SUPERSYMMETRY PARAMETER ANALYSIS

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Outline:

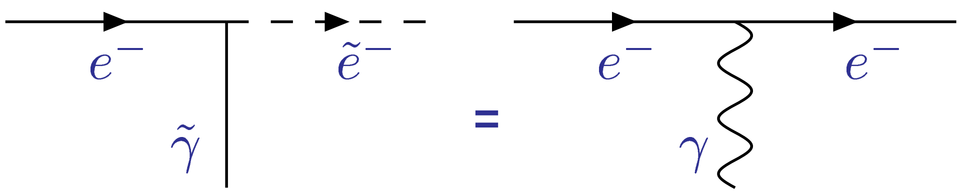
- **Introduction/motivation**
- **Experimental input**
- **Reconstructing fundamental parameters**
- **Extrapolations to high scales**
- **Conclusions**

Supersymmetry – a well motivated extension of the SM:

- **stabilizes the electroweak scale**
- **leads to unification of gauge couplings**
- **accommodates large top quark mass**
- **provides dark matter candidate**

Exact SUSY – no new parameters:

- **we know spartners:** $p \Leftrightarrow \tilde{p}$
- **and their Yukawa couplings $=$ gauge couplings**



However SUSY – must be broken:

- **its mechanism unknown**
- **many models: SUGRA, GMSB, AMSB, $\tilde{g}$MSB, . . .**
- **each model has a few parameters (at high scale)**

But what if all xMSB models are wrong?

- **experiments will tell!**

Unconstrained low-energy SUSY > 100 parameters:

- **gaugino masses** $m_{\tilde{g}}\, \bar{\tilde{g}}\, \tilde{g}$
- **sfermion masses** $m^2_{\tilde{f}}\, \tilde{f}^* \tilde{f}$
- **trilinear scalar couplings** $A_f H \tilde{f} \tilde{f}$

Lagrangian parameters $m_{\tilde{g}}$, $m_{\tilde{f}}$, A_f are not directly measurable:

- $m_{\tilde{g}}$, $m_{\tilde{f}}$, $A_f \Longleftrightarrow$ **masses, mixing angles and CP phases**
- **many measurements and**
- **considerable ingenuity will be needed**

to reconstruct a complete low-energy theory.

After SUSY discovered:

- verify **quantum numbers** $p \Leftrightarrow \tilde{p}$
- check for SUSY relations for **couplings**
- measure masses, mixing angles, CP phases in a **model independent** way
- look for specific **reactions, mass patterns**
- reconstruct low-energy Lagrangian parameters
- in (top-motivated) bottom-up approach **unravel SUSY breaking mechanism**

Need strategy:

- start with charginos $\Longleftrightarrow M_2, \mu, \tan\beta$
- add neutralinos $\Longleftrightarrow \oplus M_1$
- include sleptons $\Longleftrightarrow \oplus m_{\tilde{l}}, A_l$
- and squarks and gluinos $\Longleftrightarrow \oplus m_{\tilde{q}}, A_q$ and M_3
- to reconstruct at tree level the **basic structure** of SUSY Lagrangian

In reality:

- **might be difficult to separate a specific sector, *e.g.* sleptons enter via t-channel in chargino production processes**
- **many production channels simultaneously open, in particular in hadron colliders**
- **SUSY constitutes important background to SUSY processes**
- **sizable loop corrections will mix all sectors**

Precision measurements will require:

- **loop corrections to masses and mixing angles**
- **finite width effects**
- **loop corrections to production and decay processes**

for final global analyses of all data.

Some one-loop results available, much more work to do.

For the status: see plenary talk by W. Majerotto

and Roth, Guasch and Melles in Session 1B

Present: no clear signal of SUSY seen

- **direct searches at HERA, Tevatron, LEP and SLC $\Rightarrow$ constraints on model parameters**
- **indirect information from rare decays, EDMs, (g-2)$_\mu$ etc. $\Rightarrow$ more model dependence**

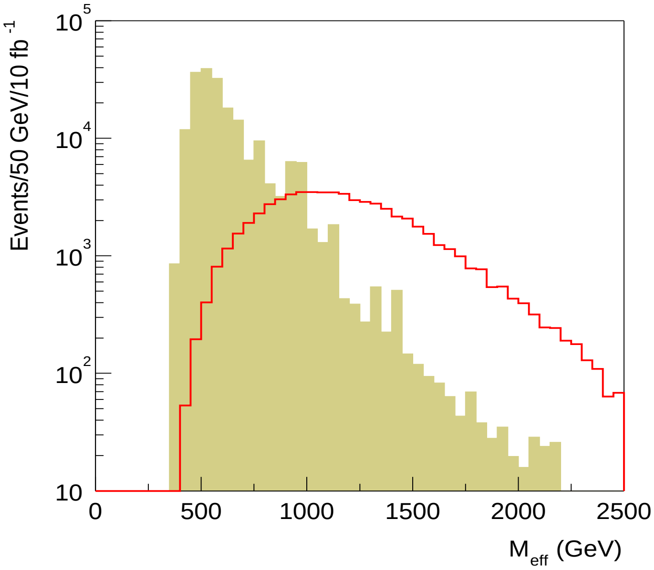
Future:

- **Tevatron Run II:**
  - ◇ **reasonable reach for SUSY with stable LSP using trilepton signals from $\tilde{\chi}_1^{\pm}\tilde{\chi}_2^0 \to l^{\pm}l\bar{l}$**
  - ◇ **be ready for surprises**

    $\Rightarrow$ Pagliarone, Allanach in Session 1B
  - ◇ **try new reference points suggested by Kane et al.**
- **LHC:**
  - ◇ **LHC will certainly see SUSY if it is at the weak scale**
  - ◇ **many different channels from squark and gluino decays**
  - ◇ **but easy to separate if the SUSY decays are distinctive, e.g. missing E^T from stable LSP**

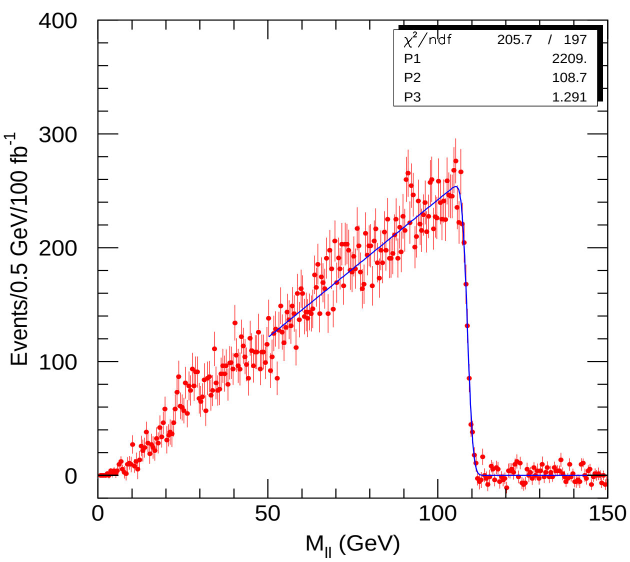
◇ **early indication: excess in**

$M_{eff} = E^T_{miss} + p^T_1 + p^T_2 + \ldots$ **[Hinchliffe, Paige '00]**



◇ **identify particular channels and measure endpoints e.g. in** $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 l^+ l^-$

$$M^{max}_{ll} = [(m^2_{\tilde{\chi}_2} - m^2_{\tilde{l}})(m^2_{\tilde{l}} - m^2_{\tilde{\chi}_1})/m^2_{\tilde{l}}]^{-1/2}$$



[ATLAS TDR]

to determine mass differences

- ◇ measure cross sections and branching ratios
- ◇ large amount of information on squarks, gluinos and their main decay products

$\Rightarrow$ Tricomi, Nojiri, Kazana in Session 1B

- ◇ theoretical interpretation possible in specific models

But if all our models are wrong?

Concurrent running of an e^+e^- LC very much welcome

- complementary information to the LHC

$\Rightarrow$ Weiglein in 1B

- **precise determination** of masses, couplings, quantum numbers, mixing angles, CP phases thanks to
  - ◇ clean environment
  - ◇ tunable energy
  - ◇ polarized beams
  - ◇ additional modes: e^-e^-, $e\gamma$ and $\gamma\gamma$
- better **model independent** reconstruction of low energy SUSY parameters
- connect low scale phenomenology with high scale physics $\Rightarrow$ plenary talk by Kane

The chargino sector:

- **chargino mass matrix in the $(\tilde{W}^-, \tilde{H}^-)$ basis**

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & |\mu| e^{i\Phi_\mu} \end{pmatrix}$$

- **to diagonalize, two unitary matrices needed parameterized by two mixing angles $\phi_{L,R}$, and three CP phases $\beta_{L,R}$ and $\gamma_1 - \gamma_2$**

From $\{M_2, \mu, \tan\beta\} \Longrightarrow \{m_{\tilde{\chi}^\pm_{1,2}},\ \cos 2\phi_{L,R}\}$

- **the chargino masses**

$$m^2_{\tilde{\chi}^\pm_{1,2}} = \frac{1}{2}\left[M_2^2 + |\mu|^2 + 2m_W^2 \mp \Delta_C\right]$$

$$\Delta_C = [(M_2^2 - |\mu|^2)^2 + 4m_W^4 \cos^2 2\beta + 4m_W^2 (M_2^2 + |\mu|^2) + 8m_W^2 M_2 |\mu| \sin 2\beta \cos \Phi_\mu]^{1/2}$$

- **the mixing angles**

$$\cos 2\phi_{L,R} = -\left[M_2^2 - |\mu|^2 \mp 2m_W^2 \cos 2\beta\right] / \Delta_C$$

experimentally

masses $\Leftarrow$ **threshold scans, or in continuum**

mixing angles $\Leftarrow$ **cross sections with polarized beams**

Inverting [Choi et al. '98-'02, Kneur, Moultaka '99,'00]

From $\{m_{\tilde{\chi}^\pm_{1,2}},\ \cos 2\phi_{L,R}\} \Longrightarrow \{M_2, |\mu|, \cos\Phi_\mu, \tan\beta\}$

- **the mass parameters**

$$M_2 = [(m^2_{\tilde{\chi}^\pm_2} + m^2_{\tilde{\chi}^\pm_1} - 2m^2_W)/2 - \Delta_C(c_{2L} + c_{2R})/4]^{1/2}$$

$$|\mu| = [(m^2_{\tilde{\chi}^\pm_2} + m^2_{\tilde{\chi}^\pm_1} - 2m^2_W)/2 + \Delta_C(c_{2L} + c_{2R})/4]^{1/2}$$

- $\tan\beta = \sqrt{\frac{4m^2_W - \Delta_C(\cos 2\phi_L - \cos 2\phi_R)}{4m^2_W + \Delta_C(\cos 2\phi_L - \cos 2\phi_R)}}$

- **and the phase Φ_μ (or sign of μ in CP–invariant case)**

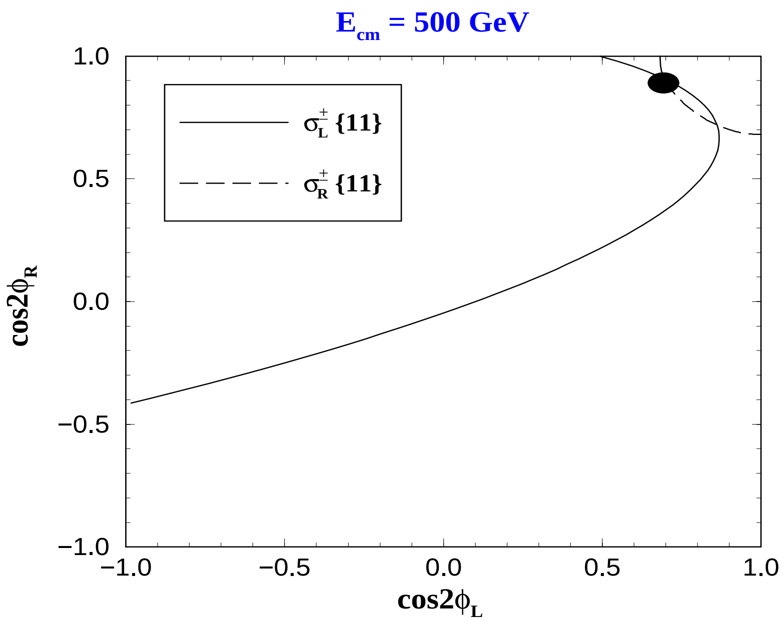
$$\cos\Phi_\mu = [\Delta^2_C(2 - c^2_{2L} - c^2_{2R}) - 8m^2_W(m^2_{\tilde{\chi}^\pm_2} + m^2_{\tilde{\chi}^\pm_1} - 2m^2_W)] \times [16m^4_W - \Delta^2_C(c_{2L} - c_{2R})^2]^{-1/2}[4(m^2_{\tilde{\chi}^\pm_2} + m^2_{\tilde{\chi}^\pm_1} - 2m^2_W)^2 - \Delta^2_C(c_{2L} + c_{2R})^2]^{-1/2}$$

CP phases in SUSY $\Rightarrow$ plenary talk by Nath

Question:

Can we invert without knowing heavy chargino mass?

Early stage of the LC, only $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ [Choi et al., '01]



twofold ambiguity (assuming $m_{\tilde{\nu}}$ known)

$\{\cos 2\phi_L, \cos 2\phi_R\} = \{0.70, 0.91\}$ **and** $\{0.86, 0.72\}$

With the mass of $\tilde{\chi}_2^\pm$ still unknown:

- **in CP–conserving case $\Longrightarrow$ two possible solutions**
  $\{\tan\beta, M_2, \mu\} = \{10, 191\,\mathrm{GeV}, 365\,\mathrm{GeV}\}$
  $\{\tan\beta, M_2, \mu\} = \{0.35, 198\,\mathrm{GeV}, 387\,\mathrm{GeV}\}$
  to be resolved using transverse beam polarization
  or info from other sectors: Higgs, neutralino etc.
- **in CP–violating case $\Rightarrow$ two trajectories in**
  $\{\tan\beta, M_2, \mu\}$ **space parameterized by unknown** $m_{\tilde{\chi}_2^\pm}$

$\Longrightarrow$ **exploit neutralinos**

The neutralino sector:

- **neutralino mass matrix in the $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ basis**

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix}$$

$$M_1 = |M_1|\, e^{i\Phi_1}, \mu = |\mu|\, e^{i\Phi_\mu}$$

- **the diagonalization matrix N: 6 angles and 10 phases**

$$N = \text{diag}\left\{e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}, e^{i\alpha_4}\right\} \mathsf{D}$$
$$\mathsf{D} = \mathsf{R}_{34}\, \mathsf{R}_{24}\, \mathsf{R}_{14}\, \mathsf{R}_{23}\, \mathsf{R}_{13}\, \mathsf{R}_{12}$$

R_{jk}: **2-dim complex rotation by** $(cos\theta_{jk}, \sin\theta_{jk}\, e^{i\delta_{jk}})$

- **if $\delta_{ij} = 0$ and $\alpha_i = 0$, CP conserved**
- **unitarity constraints $\longrightarrow$ quadrangles built up by**

$\oplus$ **the links $N_{ik} N_{jk}^*$ connecting two rows i and j**

$\oplus$ **the links $N_{ki} N_{kj}^*$ connecting two columns i and j**

- **unlike in the CKM or MNS cases of quark and lepton mixing, the orientation of all quadrangles is physical and determined by the phases**

Back to resolving the light chargino case with CP violation: [Choi et al. '02]

- neutralino masses satisfy the characteristic equation

$$m^8_{\tilde{\chi}^0_i} - a\, m^6_{\tilde{\chi}^0_i} + b\, m^4_{\tilde{\chi}^0_i} - c\, m^2_{\tilde{\chi}^0_i} + d = 0$$

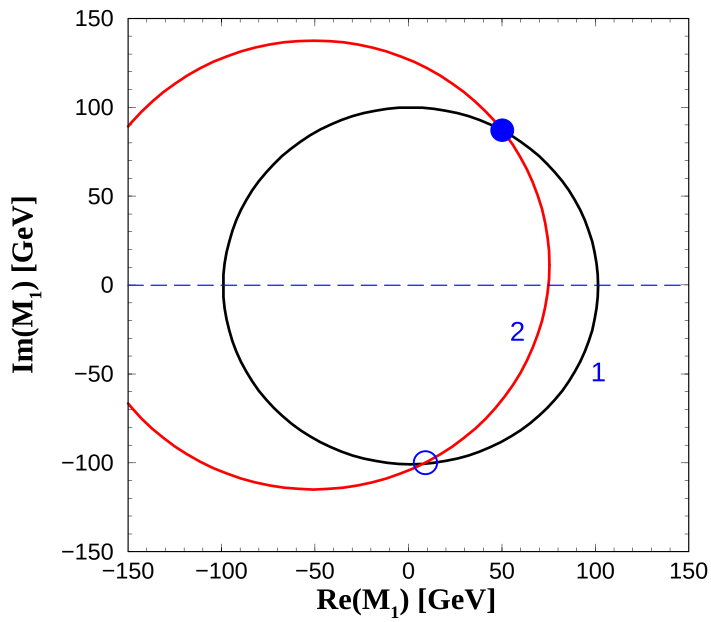
  a, b, c and d are binomials of $\Re\mathrm{e} M_1$ and $\Im\mathrm{m} M_1$

- equation for each $m^2_{\tilde{\chi}^0_i}$ has the form

$$(\Re\mathrm{e} M_1)^2 + (\Im\mathrm{m} M_1)^2 + u_i\, \Re\mathrm{e} M_1 + v_i\, \Im\mathrm{m} M_1 = w_i$$

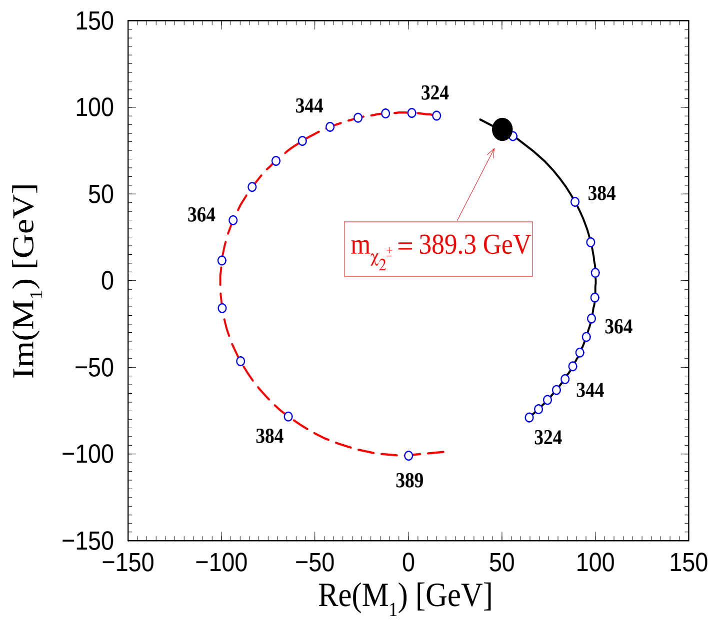
- each neutralino mass defines a circle in the $\{\Re\mathrm{e} M_1, \Im\mathrm{m} M_1\}$ plane

- for two light neutralino masses: two crossing points



From chargino: $\{\tan\beta, M_2, \mu\}$ **parameterized by unknown** $m_{\tilde{\chi}_2^\pm}$

- **crossing points will migrate with** $m_{\tilde{\chi}_2^\pm}$



- **use the measured cross section for** $\tilde{\chi}_1^0\tilde{\chi}_2^0$ **to select a unique solution for** M_1 **and predict the heavy chargino mass**
- **if LC runs concurrently with LHC, ask LHC friends to look for** $\tilde{\chi}_2^\pm$

If energy above the heavy -inos,

- **threshold behavior of non-diagonal neutralino production – clear signal of nontrivial CP phases**
- **normal neutralino polarization – a unique probe of Majorana CP phases**
- **sum rules can be exploited to verify closure of the chargino and neutralino sectors**
- **analyses of SUSY relations between Yukawa and gauge couplings**
- **extract information on slepton sector**
- **and many more ..**

see talks by:

Brandenburg, Moortgat-Pick, Hesselbach, Hensel

in Session 1B

The sfermion sector:

- **sfermion mass matrix in the $(\tilde{f}_L, \tilde{f}_R)$ basis**

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}_L} & a^*_f m_f \\ a_f m_f & m^2_{\tilde{f}_R} \end{pmatrix}$$

$$m^2_{\tilde{f}_i} = M^2_{\tilde{F}_i} + m^2_Z \cos 2\beta (I_{f_i} - Q_f \sin^2\theta_W) + m^2_f$$

$$a_f m_f = (A_f - \mu^* (\cot\beta)^{2I_f}) m_f = |a_f m_f| e^{i\Phi_f}$$

- **large $(\tilde{f}_L, \tilde{f}_R)$ mixing if $|m^2_{\tilde{f}_L} - m^2_{\tilde{f}_R}| \leq |a_f m_f|$**

$$U_f = \begin{pmatrix} e^{i\Phi_f}\cos\theta_f & \sin\theta_f \\ -\sin\theta_f & e^{-i\Phi_f}\cos\theta_f \end{pmatrix}$$

- **the sfermion masses**

$$m^2_{\tilde{f}^{\pm}_{1,2}} = \frac{1}{2}(m^2_{\tilde{f}_L} + m^2_{\tilde{f}_R} \mp \Delta_f)$$

$$\Delta_f = \left[(m^2_{\tilde{f}_L} - m^2_{\tilde{f}_R}) + 4|a_f m_f|^2\right]^{1/2}$$

- **the mixing angle**

$$\tan\phi_f = |a_f m_f|/(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_R})$$

For $\tilde{e}$ and $\tilde{\mu}$ mixing usually neglected

$\Rightarrow$ **mass eigenstates are chiral $\tilde{f}_L$ and $\tilde{f}_R$**

masses can be measured in threshold scans

- **smuons:**

$$e^+e^- \to \tilde{\mu}^+_L\tilde{\mu}^-_L \;,\; \tilde{\mu}^+_R\tilde{\mu}^-_R \qquad [\gamma, Z] \qquad \text{P-wave}$$

- **selectrons:**

$$e^+_{\rm L}e^-_{\rm R} \,/\, e^+_{\rm R}e^-_{\rm L} \;\to\; \tilde{e}^+_{\rm L}\tilde{e}^-_{\rm L} \;,\; \tilde{e}^+_{\rm R}\tilde{e}^-_{\rm R} \qquad [\gamma, Z;\; \tilde{\chi}^0] \qquad \text{P-wave}$$

$$e^+_{\rm L}e^-_{\rm L} \,/\, e^+_{\rm R}e^-_{\rm R} \;\to\; \tilde{e}^+_{\rm R}\tilde{e}^-_{\rm L} \,/\, \tilde{e}^+_{\rm L}\tilde{e}^-_{\rm R} \qquad [\tilde{\chi}^0] \qquad \text{S-wave}$$

$$e^-_{\rm L}e^-_{\rm R} \,/\, e^-_{\rm R}e^-_{\rm L} \;\to\; \tilde{e}^-_{\rm L}\tilde{e}^-_{\rm R} \qquad [\tilde{\chi}^0] \qquad \text{P-wave}$$

