

# Radiative B Decays: Standard Candles of Flavor Physics

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## Abstract

Rare radiative decays based on  $b \rightarrow s\gamma$  transitions are among the most prominent examples of flavor-changing neutral current processes. They benefit from good theoretical control and experimental accessibility, large sensitivity to physics beyond the Standard Model, and the availability of many observables. In this talk I summarize the status of the theoretical understanding of these decays and review how they may be used to constrain extensions of the Standard Model, with particular focus on supersymmetric models.

## 1 Introduction

Rare radiative decays of  $B$  mesons mediated by the quark decay  $b \rightarrow s\gamma$  are prime examples of flavor-changing neutral current (FCNC) transitions. They are forbidden in the Standard Model (SM) at tree level and so are sensitive to the contributions of heavy particles in loop diagrams. Figure 1 illustrates this fact for the case of the SM, showing the strong dependence of the inclusive  $B \rightarrow X_s\gamma$  branching ratio on the mass of the top quark.

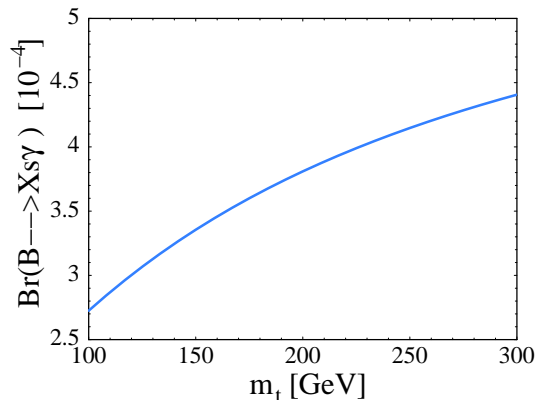


Figure 1: Sensitivity of the  $B \rightarrow X_s\gamma$  branching ratio to the top-quark mass.

$B \rightarrow X_s\gamma$  decays are an excellent probe for physics beyond the SM because their rate is small yet well measured experimentally, this rate can be calculated with high precision, and it shows large sensitivity to non-standard sources of flavor violation and CP violation. Thus, the study of these decays provides powerful constraints on many New Physics scenarios, including models with supersymmetry (SUSY).

## 2 Inclusive $B \rightarrow X_s\gamma$ Decay Rate

Starting point of the most sophisticated calculation in flavor physics is the effective weak Hamiltonian

$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_i C_i(\mu) