as higher order effects.

As input SM parameters for our theoretical predictions, we choose the most accurately measured ones, namely the  $Z$ -boson mass  $M_Z$ , the electromagnetic fine structure constant  $\alpha$  and the Fermi constant  $G_F$ . While  $\alpha$  is not affected in the models under study,  $M_Z$  and  $G_F$  generally deviate from their SM form when expressed in terms of couplings and VEV's. To first order in  $X = \frac{1}{3}\pi^2 m_Z^2 R^2$ ,  $M_Z$  and  $G_F$  may be parameterized as

$$M_Z = M_Z^{\text{SM}} (1 + \Delta_Z X), \quad G_F = G_F^{\text{SM}} (1 + \Delta_G X), \quad (4.2)$$

where  $\Delta_Z$  and  $\Delta_G$  are model-dependent parameters. For example, one finds

$$\Delta_Z = \left\{ -\frac{1}{2} \sin^4 \beta, -\frac{1}{2} \sin^2 \hat{\theta}_W, -\frac{1}{2} \cos^2 \hat{\theta}_W \right\}. \quad (4.3)$$

for the bulk-bulk, brane-bulk and bulk-brane models, with respect to the  $SU(2)_L$  and  $U(1)_Y$  gauge groups.

The relation between the weak mixing angle  $\theta_W$  and the input variables is also affected by the fifth dimension. Hence, it is useful to define an effective mixing angle  $\hat{\theta}_W$ , which still fulfills the tree-level relation

$$G_F = \frac{\pi\alpha}{\sqrt{2} \sin^2 \hat{\theta}_W \cos^2 \hat{\theta}_W M_Z^2} \quad (4.4)$$

of the Standard Model, and relate it to  $\theta_W$  by  $\sin^2 \hat{\theta}_W = \sin^2 \theta_W (1 + \Delta_{\theta} X)$ .

For the tree-level calculation of  $\Delta_{\mathcal{O}}^{\text{HDSM}}$ , it is necessary to consider the mixing effect of the Fourier modes on the masses of the Standard-Model gauge bosons as well as on their couplings to fermions. All the encountered shifts can be expanded in powers of  $X$  and are calculated to first order. For the precision measurement at the  $Z$  pole or at lower energies these effects are dominant. For cross sections at LEP2 energies, the dominant higher dimensional contributions stem from the interference of the Standard Model with virtual KK modes which roughly scales like  $s/M^2$ .

Within the framework outlined above, we compute  $\Delta_{\mathcal{O}}^{\text{HDSM}}$  for an extensive list of precision observables [15]. In addition, we consider fermion-pair production at LEP2 [22]. Employing the results of  $\Delta_{\mathcal{O}}^{\text{HDSM}}$  and calculating all the observables considered in our analysis by virtue of (4.1), we confront these predictions with the respective experimental values and calculate the corresponding  $\chi^2(X)$  where it is important to include correlations between some of the observables. The bounds on  $X$  can be derived by requiring  $\chi^2(X) -$

	SU(2) <sub>L</sub> -brane, U(1) <sub>Y</sub> -bulk	SU(2) <sub>L</sub> -bulk, U(1) <sub>Y</sub> -brane	SU(2) <sub>L</sub> -bulk, U(1) <sub>Y</sub> -bulk (brane Higgs)	SU(2) <sub>L</sub> -bulk, U(1) <sub>Y</sub> -bulk (bulk Higgs)
prec. obs.	4.2	2.9	4.6	4.6
$\mu^+\mu^-$	2.0	1.5	2.5	2.5
$\tau^+\tau^-$	2.0	1.5	2.5	2.5
hadrons	2.6	4.7	5.4	5.8
$e^+e^-$	3.0	2.0	3.6	3.5
combined	4.7	4.3	6.1	6.4

**Table 1:** Bounds on the compactification scale at the  $2\sigma$  confidence level from precision observables and the different fermion-pair production channels at LEP2.

$\chi_{\min}^2 < n^2$  for  $X$  being not excluded at the  $n\sigma$  confidence level. Here,  $\chi_{\min}^2$  is the minimal  $\chi^2$  in the physical region  $X \geq 0$ . Using slightly different definitions for the bounds does not lead to significantly different results.

Table 1 summarizes the lower bounds on the compactification scale  $M = 1/R$  coming from different observables. For the bulk-bulk model we consider the two extreme cases, a pure bulk Higgs and a pure brane Higgs.

## 5 Discussion and Conclusions

By performing  $\chi^2$ -tests, we obtain different sensitivities to the compactification radius  $R$  for the three models under consideration: (i) the SU(2)<sub>L</sub>⊗U(1)<sub>Y</sub>-bulk model, where all SM gauge bosons are bulk fields; (ii) the SU(2)<sub>L</sub>-brane, U(1)<sub>Y</sub>-bulk model, where only the SU(2)<sub>L</sub> fields are restricted to the brane, and (iii) the SU(2)<sub>L</sub>-bulk, U(1)<sub>Y</sub>-brane model, where only the U(1)<sub>Y</sub> gauge field is confined to the brane. The strongest bounds hold for the often-discussed bulk-bulk model no matter if the Higgs boson is living in the bulk or on the brane. For the bulk-brane models, we observe that the combined bounds on  $1/R$  are reduced by roughly 20 to 30%.

The lower limits on the compactification scale derived by the present global analysis indicate that resonant production of the first KK state may be at the edge of the LHC

reach, at which heavy KK masses up to 6–7 TeV [7, 12] might be explored. One probably will not be able to probe resonant effects originating from the second KK state, and so more phenomenological work has to be done to differentiate the model from other 4-dimensional new-physics scenaria.

In addition, we have paid special attention to consistently quantize the higher-dimensional models in the generalized  $R_\xi$  gauges. Specifically, we have been able to identify the appropriate higher-dimensional gauge-fixing conditions which should be imposed on the theories so as to yield the known  $R_\xi$  gauge after the fifth dimension has been integrated out [15, 23].

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