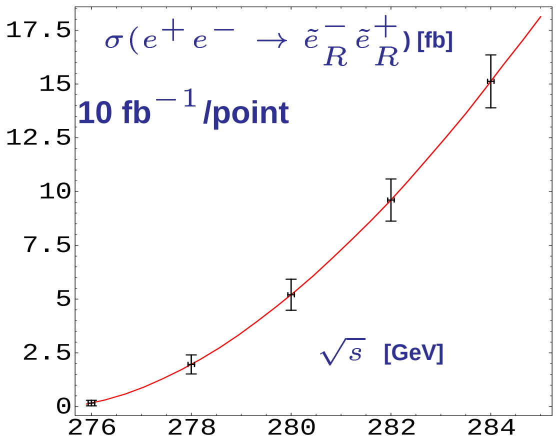
$$e^-_{\rm L}e^-_{\rm L} \,/\, e^-_{\rm R}e^-_{\rm R} \;\to\; \tilde{e}^-_{\rm L}\tilde{e}^-_{\rm L} \,/\, \tilde{e}^-_{\rm R}\tilde{e}^-_{\rm R} \qquad [\tilde{\chi}^0] \qquad \text{S-wave}$$

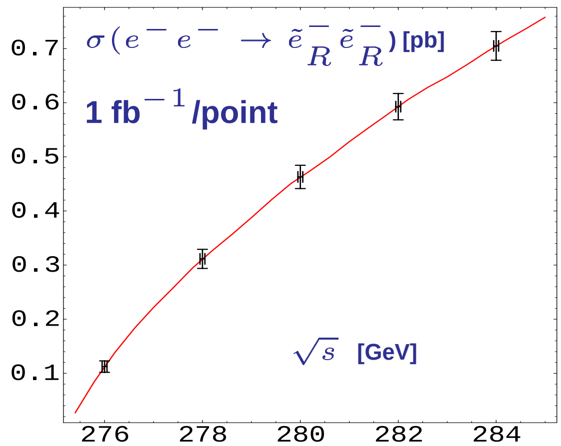
the S-wave more suitable for selectron mass measurement

- e^-e^- **collider mode more suitable for selectrons because of smaller SM and SUSY background**

[Heusch, Feng '98, Feng and Peskin '01, Freitas et al. '01]

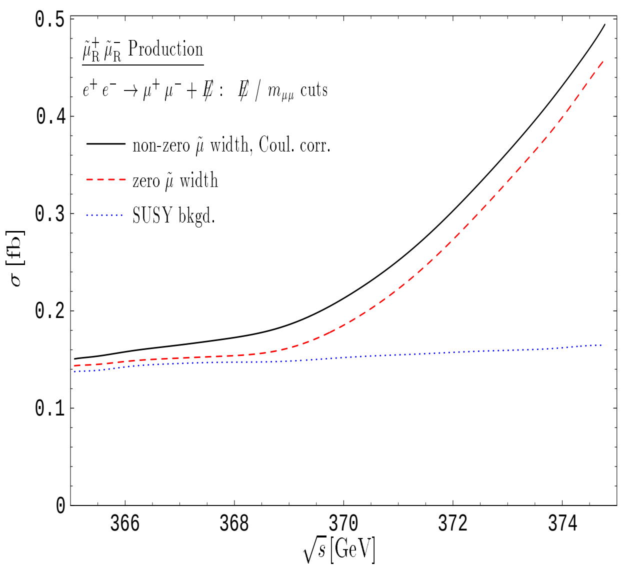
[Blöchinger et al. '02]

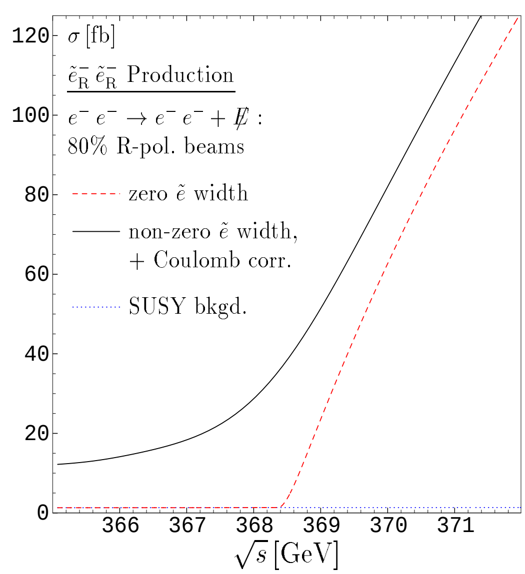




- **finite slepton width and Coulomb rescattering important for mass determination**

$\Rightarrow$ **Freitas in Session 1B**





moreover

- verify SU(2)xU(1) quantum numbers of $\tilde{f}_R$ by measuring σ_{tot} with different beam polarization

  [Tsukamoto et al. '95]

- verify SUSY relations between Yukawa and gauge couplings

- check (in)equality of $\tilde{e}_R$ and $\tilde{\mu}_R$ masses

and derive useful informations on neutralino sector

- from energy spectrum in, e.g., $\tilde{e}_R \to e\tilde{\chi}_1^0$, measure $m_{\tilde{\chi}_1^0}$

- total and differential cross section for $e^+e^- \to \tilde{e}_R^+\tilde{e}_R^-$ sensitive to M_1

  [Feng et al. 95, Nojiri '95, Blöchinger '00-'02]

