

Extended Higgs Sectors

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Abstract

I give a brief overview of the motivations for and experimental probes of extended Higgs sectors containing more than the single Higgs doublet field of the Standard Model.

We are now approaching the 40th anniversary of the introduction [1, 2] of the idea of a Higgs mechanism for electroweak symmetry breaking and mass generation using an elementary scalar field [3]. Remarkably, we still have no experimental information that either confirms or excludes the elementary scalar Higgs boson(s) that would arise if this mechanism is correct. All current experimental data is consistent with the Standard Model (SM) and its single CP-even Higgs boson (h_{SM}) provided $m_{h_{\text{SM}}} \lesssim 200$ GeV, for consistency with precision electroweak analyzes [4], and $m_{h_{\text{SM}}} \gtrsim 114$ GeV, for consistency with direct limits from LEP2 [5]. The Tevatron and LHC will be probing the remaining allowed $m_{h_{\text{SM}}}$ mass region over the next decade. However, there are many theoretical reasons supporting the idea that the SM with its simple one-doublet Higgs sector is not the whole story. In particular, the SM has difficulty explaining a light h_{SM} (the naturalness and hierarchy problems) and does not predict gauge coupling unification. Thus, theorists have spent many years constructing extensions of the SM that rectify these two inadequacies. Most involve an extension of the Higgs sector to include at least two Higgs doublets, and perhaps singlets and/or triplet and other representations. Here, I will survey some of the ideas and associated experimental challenges. The most important conclusion will be that only a combination of the LHC and a future linear collider (LC) is guaranteed to find a Higgs boson and that full exploration of the Higgs sector might require both machines plus a $\gamma\gamma$ collider facility (γC) at the LC, and possibly a muon collider (μC).

We begin with a discussion of whether or not we need supersymmetry for gauge coupling unification and a solution to the fine-tuning problem. This will lead to some specific topics regarding complicated Higgs sectors and extra-dimensional theories. We then consider what it will take to fully explore the Higgs sector of the minimal supersymmetric model (MSSM), the scalar sector for which is a highly constrained two-Higgs-doublet model (2HDM). Next, we review the strongly motivated next-to-minimal supersymmetric model (NMSSM) in which one singlet superfield is added to the MSSM, and the experimental consequences of the extra Higgs bosons that arise. The fully general case of multiple Higgs singlets, as often found in string-motivated models, is then considered. We end with a reminder regarding the somewhat overlooked, but interesting, left-right supersymmetric models that allow for automatic solutions of several important problems.

1 Extensions of the SM Higgs sector

In this section, we discuss various models and their motivations in which the only extension of the SM is the addition of more Higgs representations to the Higgs sector.

Table 1: Choices of Higgs representations that allow for coupling constant unification without any other extension of the SM. $N_{T,Y}$ is the number of representations with indicated T and Y . The tabulated $\alpha_s(m_Z)$ values are those that allow for unification at the tabulated M_U scales.

$N_{1/2,1}$	$N_{1/2,3}$	$N_{0,2}$	$N_{0,4}$	$N_{1,0}$	$N_{1,2}$	$\alpha_s(m_Z)$	M_U (GeV)
1	0	0	2	0	0	0.106	4×10^{12}
1	0	4	0	0	1	0.112	7.7×10^{12}
1	0	0	0	0	2	0.120	1.6×10^{13}
2	0	0	0	1	0	0.116	1.7×10^{14}
2	0	2	0	0	2	0.116	4.9×10^{12}
2	1	0	0	0	2	0.112	1.7×10^{12}
3	0	0	0	0	1	0.105	1.2×10^{13}

Motivations from coupling constant unification

Coupling constant unification can be achieved simply by introducing additional Higgs representations in the SM [6, 7]. For $\rho = 1$ to be natural, the neutral field member (if there is one) of representations other than $T = 1/2, |Y| = 1$ should have zero vacuum expectation value (vev) [8]. Some simple choices for representations that yield coupling unification are shown in Table 1. There, $N_{T,Y}$ gives the number of representations with the indicated weak-isospin T and hypercharge Y . From the table, we observe that achieving coupling constant unification in this way requires a lower unification scale, M_U , than comfortable for proton decay. This need not be a problem if the coupling unification is not associated with true group unification (*i.e.* if there are no extra X, Y gauge bosons to mediate proton decay), as is possible in some string theory models. The solution with the largest M_U is $N_{1/2,1} = 2$ and $N_{1,0} = 1$. Many of the most attractive solutions contain several doublets and one or more triplets. With sufficiently complicated Higgs sectors, we can even achieve coupling unification at very low M_U values, as possibly appropriate in large-scale extra-dimension models. Another motivation for models with two or more Higgs doublets is that both explicit and spontaneous CP violation in the Higgs sector is possible (see, for example, [3, 9, 10]). Of course, once one has two or more doublets in the Higgs sector, there will be many Higgs potential parameters. These must be constrained so that the potential minimum is such that all Higgs bosons have positive mass-squared. In particular, $m_{H^\pm}^2 > 0$ is required in order to avoid breaking of electromagnetism.

The light CP-odd Higgs boson scenario in a general two-Higgs-doublet model

Even the simple CP-conserving 2HDM extension of the SM Higgs sector allows for some unusual scenarios. In particular, suppose that the A^0 of the 2HDM is moderately light and all other Higgs bosons are heavy. Remarkably, this type of scenario can be consistent with precision electroweak constraints [11]. If m_{A^0} is small, the best fit to the precision electroweak data is achieved by choosing the lighter CP-even Higgs boson, h^0 , to be SM-like. A good fit is possible even for $m_{h^0} \sim 1$ TeV. Of course, such a heavy SM-like h^0 leads to large $\Delta S > 0$ and large $\Delta T < 0$ contributions, which on their own would place the S, T prediction of the 2HDM model well outside the current 90% CL ellipse — see the stars in Fig. 1, taken from [12]. However, the large $\Delta T < 0$ contribution from the

SM-like h^0 can be compensated by a large $\Delta T > 0$ from a small mass non-degeneracy (weak isospin breaking) of the still heavier H^0 and H^\pm Higgs bosons. In detail, for a moderate light A^0 (roughly $m_{A^0} \lesssim \frac{1}{2}m_{h^0}$) and SM-like h^0 one finds

$$\Delta\rho = \frac{\alpha}{16\pi m_W^2 c_W^2} \left\{ \frac{c_W^2}{s_W^2} \frac{m_{H^\pm}^2 - m_{H^0}^2}{2} - 3m_W^2 \left[\log \frac{m_{h^0}^2}{m_W^2} + \frac{1}{6} + \frac{1}{s_W^2} \log \frac{m_W^2}{m_Z^2} \right] \right\} \quad (1)$$

from which we see that the first term can easily compensate the large negative contribution to $\Delta\rho$ from the $\log(m_{h^0}^2/m_W^2)$ term. In Fig. 1, the blobs correspond to 2HDM parameter choices for which: (a) $m_{h^0} = \sqrt{s}$ (either 500 GeV or 800 GeV) of a linear e^+e^- collider (*i.e.* m_{h^0} is such that the h^0 cannot be observed at the LC); (b) $m_{H^\pm} - m_{H^0} \sim \text{few GeV}$ has been chosen (with both $m_{H^\pm}, m_{H^0} \gtrsim 1$ TeV) so that the S, T prediction is well within the 90% CL ellipse of the precision electroweak fits; and (c) m_{A^0} and $\tan\beta$ are in the ‘wedge’ of $[m_{A^0}, \tan\beta]$ parameter space characterized by moderate $\tan\beta$ values and $m_{A^0} \gtrsim 250$ GeV for which the LHC and e^+e^- LC operation at $\sqrt{s} = 500$ GeV or 800 GeV would not allow discovery of the A^0 through $b\bar{b}A^0$ or $t\bar{t}A^0$ production [13] (see also [14]) and the LC $e^+e^- \rightarrow ZA^0A^0$ and $e^+e^- \rightarrow \nu\bar{\nu}A^0A^0$ rates are too small to be detected (as is the case for $m_{A^0} \gtrsim 150$ GeV at $\sqrt{s} = 500$ GeV and $m_{A^0} \gtrsim 270$ GeV at $\sqrt{s} = 800$ GeV) [15, 16, 20]. However, this scenario can only be pushed so far. In order to maintain perturbativity for all the Higgs self couplings, it is necessary that the quartic couplings of the 2HDM potential obey $|\lambda_i|/(4\pi) \lesssim \mathcal{O}(1)$ [10, 17–19]. This in turn implies that the h^0, H^0 and H^\pm masses should obey $m_{h^0}, m_{H^0}, m_{H^\pm} \sim |\lambda_i|^{1/2}v \lesssim 800 - 900$ GeV. This bound on m_{h^0} also ensures the absence of strong WW scattering — see [3]. Thus,

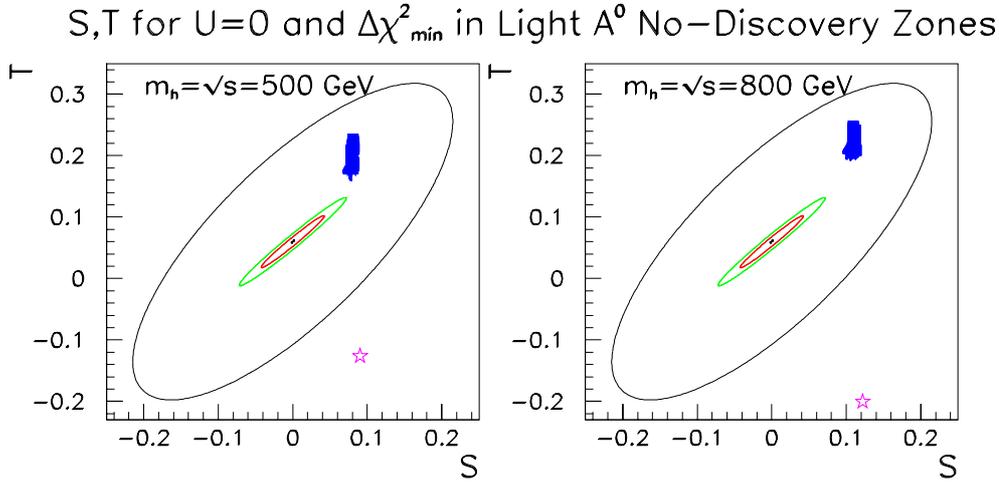


Figure 1: The outer ellipses show the 90% CL region from current precision electroweak data in the S, T plane for $U = 0$ relative to a central point defined by the SM prediction with $m_{h_{\text{SM}}} = 115$ GeV. The blobs of points show the S, T predictions for 2HDM models with a light A^0 and with $\tan\beta$ such that the A^0 cannot be detected in $b\bar{b}A^0$ or $t\bar{t}A^0$ production at either the LC or the LHC; the mass of the SM-like h^0 is set equal to $\sqrt{s} = 500$ GeV (left) or 800 GeV (right) and m_{H^\pm} and m_{H^0} have been chosen to minimize the χ^2 of the full precision electroweak fit. The innermost (middle) ellipse shows the 90% (99.9%) CL region for $m_{h_{\text{SM}}} = 115$ GeV after Giga- Z LC operation and a $\Delta m_W \lesssim 6$ MeV threshold scan measurement. The stars to the bottom right show the S, T predictions in the case of the SM with $m_{h_{\text{SM}}} = 500$ GeV (left) or 800 GeV (right). This figure is from [12].

the SM-like h^0 would be detected at the LHC. If it should happen that a heavy SM-like Higgs boson is detected at the LHC, but no other new particles (supersymmetric particles, additional Higgs bosons, *etc.*) are observed, the precision electroweak situation could only be resolved by Giga- Z operation and a $\Delta m_W = 6$ MeV WW threshold scan at the LC (yielding the 90% CL Giga- Z ellipse sizes illustrated in Fig. 1). The resulting determination of S, T would be sufficiently precise to definitively check for values like those of the blobs of Fig. 1. If no other new physics was detected at the LC or LHC that could cause the extra $\Delta T > 0$, searching for the other Higgs bosons of a possible 2HDM Higgs sector, especially a relatively light decoupled A^0 , would become a high priority. Interestingly, the current discrepancy with SM predictions for a_μ can be explained in whole or part * by two-loop diagrams involving a light A^0 [21, 22].

Special cases in which Higgs discovery would be complicated and/or difficult

Some additional complications that would make Higgs discovery more difficult in the case of the general 2HDM or a still more extended Higgs sector are:

- (i) The Higgs sector could be CP-violating.

Both spontaneous and explicit CP-violation is possible for a general 2HDM (see, *e.g.* [3, 9, 10]). If CP-violation is present, the three neutral Higgs bosons mix to form three mass eigenstates of mixed CP nature, $h_{1,2,3}$, which share the WW/ZZ coupling strength squared: $\sum_i g_{VVh_i}^2 = g_{VVh_{SM}}^2$. In this case, the signal for any one of them would be weakened, perhaps dramatically so. Such sharing would be particularly devastating for the LHC $gg \rightarrow h_i \rightarrow \gamma\gamma$ signals. While this would reduce the LC $e^+e^- \rightarrow Zh_i$ signals, the above sum rule and the fact that the h_i with large $g_{VVh_i}^2$ would need to be light ($\lesssim 200$ GeV) in order to agree with precision electroweak data (modulo the type of special situation described in the previous subsection) imply that at least one of the signals would always be visible.

- (ii) The (possibly mixed) Higgs bosons could be sufficiently close in mass that their resonance peaks, which have finite (decay-channel-dependent) width because of experimental resolution, would overlap.

At the LC, this would smear out the $e^+e^- \rightarrow Zh_i$ signals. As discussed later, the Higgs signal would still be revealed as a broad enhancement (from the composite of the overlapping signals) in the M_X spectrum observed in $e^+e^- \rightarrow ZX$ events. At the LHC, if the $gg \rightarrow h_i \rightarrow \gamma\gamma$ signal for one or more of the h_i is of observable strength, the excellent experimental $m_{\gamma\gamma}$ resolution would make it likely that each individual signal could be seen. However, this would not be the case for $h_i \rightarrow \tau^+\tau^-$ and $b\bar{b}$ discovery channels where the experimental resolution is not very good. The related Higgs signals would be very difficult to extract.

- (iii) All the Higgs bosons with substantial $g_{VVh_i}^2$ could decay to a pair of lighter Higgs bosons or to a light Higgs boson and a gauge boson. For example, in the CP-conserving case, we could have large $h^0, H^0 \rightarrow A^0 A^0$ branching ratios. At the LHC,

*For the rather low m_{A^0} and high $\tan\beta$ values required in order that the A^0 be the *full* explanation of the discrepancy, the A^0 would be seen at the LC in $e^+e^- \rightarrow ZA^0 A^0$ and $e^+e^- \rightarrow b\bar{b}A^0$ production if not earlier at the LHC.

this would greatly weaken the $gg \rightarrow h^0, H^0 \rightarrow \gamma\gamma$ signals, which then might not be detectable. The $WW \rightarrow h^0, H^0 \rightarrow \tau^+\tau^-, b\bar{b}$ signals would also be very weak. Searches for the CP-even Higgs bosons would have to rely on the $A^0A^0 \rightarrow \tau^+\tau^-\tau^+\tau^-$ and $\tau^+\tau^-b\bar{b}$ final states, which have not been shown to lead to observable signals at the LHC. Existing LHC studies [23, 24] suggest that single A^0 detection in the $\tau^+\tau^-$ or $b\bar{b}$ final states would be very difficult. In contrast, the h^0 or H^0 would be detectable at the LC using the $e^+e^- \rightarrow ZX$ search for a resonant bump in M_X . Once found, the A^0A^0 decays of the h^0 and H^0 could be studied.

Finally, there is nothing to rule out a combination of the above difficulties. In such a case, only the LC would have the ability to detect at least one of the Higgs bosons of the general 2HDM. Still more complicated Higgs sectors, for example one containing many doublets and a number of singlets, would lead to still greater difficulties. A common theme in all the above scenarios, and in the preceding light- A^0 2HDM scenario, is the probable importance of directly detecting a light A^0 . As outlined later, the γC and μC are likely to be the machines of choice for this purpose unless $\tan\beta$ is very large. More detailed examples of these general complexities/difficulties will be outlined in the supersymmetric Higgs portion of this talk.

Triplet Higgs representations

$SU(2)_L$ triplet Higgs representations (Δ_L) with zero vev for their neutral members have significant motivation. They are especially well-motivated in the context of left-right (LR) symmetric and related models (see [3] for discussion and references). The 2×2 notation for a $T = 1, |Y| = 2$ triplet is $\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$. The non-supersymmetric LR model contains both Δ_R and Δ_L triplets. A Majorana lepton-number-violating coupling is introduced for the Δ_R so that a large $\langle\Delta_R^0\rangle$ will yield a large Majorana ν_R mass as well as large m_{W_R} . The LR symmetry requires that the Δ_L triplet have an equivalent lepton-number-violating coupling. However, symmetry breaking can be arranged so that $\langle\Delta_L^0\rangle = 0$ (to keep $\rho = 1$ natural). More generally, there is no reason not to consider the possibility of a Δ_L with $\langle\Delta_L^0\rangle = 0$. Coupling constant unification can be arranged in models with triplets, but might require other types of matter fields at intermediate scales and/or extra dimensional physics [25–27].

The interesting phenomenology of triplets is illustrated by focusing on the case of a $|Y| = 2$ triplet representation, for which the lepton-number-violating coupling Lagrangian is:

$$\mathcal{L}_Y = ih_{ij}\psi_i^T C\tau_2\Delta\psi_j + \text{h.c.}, \quad i, j = e, \mu, \tau. \quad (2)$$

which leads to lepton-number-violating $e^-e^-, \mu^-\mu^-, \tau^-\tau^- \rightarrow \Delta^{--}$ couplings. If we write $|h_{\ell\ell}^{\Delta^{--}}|^2 \equiv c_{\ell\ell}m_{\Delta^{--}}^2$ (GeV), the strongest limits are $c_{ee} < 10^{-5}$ (from Bhabha scattering) and $c_{\mu\mu} < 10^{-6}$ [noting that a triplet gives the wrong sign for the observed $(g-2)_\mu$ deviation]. For $\langle\Delta_L^0\rangle = 0$, $\Gamma_{\Delta_L^{--}}^T$ would be small and $\Delta_L^{--} \rightarrow \ell^-\ell^-$ decays could dominate. For $m_{\Delta_L^{--}} \lesssim 1$ TeV, one would discover the Δ_L^{--} in $pp \rightarrow \Delta_L^{--}\Delta_L^{++}$ with $\Delta_L^{--} \rightarrow \ell^-\ell^-, \Delta_L^{++} \rightarrow \ell^+\ell^+$ ($\ell = e, \mu, \tau$) at the LHC (or earlier at the Tevatron if $m_{\Delta_L^{--}} \lesssim 350$ GeV) [28]. Thus, the pp colliders will tell us if such a Δ_L^{--} exists in the mass range accessible to a LC or possible μC and how it decays. However, only the relative $c_{\ell\ell}$

values for those ℓ 's observed in $\Delta_L^{--} \rightarrow \ell^- \ell^-$ decays could be determined. The next step would be to produce and study the Δ_L^{--} in $\ell^- \ell^-$ s -channel collisions. If c_{ee} ($c_{\mu\mu}$) is near its current upper limit, event rates in $e^- e^-$ ($\mu^- \mu^-$) collisions would be enormous [6,7] for the expected small values of $\Gamma_{\Delta_L^{--}}^T$ and would provide a direct measure of the corresponding $c_{\ell\ell}$. Since backgrounds are very small, observable signals would be present for even very small $c_{\ell\ell}$ — *e.g.* a c_{ee} value as small as $\sim 10^{-16}$ could be probed at an $e^- e^-$ collider with $L = 300 \text{ fb}^{-1}$. This would cover essentially the entire range of coupling relevant for the see-saw mechanism.

2 Higgs-radion mixing in the Randall-Sundrum model

Although models in which only the Higgs sector of the SM is extended lead to interesting new phenomenological possibilities, they do not solve the hierarchy problem — there is no natural reason for Higgs boson masses to be below $\sim 1 \text{ TeV}$. Large-scale extra dimensions appear to be required in order to solve the hierarchy problem without the introduction of supersymmetry. One model of this type is the Randall-Sundrum (RS) model [29], wherein a single extra (5th) dimension is introduced with a warped metric between two 3-branes (*i.e.* branes with 3 spatial dimensions and 1 time dimension). In the simplest version, all SM fields are confined to the “visible” brane; only gravity propagates in the 5th dimension. The TeV scale on the visible brane arises as an exponential suppression warp factor times the Planck scale on the “invisible” brane. The RS approach gives rise to many fascinating new phenomena. Of particular interest are the possibly dramatic implications of such a model for the Higgs sector. If all matter (in particular the one Higgs doublet of the SM) is on the TeV brane, the most interesting deviations from SM Higgs physics arise if there is mixing of the Higgs doublet with the radion [30–37]. The mixing arises from the allowed action form:

$$S_\xi = \xi \int d^4x \sqrt{g_{\text{vis}}} R(g_{\text{vis}}) \widehat{H}^\dagger \widehat{H}. \quad (3)$$

Here, $R(g_{\text{vis}})$ is the Ricci scalar for the metric induced on the visible brane, \widehat{H} is the Higgs field (before rescaling to canonical normalization on the brane), and $g_{\text{vis}}^{\mu\nu} = \Omega_b^2(x)(\eta^{\mu\nu} + \epsilon h^{\mu\nu})$, where the quantum fluctuations in $\Omega_b(x)$ define the radion field (before rescaling) and the $h^{\mu\nu}$ are the fluctuations about the locally flat 4-d metric. For $\xi \neq 0$, the Higgs and radion mix and one must re-diagonalize and rescale to canonically-normalized mass eigenstates h and ϕ .

The basic parameters determining the Higgs-radion phenomenology are m_h , m_ϕ , Λ_ϕ (the new physics scale characterizing the radion interactions) and ξ . A complicated inversion process relates these to the bare parameters of the Lagrangian needed to compute the couplings of the h and ϕ . We very briefly outline the consequences of $\xi \neq 0$ as obtained in [36]. While it is possible to have $m_h \sim 112 \text{ GeV}$ (*i.e.* somewhat below the SM lower limit of 114 GeV) without violating LEP constraints on g_{ZZh}^2 , let us focus on the case of $m_h = 120 \text{ GeV}$. The h and ϕ will typically be detected in the same modes as have been studied for the SM Higgs boson. For allowed ξ values, the h and ϕ discovery mode rates at the LHC and at the LC can be dramatically different as compared to the rates predicted for a h_{SM} of the same mass. This is illustrated in Fig. 2. This figure shows that

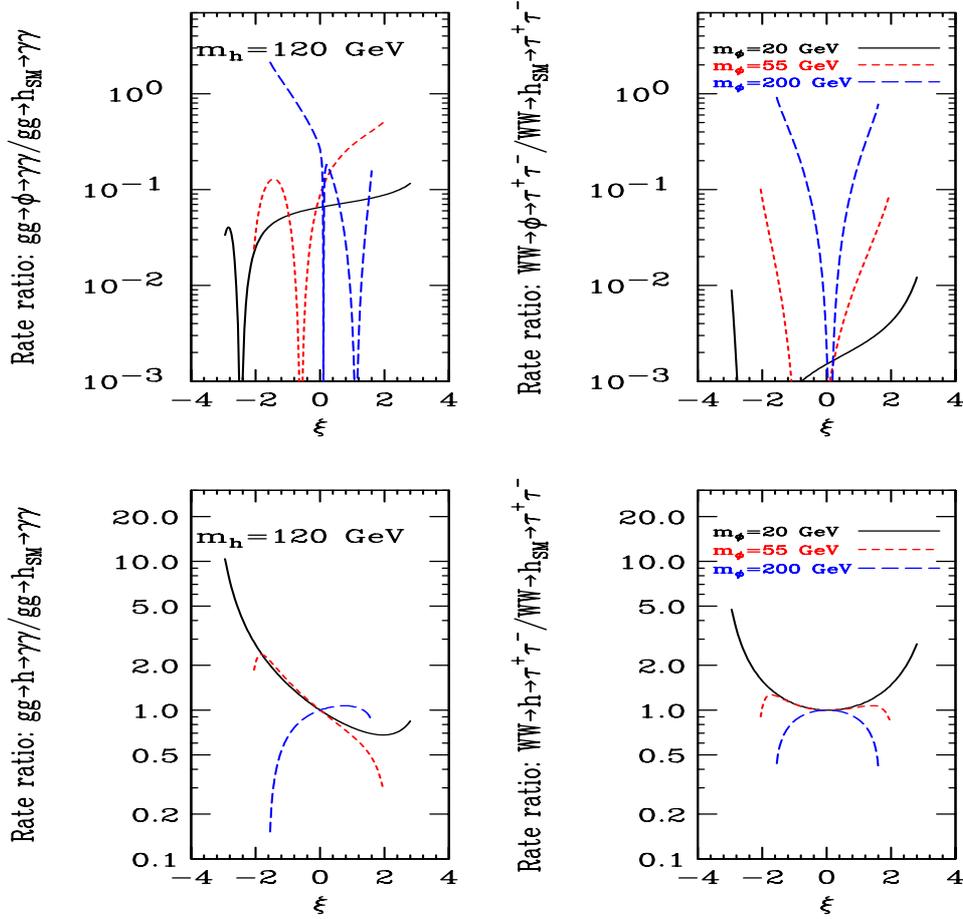


Figure 2: The ratio of important LHC h and ϕ production/decay discovery channels to the prediction for the h_{SM} as a function of the mixing parameter ξ , assuming $m_h = 120$ GeV and for $m_\phi = 20, 55, 200$ GeV. The h (ϕ) comparisons assume $m_{h_{SM}} = m_h$ ($m_{h_{SM}} = m_\phi$), respectively. The upper and lower limit for ξ of each curve is determined by theoretical constraints within the RS model. From [36].

for most values of ξ the ϕ rates will be much smaller than expected for the h_{SM} (when $m_{h_{SM}} = m_\phi$). However, for some values of ξ the LHC rates for the ϕ are closer to being SM-like than those of the h . Typically, a LC will be required to fully unravel what is going on. Where rates in the plotted LHC discovery modes are small for the ϕ , it could be that the LHC would still be able to discover the ϕ in $h \rightarrow \phi\phi \rightarrow b\bar{b}b\bar{b}, b\bar{b}g\bar{g}$ decays. The decay $h \rightarrow \phi\phi$ can have a sizable branching ratio and would provide a definitive signature that $\xi \neq 0$ mixing is present. If this and other LHC signatures for the ϕ are too weak to be detected, an LC will be needed to discover the ϕ . Indeed, an LC with $L = 500 \text{ fb}^{-1}$ can detect $e^+e^- \rightarrow Z^* \rightarrow Z\phi$ events for very small $g_{ZZ\phi}^2$ values, the precise limit depending upon m_ϕ . The only part of parameter space for which the LC could not detect the ϕ is in the vicinity of a line in (ξ, Λ_ϕ) parameter space where $g_{ZZ\phi}^2 = g_{ff\phi}^2 = 0$. This illustrates the importance of the LC to a full exploration of the RS Higgs-radion sector. For many choices of parameters, a γC would be extremely valuable for sorting out the Higgs-radion sector [38]. This is because one of the most characteristic features of the RS model is the presence of anomalous $h \rightarrow \gamma\gamma, gg$ and $\phi \rightarrow \gamma\gamma, gg$ couplings that can only be extracted in a model-independent manner using $gg \rightarrow h, \phi \rightarrow \gamma\gamma$ and $\gamma\gamma \rightarrow h, \phi \rightarrow b\bar{b}$ measurements.

The RS model does have some undesirable features. In particular, there is the new fine-tuning problem of adjusting cosmological constants on the branes and in the bulk to have exactly the right relationships. A more fundamental source for these relationships has yet to be demonstrated. Coupling unification is also problematical in that the couplings would only appear to unify (via logarithmic running) at the 4-d Planck scale or typical GUT scale if there is matter off the brane [39–41].

3 Higgs sectors in supersymmetric models

Supersymmetry is still viewed as the best approach to solving the hierarchy and naturalness problems and no other model yields coupling unification, and also electroweak symmetry breaking, in such an inherently natural way. Thus, the balance of the talk will focus on how well we can explore a supersymmetric model Higgs sector. We will begin with the MSSM and then move to the NMSSM and to LR-symmetric supersymmetric models.

MSSM Higgs sector highlights

In the case of the MSSM Higgs sector (as reviewed, for example, in [3, 42–44]), the key issue is the extent to which we will be able to completely explore the Higgs sector at the Tevatron, LHC, and future LC. The discussion here will assume the maximal-mixing scenario with a SUSY scale of 1 TeV, and the absence of CP violation in the Higgs sector.

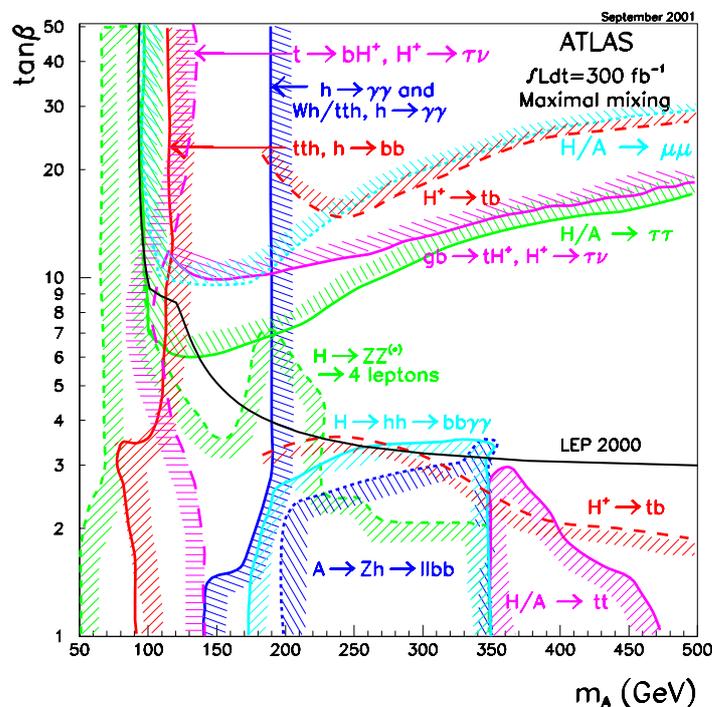


Figure 3: 5σ discovery contours for MSSM Higgs boson detection in various channels are shown in the $[m_{A^0}, \tan \beta]$ parameter plane, assuming maximal mixing and $L = 300 \text{ fb}^{-1}$ for the ATLAS detector [46].

In this case, the light CP-even h^0 has mass $m_{h^0} < 135$ GeV. Assuming that the CP-odd Higgs boson has mass $m_{A^0} \gtrsim 200$ GeV (as is probable, given typical renormalization group evolution scenarios for electroweak symmetry breaking), the Higgs sector will be in the decoupling regime [10]. In this regime, $m_{H^0} \sim m_{A^0} \sim m_{H^\pm}$, the h^0 has nearly SM-like properties, while the H^0 will have weak WW, ZZ couplings. Consequently, the h^0 is the most experimentally accessible Higgs boson of the MSSM. At the Tevatron (see [45]), integrated luminosity of order 15 fb^{-1} (20 fb^{-1}) is required to detect the h^0 if $m_{A^0} \lesssim 250$ GeV ($m_{A^0} \lesssim 400$ GeV). For $m_{A^0} = 150$ GeV (200 GeV) and $L = 15 \text{ fb}^{-1}$, the A^0, H^0 will be detected in $b\bar{b}H^0 + b\bar{b}A^0$ production if $\tan\beta > 35$ ($\tan\beta > 50$). The LHC (see [23, 24] for the CMS and ATLAS studies) is guaranteed to find one of the MSSM Higgs bosons with $L = 300 \text{ fb}^{-1}$ (roughly three years of high-luminosity operation), but there is a significant wedge of moderate $\tan\beta$ where only the h^0 will be detected unless SUSY decays of the heavier Higgs bosons have substantial branching fraction. This is illustrated by the ATLAS plot of Fig. 3. Similar results have been obtained by the CMS collaboration [23, 47].

At a LC, the h^0 will be detected using the same production/decay modes as for a light h_{SM} . In particular, the $e^+e^- \rightarrow Zh^0$ and $e^+e^- \rightarrow \nu\bar{\nu}h^0$ (Higgstrahlung and WW fusion) processes will yield tens of thousands of h^0 's per year. However, the H^0, A^0, H^\pm might not be detected in e^+e^- collisions at the LC [13]. First, the $e^+e^- \rightarrow H^0A^0$, $e^+e^- \rightarrow H^+H^-$, $e^+e^- \rightarrow ZA^0A^0$ and $e^+e^- \rightarrow \nu\bar{\nu}A^0A^0$ production mechanisms would be forbidden for $m_{A^0} \gtrsim \sqrt{s}/2$. Second, while for very high (low) $\tan\beta$ it will be possible to detect $e^+e^- \rightarrow b\bar{b}A^0, b\bar{b}H^0, tbH^\pm$ ($e^+e^- \rightarrow t\bar{t}A^0, t\bar{t}H^0, tbH^\pm$) if not too near the relevant kinematic threshold, there will be a wedge region of moderate $\tan\beta$ in which $t\bar{t}H^0 + t\bar{t}A^0$ and $b\bar{b}H^0 + b\bar{b}A^0$ both produce too few events for detection. Assuming a substantial increase in the LC \sqrt{s} is many years in the future, implementation of the γC would be called for in order to make possible the direct observation of the H^0, A^0 (through $\gamma\gamma \rightarrow H^0 + \gamma\gamma \rightarrow A^0$) [38, 48–50] for m_{A^0} up to $\sim 0.8\sqrt{s}$. A μC with the same energy as the LC would be able to detect the H^0, A^0 via the s -channel $\mu^+\mu^- \rightarrow H^0, A^0 \rightarrow b\bar{b}, \tau^+\tau^-, t\bar{t}$ resonance signals using an appropriately designed energy scan procedure [51–53].

In considering the γC option, there are two distinct scenarios. If precision h^0 measurements give a first indication of the presence of the A^0, H^0 and a rough determination of $m_{A^0} \sim m_{H^0}$ (both of which require knowing other MSSM parameters sufficiently well to determine the size of the one-loop corrections [54] to the $b\bar{b}h^0$ coupling and the extent to which premature or “exact” decoupling [10] is present), then the γC could be set up to yield a $\gamma\gamma$ luminosity spectrum peaked in the region of the expected $m_{A^0} \sim m_{H^0}$ value. Less than one year’s luminosity is needed for direct detection if you know m_{A^0} within ~ 50 GeV (so that only two or three different settings of \sqrt{s} are needed to explore the interval) [49]. However, if there is no indirect m_{A^0} determination, or if there is reason to mistrust the indirect determination (not an easy thing to assess because of the possibility of large corrections to the $b\bar{b}h^0$ coupling and/or premature decoupling), the preferred approach would be to operate at the highest \sqrt{s} available using several different γC configurations. To illustrate, we summarize the results of [49], as reanalyzed in [38], where it is supposed that the LC has $\sqrt{s} = 630$ GeV. Direct $e^+e^- \rightarrow H^0A^0$ production is assumed to exclude $m_{A^0} \sim m_{H^0} \lesssim 300$ GeV. The largest $\gamma\gamma$ energy for which good $\gamma\gamma$ luminosity could be achieved is $E_{\gamma\gamma} \sim 0.8\sqrt{s} \sim 500$ GeV. To search in

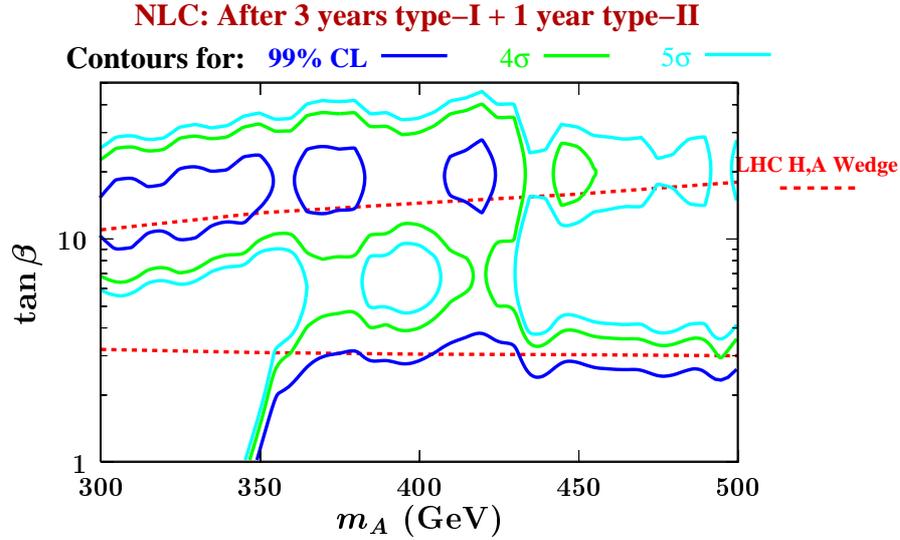


Figure 4: Regions for H^0, A^0 discovery and exclusion at a $\sqrt{s} = 630$ GeV LC based on the NLC/Livermore design, assuming 3 years of operation in type-I configuration and 1 year in type-II configuration. Higher luminosity, as possibly achievable at TESLA, would be helpful to ensure coverage. Results are from [38, 49].

the full range between 300 and 500 GeV, the optimal approach is to employ two different configurations (**I** and **II**, as defined in Ref. [49]) for the electron helicity / laser-photon polarizations. The **Type-II** configuration yields a $E_{\gamma\gamma}$ luminosity spectrum peaked at the high end and would be used to search the $m_{A^0} \sim m_{H^0} \in [450, 500]$ GeV interval. The **Type-I** configuration yields a broader $E_{\gamma\gamma}$ spectrum with ability to probe a range of lower masses, $m_{A^0} \sim m_{H^0} \in [300, 450]$ GeV. Both spectra types have substantial $\langle\lambda\lambda'\rangle$ of the back-scattered photons in the indicated mass regions, as needed to suppress the $\gamma\gamma \rightarrow b\bar{b}$ background to the $\gamma\gamma \rightarrow H^0, A^0 \rightarrow b\bar{b}$ signal. Using this approach, Fig. 4 shows that a γC based on the American/Asian NLC design could detect the H^0, A^0 throughout most of the LHC wedge region at the 4σ level, and exclude their presence at the 99%CL throughout the entire wedge, after about four years of operation. Thus, if a light CP-even Higgs boson is detected at the LHC and LC, but no heavier Higgs bosons, and if there are SUSY signals at the LHC and LC consistent with moderate $\tan\beta$, a $\gamma\gamma$ collider becomes mandatory in the absence of a timely upgrade of the LC to higher \sqrt{s} .

Determining $\tan\beta$ in the MSSM using heavy Higgs bosons

One of the most important parameters of the MSSM is $\tan\beta$. While some measurements of $\tan\beta$ will be possible using gaugino and slepton production, measurements of the Yukawa couplings of the H^0, A^0, H^\pm provide the most direct measurement of the ratio of vacuum expectation values that defines $\tan\beta$. This is because in the decoupling regime the Yukawa couplings behave as $t\bar{t}H^0, t\bar{t}A^0 \propto \cot\beta$ and $b\bar{b}H^0, b\bar{b}A^0 \propto \tan\beta$.

Simple observables sensitive to these couplings at a LC are: a) the rate for $b\bar{b}A^0, b\bar{b}H^0 \rightarrow b\bar{b}b\bar{b}$; b) the $H^0A^0 \rightarrow b\bar{b}b\bar{b}$ rate; c) a measurement of the average H^0, A^0 total width in H^0A^0 production; d) the $H^+H^- \rightarrow t\bar{t}t\bar{t}$ rate; and e) the total H^\pm width measured in $H^+H^- \rightarrow t\bar{t}t\bar{t}$ production. Because of limited experimental resolution for the width

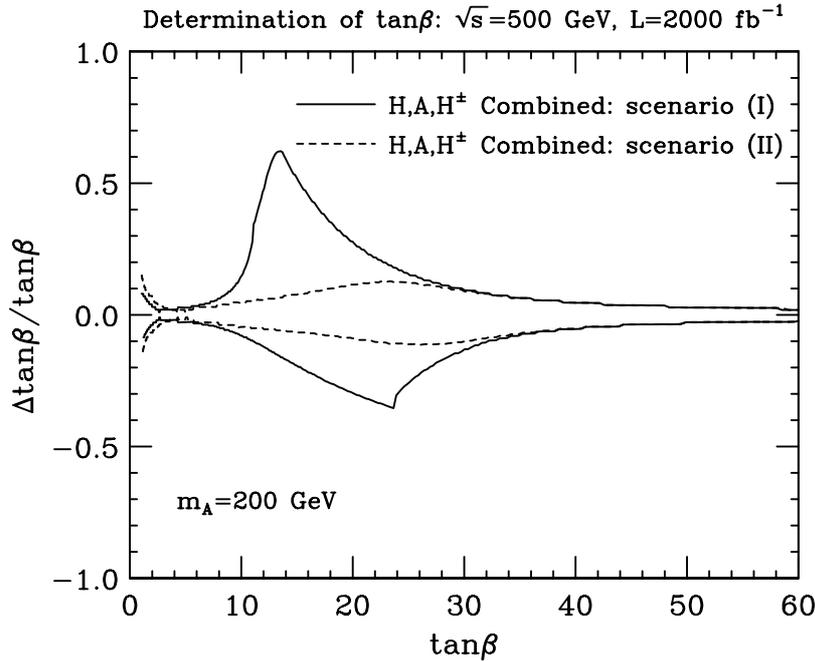


Figure 5: For the MSSM with $m_{H^\pm} \sim m_{A^0} = 200$ GeV, and assuming a LC with $L = 2000 \text{ fb}^{-1}$ at $\sqrt{s} = 500$ GeV, we plot the 1σ statistical upper and lower bounds in terms of $\Delta \tan\beta / \tan\beta$ as a function of $\tan\beta$ based on combining (in quadrature) the results from the channels listed in the text. Results are shown for two SUSY scenarios; in (I) SUSY decays of the H^0 and A^0 are not present; in (II) $H^0, A^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ decays are substantial. Results are taken from [55].

measurements, the width determinations of $\tan\beta$ are only good at high $\tan\beta$ where the intrinsic widths are large. The rate determinations are typically only accurate at lower $\tan\beta$ values for which there is substantial variation of the $H^0, A^0 \rightarrow b\bar{b}$ and $H^\pm \rightarrow tb$ branching ratios. If SUSY decays of the H^0, A^0 are present, this variation will persist to higher $\tan\beta$ values. The errors on $\tan\beta$ resulting from combining a)-e) above are shown in Fig. 5, from [55].

We note that $\gamma\gamma \rightarrow H^0, A^0$ rates also provide a reasonably good $\tan\beta$ determination [49] and would be the only way of assessing the H^0, A^0 Yukawa coupling strengths if the $[m_{A^0}, \tan\beta]$ parameter set lies in the wedge region.

Determining $\tan\beta$ at the LHC on the basis of heavy Higgs production rates has been discussed in [24, 42, 55]. The LHC determination may be superior in the $\tan\beta$ range from roughly 10 to 25 where the errors from the LC determination, illustrated in Fig. 5, are largest. Most probably, the width technique for determining $\tan\beta$ will not work at the LHC except for really large $\tan\beta$ values. This is because the $H^0, A^0 \rightarrow \tau^+\tau^-$ channel (which is detectable in $b\bar{b}H^0 + b\bar{b}A^0$ production further down in $\tan\beta$ than other channels) cannot be used for direct width reconstruction because of the poor experimental width resolution, $\sim 15\%$. Once $\tan\beta$ is very large, detection of $b\bar{b}H^0 + b\bar{b}A^0$ production with $H^0, A^0 \rightarrow b\bar{b}$ will become possible, but even better, the $H^0, A^0 \rightarrow \mu^+\mu^-$ decays will become visible and provide an excellent intrinsic width measurement. Detailed studies have not been performed.

A CP-violating MSSM Higgs sector

Generically, it is certainly possible that the soft-SUSY-breaking parameters of the MSSM are complex. If so, the one-loop corrections to the Higgs tree-level potential can give rise to CP violation in the Higgs sector. In this case, the MSSM Higgs sector becomes rather similar to the CP-violating 2HDM, except that there is still an upper bound on the mass of the lightest of the three neutral Higgs bosons, $h_{1,2,3}$. For this situation it is convenient to use m_{h_1} (in place of m_{A^0} for the CP conserving case), $\tan\beta$ and a CP-violation angle ϕ to parameterize the Higgs sector of the MSSM. A recent study [56] examines a particular MSSM scenario of this type, dubbed the CPX benchmark scenario, for which CP violation in the Higgs sector can be substantial without having electric dipole moments (EDM's) that violate current bounds. As anticipated in our discussion of special cases in the general 2HDM, for large CP violation ($\phi = 60^\circ$ or 90°) there are portions of the $(m_{H^\pm}, \tan\beta)$ parameter plane where none of the Higgs bosons of the MSSM can be detected at LEP 2, the Tevatron or the LHC. In particular, one such region is characterized by $\phi = 90^\circ$, $m_{h_1} < 60$ GeV and $\tan\beta \sim 3 - 5$. At LEP 2, the Zh_1 Higgstrahlung signal is suppressed by weak ZZh_1 coupling while the h_2 is either too heavy to be produced or decays to h_1h_1 , a signal for which existing LEP 2 analyses are not well suited. (This region might be excluded by a LEP 2 analysis focusing on 6-body final states.) At the Tevatron and LHC none of the Higgs bosons are detected by virtue of the fact that the heavier $h_{2,3}$ are the only Higgs bosons that have substantial couplings to $WW, ZZ, t\bar{t}$ and $b\bar{b}$. But, despite abundant production rates they cannot be detected because they decay to a pair of lighter Higgs bosons or a lighter Higgs boson and the Z (e.g. $h_2 \rightarrow h_1h_1, Zh_1$) — the corresponding signals associated with the resulting final states, such as $h_1h_1 \rightarrow b\bar{b}\tau^+\tau^-$, have not been shown to be observable in the presence of expected backgrounds. There are also cases in which the $h_{1,2,3}$ signals in a given discovery channel are all of similar size and overlap due to limited experimental resolution — there is no demonstration by the ATLAS and CMS collaborations that the resulting broad enhancement would be distinguished from the background.

An LC with $\sqrt{s} \sim 500$ GeV would be guaranteed to find at least one of the $h_{1,2,3}$ since the model constrains the h_i with the largest $g_{ZZh_i}^2$ coupling to be fairly light. If it decays substantially to a still lighter Higgs boson pair, then the latter could also be studied. On the other hand, there is a distinct possibility that one of the three h_i does not have highly enhanced $b\bar{b}$ coupling and does not appear in the decays of a heavier Higgs boson; to detect it would probably require the γC or a μC — see the earlier discussion regarding the MSSM A^0 [49, 52].

The Next to Minimal Supersymmetric Model, NMSSM

Let us now turn to the NMSSM model in which one adds an extra singlet superfield to the MSSM (see [3] for a summary of the NMSSM). This provides an extremely natural source for the μ term of the MSSM via the superpotential term $W \ni \lambda \widehat{H}_1 \widehat{H}_2 \widehat{N}$. When $\langle (\widehat{N})_{\text{scalar component}} \rangle = n$, where n of order the electroweak scale is natural in many cases, an effective $\mu_{\text{eff}} \sim \lambda n$ results. (Note that n can be traded for μ_{eff} in describing parameter space.) Another possible superpotential terms is $\kappa \widehat{N}^3$. Assuming no CP violation, the NMSSM Higgs sector will have an extra complex scalar field in addition to the usual two doublet fields, resulting in three CP-even Higgs bosons, $h_{1,2,3}$, two CP-odd Higgs bosons, $a_{1,2}$, and a charged Higgs pair, H^\pm .

Many groups have shown that a LC will find at least one of the CP-even Higgs bosons of the NMSSM (*e.g.* via the Higgstrahlung process) for any choice of λ and κ consistent with perturbativity up to high scales. A recent study appears in [57]. The keys are that the Higgs bosons must share the net VV coupling squared of the SM Higgs boson and that the sum of the Higgs masses squared times their VV couplings-squared has a strong upper bound in the perturbative NMSSM context. However, the situation at the LHC is far more uncertain. At the time of Snowmass96, it was demonstrated [58] that one could find parameter choices for Higgs masses and mixings such that the LHC would find no Higgs boson using just the production/detection modes explored up to that time. Since then, there have been some improvements in LHC simulations and new discovery channels have been added. In [59], it was shown that Higgs discovery for all of the difficult parameter choices identified in the Snowmass96 work would be possible in the newly analyzed $t\bar{t}h \rightarrow t\bar{t}b\bar{b}$ mode [60–62]. Ref. [59] also shows that the addition of WW fusion discovery modes (as studied for the SM Higgs boson in [63]) will allow detection of at least one NMSSM Higgs boson for all parameter choices, *provided we exclude choices for which a heavier Higgs boson decays primarily to a pair of lighter Higgs bosons*.

In more detail, the modes employed in 1996 were: (1) $gg \rightarrow h \rightarrow \gamma\gamma$ at LHC; (2) $Wh, t\bar{t}h \rightarrow \ell + \gamma\gamma$ at LHC; (3) $gg \rightarrow h, a \rightarrow \tau^+\tau^-$ plus $b\bar{b}h, b\bar{b}a \rightarrow b\bar{b}\tau^+\tau^-$ at LHC; (4) $gg \rightarrow h \rightarrow ZZ^*$ or $ZZ \rightarrow 4\ell$ at LHC; (5) $gg \rightarrow h \rightarrow WW^*$ or $WW \rightarrow 2\ell 2\nu$ at LHC; (6) $Z^* \rightarrow Zh$ and $Z^* \rightarrow ha$ at LEP2. To these, [59] added (7) $gg \rightarrow t\bar{t}h \rightarrow t\bar{t}b\bar{b}$; (8) $WW \rightarrow h \rightarrow \tau^+\tau^-$; and (9) $WW \rightarrow h \rightarrow WW^{(*)}$. If one avoids regions of parameter space where (a) $h \rightarrow aa$, (b) $h \rightarrow h'h'$, (c) $h \rightarrow H^+H^-$, (d) $h \rightarrow aZ$, (e) $h \rightarrow H^+W^-$, (f) $a \rightarrow ha'$, (g) $a \rightarrow Zh$, and (h) $a \rightarrow H^+W^-$ are present, and where (i) $a, h \rightarrow t\bar{t}$, (j) $t \rightarrow H^\pm b$ decays are possible then discovery of at least one NMSSM Higgs boson is always possible at the LHC. The parameters varied comprised $\lambda, \kappa, \mu_{\text{eff}}, \tan\beta, A_\lambda, A_\kappa$. Constraints from renormalization group evolution and perturbativity were imposed. The most difficult points found for the LHC have marginal rates for the following reasons. First, the WW, ZZ coupling-squared is shared among the h_i ($\sum_i g_{VVh_i}^2 = g_{VVh_{\text{SM}}}^2$ is required). This decreases the decays and production processes that rely on the VVh_i coupling. In particular, it is easy to make the $\gamma\gamma$ coupling and decays small since the reduced W loop cancels strongly against t, b loops. Second, since $\tan\beta$ is not very large one is well inside the ‘LHC wedge’ (as discussed earlier) for all Higgs bosons. As a result, one needs the full $L = 300 \text{ fb}^{-1}$ for ATLAS and CMS and the WW fusion modes to achieve an observable signal. In making the claim of observability here, the partonic level WW -fusion results of [63] were employed. These channels are still being studied by the LHC collaborations.

Of course, there is much more work to do on how to detect Higgs bosons in Higgs pair or Z +Higgs decay modes at the LHC. The parton-level study of [64] suggested that in the MSSM the $H^0 \rightarrow A^0A^0 \rightarrow 4b$ process could be detected by using 3 or 4 b tagging, reconstructing the double A^0 mass peak, and reconstructing the H^0 mass peak. Studies by the LHC experimental collaborations are casting doubt that this signal will actually be observable [65]. In any case, the MSSM results also need to be translated into the NMSSM context. The $WW \rightarrow h_i \rightarrow a_j a_j, h_k h_k$ modes could also prove extremely valuable, but have not yet been simulated.

A continuum of Higgs resonances

One of the most difficult cases [66] for Higgs discovery is when there is a series of Higgs bosons separated by the mass resolution in the discovery channel(s) — *e.g.* in $e^+e^- \rightarrow Z+\text{Higgs}$ there would be one Higgs boson every ~ 10 GeV (the detector resolution in the recoil mass spectrum). Since extra singlet and doublet representations (beyond the minimal two-doublets required in SUSY models) are abundant in string models, this possibility deserves serious consideration. In general, all the extra neutral Higgs bosons would mix with the normal SM Higgs (or the MSSM scalar Higgs bosons) in such a way that the physical Higgs bosons share the WW/ZZ coupling and decay to a variety of channels. The only iron-clad approach would then be to use $e^+e^- \rightarrow Z + X$ production and look for a broad excess in the recoil mass, M_X . Fortunately, there are significant constraints on this scenario. Adopting a continuum notation, we have

$$\int_0^\infty dm K(m) m^2 = m_C^2, \quad \text{where} \quad \int_0^\infty K(m) = 1 \quad (4)$$

where $K(m)(gm_W)^2$ is the (density in Higgs mass of the) strength of the hWW coupling-squared. Precision electroweak data suggests $m_C^2 \lesssim (200 - 250 \text{ GeV})^2$ in the absence of compensating $\Delta T > 0$ contributions from some heavy Higgs bosons or other new physics. Further, for multiple Higgs representations of any kind in the most general SUSY context, the RGE equations plus perturbativity up to $M_U \sim 2 \times 10^{16}$ GeV gives the same constraint on m_C . An OPAL analysis [67] of their LEP2 data in a decay mode independent fashion imposes strong constraints on possible weight $K(m)$ in the region $K(m) < 80$ GeV. In particular, using data for $e^+e^- \rightarrow ZX$ with $Z \rightarrow e^+e^-$ or $\mu^+\mu^-$, they obtain an upper limit (at 95% CL) on $\int_{m_A}^{m_B} dm K(m)$ for any choice of m_A and m_B . For example, for $m_A = 0$, they have eliminated almost the full interval up to $m_B \sim 350$ GeV assuming $m_C = 200$ GeV. But, for $m_A \geq 80$ GeV, they have not eliminated any interval.

To go further, requires higher energy. A LC energy of $\sqrt{s} = 500$ GeV is more or less ideal. The required analysis is given in [66]. If we assume that $K(m)$ is constant, that $m_C = 200$ GeV, and that $m_A = 70$ GeV, then $m_B = 300$ GeV. A fraction $f = 100 \text{ GeV}/230 \text{ GeV} \sim 0.43$ of the continuum Higgs signal then lies in the 100 – 200 GeV region. (This interval is chosen to avoid the Z peak region with largest background while avoiding kinematic suppression of the $Z+\text{Higgs}$ cross section when $\sqrt{s} = 500$ GeV.) Summing $Z \rightarrow e^+e^- + \mu^+\mu^-$ leads to an integrated signal rate of $S \sim 540f$ with a background rate of $B = 1080$ for the 100 – 200 GeV window, assuming $L = 200 \text{ fb}^{-1}$. The result is $\frac{S}{\sqrt{B}} \sim 16f \left(\frac{L}{200 \text{ fb}^{-1}} \right)$ for $m \in [100 - 200] \text{ GeV}$. This is a robust signal that would be easily detected. With $L = 500 \text{ fb}^{-1}$, one can determine the magnitude of the signal with reasonable error ($\sim 15\%$) in each 10 GeV interval of M_X . This is a clear case in which the LC would be essential for observing and studying Higgs bosons since detection of this kind of continuum signal at a hadron collider appears to be almost certainly impossible.

Left-right symmetric supersymmetric models

Finally, let us consider the left-right symmetric supersymmetric model (LRSSM) [68–71]. In general, the LR-symmetric models assume that nature has an underlying parity invariance and it is Higgs fields that break the parity at some high scale. The group

structure prior to breaking is typically taken to be $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ and it is the $SU(2)_R \times U(1)_{B-L}$ symmetry that is broken down to $U(1)_Y$ at scale m_R . The above groups are naturally contained within $SO(10)$, the fundamental representations of which automatically contain ν_R fields as well as the $SU(5)$ representations for the observed fermions. Higgs fields are easily introduced in such a way that a large Majorana mass is induced for the ν_R when parity is broken, leading to the see-saw mechanism for neutrino masses. Further, the LRSSM context guarantees that, at scale m_R , there is no strong CP problem and no SUSY-CP problem (*i.e.* the generic problem of SUSY phases for the μ parameter and for the gluino mass that would yield large EDM's unless cancellations are carefully arranged). It is then a matter of making sure that evolution from m_R down to the TeV scale does not destroy these latter two properties.

In fact, there are two LRSSM's on the market. In one, there is Majorana lepton-flavor-violation (LFV) as referred to above while in the other the LFV is Dirac in nature. In the former, the superpotential includes the generic terms (I will drop the $\hat{}$ notation for superfields) $W \ni f\nu^c\nu^c\Delta + Y_\nu\nu^c LH^u$, where the fermionic component of ν^c is the ν_R , Δ is a $B - L = 2$ Higgs superfield the scalar component of which is the Δ_R Higgs triplet representation discussed earlier, and H^u is the superfield whose scalar component is the Higgs doublet field, with neutral component vacuum expectation value v_L , responsible for up-type quark masses. For $\langle\Delta_R^0\rangle \sim m_R$, one generates the required Majorana ν_R mass and at scales $\ll v_R$ the ν_L masses will be of order $Y_\nu^2 v_L^2/m_R$, *i.e.* very small. In the Dirac LFV models, the mass matrix containing m_R is either put in "by hand" as a bare mass terms in the Lagrangian (such terms are super renormalizable) or arises from non-renormalizable operators involving a $B - L = 1$ Higgs boson χ^c via couplings of the type $(\nu^c\chi^c)^2/M$. Thus, Majorana neutrino mass generation is rather ad hoc in the Dirac LFV models. Further, the Dirac LFV models are most attractive for a large $m_R \sim M_U$ scale (which allows for MSSM-like coupling constant unification). This makes the model of less interest for TeV scale experimentation. Thus, I will focus on the Majorana LFV LRSSM case, in which the scale m_R is (given current theoretical results) required to be of order a TeV, in which case all the exotic Higgs bosons would be potential accessible.

The matter (*i.e.* not related to the gauge bosons of the model) superfields required in the Majorana LFV LRSSM model are as follows. (Here, I will drop the L, R subscripts and use the notation $\Delta \equiv \Delta_L$, $\Delta^c \equiv \Delta_R$ of the references given earlier.) (a) Two bi-doublets $\Phi_{1,2}$, with $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ quantum numbers $(2, 2, 0)$, are required in order that the vacuum expectation values of the neutral spin-0 components lead to a CKM matrix that is not simply proportional to the identity matrix. (b) An $SU(2)_R$ triplet $\Delta^c (1, 3, +2)$ is required, whose neutral spin-0 component breaks $SU(2)_R$ symmetry when it acquires a vev of order m_R . (c) The corresponding $SU(2)_L$ triplet $\Delta (3, 1, +2)$ is required by L-R symmetry. (d) In addition, the Δ and Δ^c must have anti-field partners, $\overline{\Delta^c} (1, 3, -2)$ and $\overline{\Delta} (3, 1, -2)$ in order that all anomalies cancel. (e) We also require the quark and lepton superfields, $Q (2, 1, 1/3)$, $Q^c (1, 2, -1/3)$, $L (2, 1, -1)$, and $L^c (1, 2, +1)$, whose spin-1/2 components are the normal quarks and leptons. (f) Finally, there may be a CP-odd singlet $(1, 1, 0)$ superfield that breaks the parity symmetry when its scalar component acquires a vev and a CP-even partner singlet. (The alternative for achieving breaking of P is certain nonrenormalizable interactions.)

We give a very brief summary of how the LRSSM models avoid the strong CP and SUSY CP problems. This is accomplished as follows. Consider first the strong CP problem. The standard strong CP quantity is

$$\bar{\Theta} = \Theta + \text{Arg det}(M_u M_d) - 3\text{Arg}(m_{\tilde{g}}) \quad (5)$$

where Θ is the coefficient of the $F_{\mu\nu}\tilde{F}^{\mu\nu}$ term (which is P violating) and $\bar{\Theta}$ must be very small to solve the strong CP problem. The P invariance for scales above m_R guarantees that $\Theta = 0$ above m_R . The L-R symmetry requires that $m_{\tilde{g}}$ be real above m_R . Finally, the Yukawa coupling matrices are required to be hermitian by the L-R symmetry transformations. Then, if the bi-doublet Higgs vevs are real the quark mass matrices will be hermitian, which in turns implies that the determinant of the 2nd term is real. Note that it is necessary to show that the required Higgs potential does not give rise to spontaneous CP violation. It turns out that this is not really automatic [69]; problematical phases develop at one loop unless the scale m_R is of order $m_{\text{SUSY}} \sim \text{TeV}$. The other possibly unnatural feature of the Majorana LFV approach is that a single non-renormalizable operator $\frac{\lambda}{M}[\text{Tr}(\Delta^c \tau_m \bar{\Delta}^c)]^2$ (M is of order M_{Pl} or m_R) is needed in order that the vacuum state of the model have $\langle \tilde{\nu}_R \rangle = 0$ (so that R-parity is conserved).

Regarding the SUSY CP issue, we first note that, generically speaking, it is necessary to have small phases for $Am_{\tilde{g}}$ and $\mu v_u m_{\tilde{g}}/v_d$. At scales above m_R , the hermiticity of A_u and A_d (the soft-SUSY-breaking parameter matrices) and of the Yukawa coupling matrices, along with reality of $m_{\tilde{g}}$, guarantees the required reality.

The result is that the naturalness, strong CP and SUSY CP problems can all be solved in the context of the LRSSM without R-parity violation. Further, in the Majorana LFV case this requires many Higgs fields, including $SU(2)_L$ and $SU(2)_R$ triplets as well as doublets, and a low scale for m_R that would imply TeV scale masses for all these new Higgs bosons (as well as for the W_R). Measuring their properties would be key to understanding the full structure of the LRSSM model. The two downsides of having m_R of order a few TeV are: (i) generating small neutrino masses via the see-saw mechanism requires careful adjustment, *i.e.* small values, of the associated lepton-number violating couplings; and (ii) coupling unification is hard to arrange and would typically require extra matter and/or extra dimensions.

4 Determining the CP nature of a Higgs boson

In essentially all of the extended Higgs scenarios considered above, either there are one or more CP-odd Higgs bosons (for the case of a parity conserving Higgs sector) or a collection of Higgs bosons of mixed CP nature. Direct determination of the CP nature of any observed Higgs boson will probably be critical to disentangling any but the simplest SM Higgs sector. For this the γC facility would be ideal [49, 72–74]. A muon collider would also be of great value for determining the CP nature of observed Higgs bosons [51, 75, 76].

Let us focus on the γC . We recall that the $\sigma(\gamma\gamma \rightarrow H^0) \propto \vec{\epsilon}_1 \cdot \vec{\epsilon}_2$ while $\sigma(\gamma\gamma \rightarrow A^0) \propto \vec{\epsilon}_1 \times \vec{\epsilon}_2$. Thus, if you could produce 100% transversely polarized back scattered photons, only the A^0 (H^0) would be produced for perpendicular (parallel) polarizations, respectively. In practice, there is always some circular polarization for the back-scattered

photons, even for 100% transversely polarized laser photons. Also, it could be that the Higgs bosons are of mixed CP parity. (Although, in the decoupling limit the light Higgs boson is guaranteed to be CP-even [77].) Thus, to fully explore the CP parity of a Higgs boson, measurements of three asymmetries, $\mathcal{A}_{1,2,3}$ would be ideal. These are defined as

$$\begin{aligned}\mathcal{A}_1 &= \frac{|\mathcal{M}_{++}|^2 - |\mathcal{M}_{--}|^2}{|\mathcal{M}_{++}|^2 + |\mathcal{M}_{--}|^2}, & \mathcal{A}_2 &= \frac{2\text{Im}(\mathcal{M}_{++}\mathcal{M}_{--}^*)}{|\mathcal{M}_{++}|^2 + |\mathcal{M}_{--}|^2}, \\ \mathcal{A}_3 &= \frac{2\text{Re}(\mathcal{M}_{++}\mathcal{M}_{--}^*)}{|\mathcal{M}_{++}|^2 + |\mathcal{M}_{--}|^2} = \frac{|\mathcal{M}_{\parallel}|^2 - |\mathcal{M}_{\perp}|^2}{|\mathcal{M}_{\parallel}|^2 + |\mathcal{M}_{\perp}|^2}.\end{aligned}\quad (6)$$

The first two asymmetries are typically quite substantial for a large range of 2HDM parameter space for which CP violation occurs. $\mathcal{A}_3 = +1$ (-1) for a purely CP-even (CP-odd) Higgs boson. In terms of the Stokes parameters specifying the polarizations of the back-scattered photons

$$dN = dL_{\gamma\gamma} dPS \frac{1}{4} (|\mathcal{M}_{++}|^2 + |\mathcal{M}_{--}|^2) \times [(1 + \langle \xi_2 \xi_2' \rangle) + (\langle \xi_2 \rangle + \langle \xi_2' \rangle) \mathcal{A}_1 + (\langle \xi_3 \xi_3' \rangle + \langle \xi_1 \xi_1' \rangle) \mathcal{A}_2 + (\langle \xi_3 \xi_3' \rangle - \langle \xi_1 \xi_1' \rangle) \mathcal{A}_3]. \quad (7)$$

The actually measured asymmetries are then

$$\begin{aligned}T_1 &= \frac{N_{++} - N_{--}}{N_{++} + N_{--}} = \frac{\langle \xi_2 \rangle + \langle \xi_2' \rangle}{1 + \langle \xi_2 \xi_2' \rangle} \mathcal{A}_1, & T_2 &= \frac{N(\phi = \frac{\pi}{4}) - N(\phi = -\frac{\pi}{4})}{N(\phi = \frac{\pi}{4}) + N(\phi = -\frac{\pi}{4})} = \frac{\langle \xi_3 \xi_3' \rangle + \langle \xi_1 \xi_1' \rangle}{1 + \langle \xi_2 \xi_2' \rangle} \mathcal{A}_2, \\ T_3 &= \frac{N(\phi = \frac{\pi}{2}) - N(\phi = 0)}{N(\phi = \frac{\pi}{2}) + N(\phi = 0)} = \frac{\langle \xi_3 \xi_3' \rangle - \langle \xi_1 \xi_1' \rangle}{1 + \langle \xi_2 \xi_2' \rangle} \mathcal{A}_3,\end{aligned}\quad (8)$$

where for T_1 we 100% polarize the laser photons both with + helicity and then flip both to negative helicities and for $T_{2,3}$ ϕ is the angle between the 100% linear polarizations of the laser photons. T_2 and T_3 are harder to measure than T_1 because the Stoke's parameters in the numerators are smaller for the former two. Nonetheless, excellent accuracy can be achieved. For example, at the American/Asian NLC with the LLNL laser and IP design, \mathcal{A}_3 can be measured to 10% after two years of dedicated operation in the case of a 120 GeV CP-even SM-like Higgs boson [49]. Similar accuracy can be achieved at the μC [75] using asymmetries [51, 76] obtained by varying the polarizations of the colliding μ^+ and μ^- in $\mu^+\mu^-$ s -channel Higgs production.

This accuracy should be compared to what is possible at the LHC and the LC. At the LHC, parton level studies [78] suggest some promise for determining the relative size of the CP-even and CP-odd couplings of a Higgs boson to $t\bar{t}$ by looking at angular distributions of the t , \bar{t} and Higgs boson relative to one another, where the Higgs boson is detected in the $\gamma\gamma$ (for a SM-like Higgs boson) or $b\bar{b}$ (for other types of Higgs bosons) decay mode. More realistic studies are only now being performed but show less promise.

There are numerous studies of Higgs CP-determination using e^+e^- collisions (see, *e.g.*, [74] and [79]). However, caution is necessary in interpreting the results of those that rely on angular distributions and the like in the Z +Higgs final state. Using h (a) to denote a CP-even (CP-odd) canonically normalized state, a mixed-CP Higgs state can be written in the form $h_M = \cos \phi_M h + \sin \phi_M a$. The crucial point is that the aZZ coupling is at the

one-loop level [80] compared to the tree-level hZZ coupling. The cross section $d\sigma/d\cos\theta$ for the h_M contains terms proportional to (L is a typical one-loop factor) $\cos^2\phi_M$ and $\sin^2\phi_M L^2$ that are even in $\cos\theta$ and a term proportional to $\cos\phi_M \sin\phi_M L$ that is odd in $\cos\theta$, and provides the best sensitivity to the a component. For the a component to have a strong fractional influence, requires $\tan\phi_M L \sim 1$. In this case, all the terms in $d\sigma/d\cos\theta$ will be of order L^2 , including the term odd in $\cos\theta$, and errors for the CP determination will be very large since the h_M production rate will be small. If $\cos\phi_M$ is substantial, the rate will be large but the fractional influence of the a component will be at the one-loop level and very hard to detect. This same caution applies to CP determinations related to $h_M \rightarrow W^+W^-$ or ZZ decay angular distributions (see, *e.g.*, [81–83]).

The best technique for the Zh_M final state is to employ the self-analyzing decays $h_M \rightarrow \tau^+\tau^-$. We wish to probe the relative strengths of the 1 versus γ_5 terms in the interaction $\bar{\psi}_\tau(\cos\phi_M + i\gamma_5 \sin\phi_M)\psi_\tau$. In the simple limits of $\cos\phi_M = 1, 0$, respectively, one finds $\Gamma(h_M \rightarrow \tau^+\tau^-) \propto (1 - s_{\parallel}^{\tau^+} s_{\parallel}^{\tau^-} \pm s_{\perp}^{\tau^+} s_{\perp}^{\tau^-})$, where \parallel, \perp denote components parallel/transverse to the Higgs boson momentum as seen from the respective τ^\pm rest frames. (The corresponding expression in the general case is complicated.) While these spin directions are not directly measurable, the distributions of the π^\pm or ρ^\pm from the $\tau^\pm \rightarrow \pi^\pm\nu$ or $\tau^\pm \rightarrow \rho^\pm\nu$ decays will reflect the the spin directions and one can extract the relative magnitude of the CP-even versus CP-odd coupling. This technique shows substantial promise according to theoretical studies [74, 84]. A more detailed experimental study [85], using somewhat different techniques than originally proposed, finds that the CP-even nature of a h with $m_h = 120$ GeV can be verified at the 95% CL in Zh production at $\sqrt{s} = 500$ GeV, assuming $L = 500$ fb $^{-1}$. Thus, for a CP-even h the γC initial state polarization asymmetries and the final state LC $\tau^+\tau^-$ analysis yield comparable accuracies. However, since the aZZ coupling is one-loop, $e^+e^- \rightarrow Za$ production will have low rate and only the γC (or μC) could verify the CP nature of a state that is mainly or entirely CP-odd. It should also be noted that if $\tau^+\tau^-$ decays are suppressed (*e.g.* because of competing Higgs pair final states and/or SUSY final states), the accuracy of the $\tau^+\tau^-$ technique will suffer, whereas the γC (and μC) asymmetry measurements are for production rates, and are independent of how the Higgs boson decays. Finally, we note that the $\tau^+\tau^-$ final state CP determinations performed for a Higgs produced at a γC (or μC [76, 84]) would complement the determination obtained using initial state polarization asymmetries.

5 Conclusions

There are many quite well-motivated possibilities for the Higgs sector that go far beyond the one-doublet sector of the SM. The plethora of possibilities means that it is entirely possible that the Higgs sector will prove very challenging to fully explore. The variety of models, complications due to unexpected decay modes (*e.g.* Higgs pairs or SUSY particles), overlapping of resonances, sharing of WW, ZZ coupling strength, CP violation, the possible impact of extra dimensions and Higgs-radion mixing, *etc.* make attention to multi-channel, multi-collider analysis vital. In particular, it seems we must accept the fact that there is enough freedom in the Higgs sector that we should not take Higgs discovery at the Tevatron or LHC for granted and that even at the LC Higgs detection and study

could prove quite challenging (as in the light- A^0 scenario for the general 2HDM where m_{h^0} can be as heavy as $\sim 800 - 900$ GeV without conflicting with precision electroweak data or perturbative constraints). The LHC collaborations must keep improving and working on every possible signature and the LC design must be pushed to the highest feasible energy given financial and technological constraints. Research regarding the feasibility of a μC should be continued.

The LHC ability to show that the WW sector is perturbative could be very useful. Two particular examples are the following. First, in the NMSSM we might not detect a Higgs boson using the analysis techniques considered so far, but a perturbative WW sector would imply that there are light CP-even Higgs bosons with significant WW coupling. Perhaps with that motivation, it would be possible to find new techniques capable of digging out faint signatures. Second, we can imagine a scenario in which there are a number of heavy $\sim 800 - 900$ GeV mixed-CP Higgs bosons [†] that share the WW, ZZ coupling strength and/or they decay to lighter Higgs bosons (with small ZZ coupling) and/or they give rise to overlapping resonance signals. In such a scenario, it would be impossible to absolutely guarantee discovery of a Higgs boson at the LHC or at the LC, γC or μC unless the center of mass energy of the latter machines can reach the multi-TeV level. At the LHC, the WW scattering processes would exhibit moderately perturbative behavior, and Giga- Z operation at the LC would show that the S, T values matched the expectations for such a scenario. These observations would indicate the need for sufficiently higher \sqrt{s} at the LC to make production of a pair of the CP-mixed Higgs bosons possible.

Sticking to less extreme and better-motivated cases in which one or more Higgs bosons are reasonably light, it seems very apparent that experimentation at both the LC and the LHC is needed to have a high probability of discovering even one Higgs boson and almost certainly both machines will be needed to fully study the Higgs sector. Particularly strong motivations for the LC, γC and μC include the following. The LC would possibly be necessary in the case of the NMSSM and would certainly be required to detect a continuum of strongly mixed CP-even Higgs bosons. Observation of the heavy H^0, A^0 of the MSSM will require $\gamma\gamma$ collisions if $[m_{A^0}, \tan\beta]$ are in the “wedge” region of parameter space. Once observed, the properties and rates for the H^0, A^0 will help enormously in determining important SUSY parameters, especially checking for the predicted relation between their Yukawa couplings and $\tan\beta$. Exotic Higgs representations, *e.g.* the triplet as motivated by the seesaw approach to neutrino masses and the LRSSM solutions to the strong and SUSY CP problems, will lead to exotic collider signals and possibilities that might ultimately be best explored via e^-e^- and/or $\mu^-\mu^-$ collisions. Finally, we have reviewed how important a γC (and eventual μC) could be for directly measuring the CP composition of a Higgs boson, especially one with a substantial CP-odd component.

In short, since our ability to fully explore the Higgs sector will be very important to a full understanding of the ultimate theory, it seems very clear that a full complement of collider facilities will ultimately be needed, including the LHC, a LC, a γC at the LC, and eventually a μC .

[†]Current precision electroweak constraints would be satisfied due to weak-isospin breaking arising from mass differences relative to charged Higgs bosons.

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