$\tilde{\tau}$ very interesting object

- in GUT scenarios its mass parameters depend on physics at the GUT scale
- $\tilde{\tau}$ production and decay different from $\tilde{e}$ and $\tilde{\mu}$ because of non-negligible τ Yukawa coupling appearing:
  in the mass matrix $\Rightarrow$ $\tilde{\tau}_R$–$\tilde{\tau}_L$ mixing, θ_τ
  in interactions $\Rightarrow$ $\tilde{\tau}$ couples also to the higgsino component of $\tilde{\chi}^0_i$ and $\tilde{\chi}^\pm_j$ [Bartl et al. '97-'00 Eberl et al. '96, '99]
- the τ polarization in $\tilde{\tau} \to \tilde{\chi}^0 \tau$ decay measurable, and interesting [Nojiri '96]
- τ polarization – a sensitive function of $\tilde{\tau}$ mixing, neutralino mixing and $\tan\beta$ [Boos et al., '02]

$$P_{\tilde{\tau}_{1/2}} = \frac{4-x^2 \pm (4+x^2-2w^2)\cos 2\theta_\tau \mp 2(2+x)w \sin 2\theta_\tau}{4+x^2+2w^2 \pm (4-x^2)\cos 2\theta_\tau \mp 2(2-x)w \sin 2\theta_\tau}$$

$x \sim$ neutralino mixing, and $w \sim 1/\cos\beta$

for example:

If mixing in stau sector is very small ($\xi \rightarrow 0$) and in case $m_{LL}^2 > m_{RR}^2$:

$$\tilde{\tau}_1 = \tilde{\tau}_R \quad \text{and} \quad \tilde{\tau}_2 = \tilde{\tau}_L$$

From the general formula for the tau polariztion one gets:

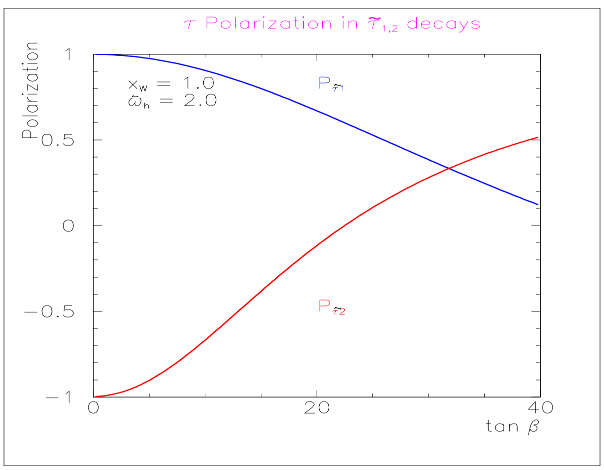
$$P_{\tilde{\tau}_1} = \frac{4 - w_h^2}{4 + w_h^2}$$

$$P_{\tilde{\tau}_2} = \frac{-x_W^2 + w_h^2}{+x_W^2 + w_h^2}$$

$P_{\tilde{\tau}_1}$ does not depend of on gaugino mixing parameter x_W

$$w_h^2 = (1 + \tan^2_\beta) \frac{m_\tau^2}{m_W^2} \tilde{w}_h^2,$$

Polarization in "No stau mixing" case



⇒ **Boos in Session 1B**

if A_τ or μ complex: **[Bartl et al. '02]**

- **the phase of the off-diagonal term**

$$a_\tau m_\tau = (A_\tau - \mu^* \tan\beta) m_\tau = |a_\tau m_\tau| e^{i\Phi_\tau}$$

modifies $\tilde{\tau}$ decay rates and τ polarization

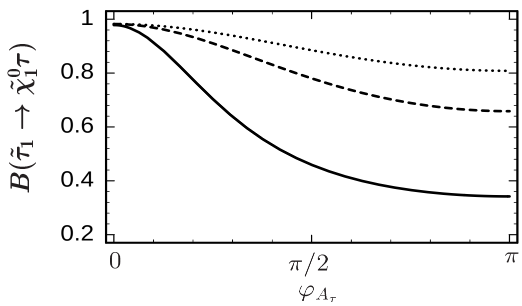


Figure 1: Branching ratio of $\tilde{\tau}_1 \to \tilde{\chi}_1^0 \tau$ as a function of φ_{A_τ} for $m_{\tilde{\tau}_1} = 240$ GeV, $m_{\tilde{\nu}_\tau} = 233$ GeV (solid line), 238 GeV (dashed line), 243 GeV (dotted line), and $\varphi_\mu = \varphi_{U(1)} = 0$, $|\mu| = 300$ GeV, $|A_\tau| = 1000$ GeV, $\tan\beta = 3$, and $M_2 = 200$ GeV.

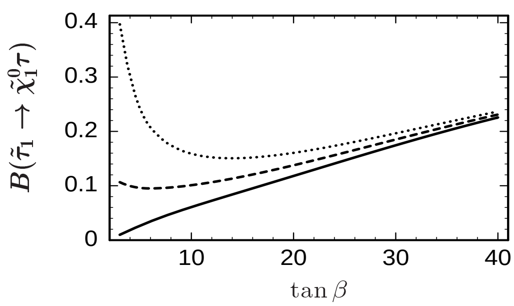


Figure 2: Branching ratio of $\tilde{\tau}_1 \to \tilde{\chi}_1^0 \tau$ as a function of $\tan\beta$ for $m_{\tilde{\tau}_1} = 240$ GeV, $m_{\tilde{\tau}_2} = 500$ GeV, $\varphi_\mu = 0$(solid line), $\pi/2$(dashed line), π(dotted line), with the other parameters $\varphi_{A_\tau} = \varphi_{U(1)} = 0$, $M_2 = 200$ GeV, $|\mu| = 150$ GeV, and $|A_\tau| = 1000$ GeV, assuming $M_{\tilde{L}} < M_{\tilde{E}}$.

much work to do

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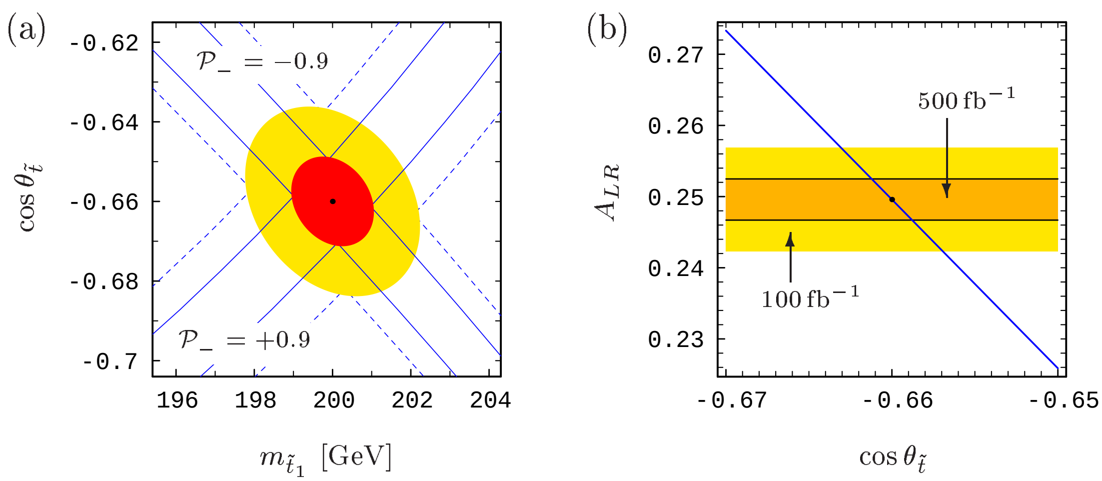
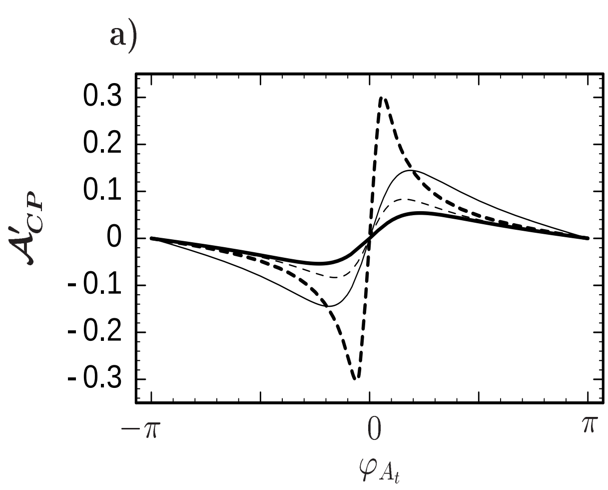
for $\tilde{t}$ and $\tilde{b}$, the $L-R$ mixing important

- measuring production cross sections with polarized beams $\Rightarrow$ determine masses and mixing angles



[Bartl et al. '00]

- if CP phases non-zero, measure τ^+ polarization $\mathcal{P}^+$ transverse to the decay plane in $\tilde{t}_1 \to b\tilde{\chi}_1^0\nu_\tau\tau^+$ and build a CP asymmetry $\mathcal{A}_{CP} = (\mathcal{P}^+ - \mathcal{P}^-)/2$

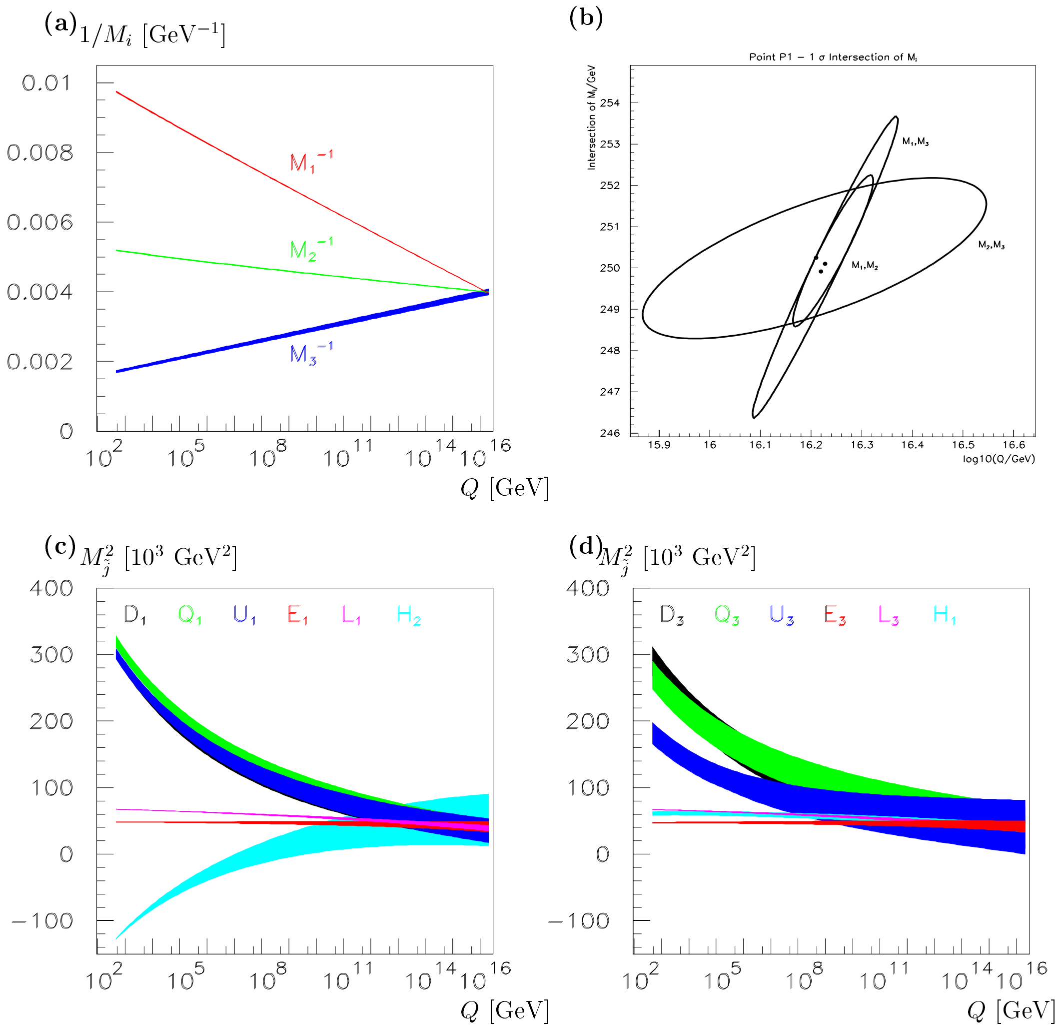


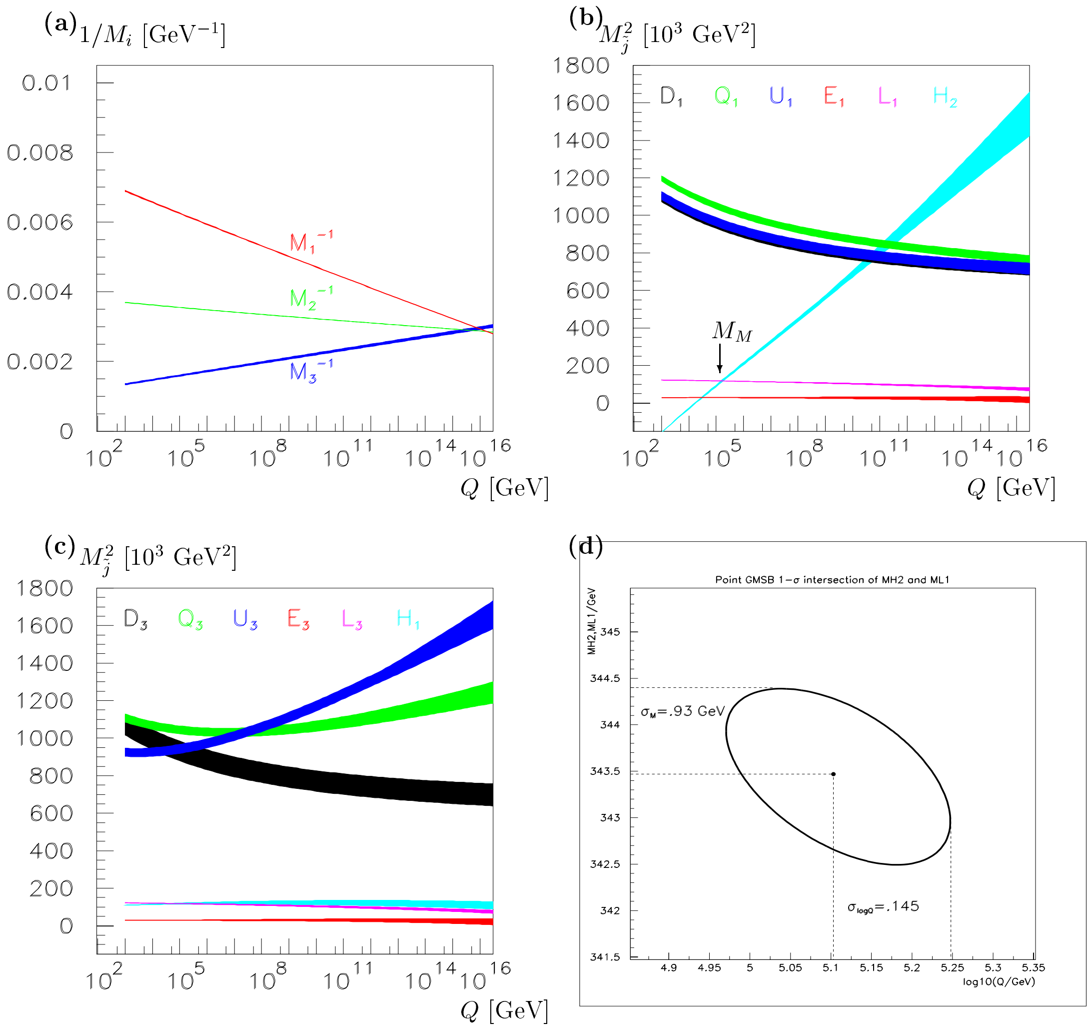
[Bartl et al. '02]

Why we need high precision measurements?

Extrapolate to high energy scale in bottom-up approach

example ⇒ **Porod in Session 5**




(a) 1/M_i [GeV^-1]
0.01
0.008
0.006
0.004
0.002
0
M_1^-1
M_2^-1
M_3^-1
Q [GeV]
(b) M_j^2 [10^3 GeV^2]
D_1 Q_1 U_1 E_1 L_1 H_2
M_M
Q [GeV]
(c) M_j^2 [10^3 GeV^2]
D_3 Q_3 U_3 E_3 L_3 H_1
Q [GeV]
(d)
Point GMSB 1−σ intersection of MH2 and ML1
MH2,ML1/GeV
σ_M=.93 GeV
σ_logQ=.145
log10(Q/GeV)

Conclusions:

- Data rules! We need them
- LHC will provide plenty of data
- their theoretical interpretation possible in specific models
- LC very much welcome
- overlap of LC running with LHC would greatly help
- critical test: quantum numbers, couplings
- precise measurements necessary to determine low-energy SUSY parameters
- the model-independent reconstruction of fundamental supersymmetric theory at the high scale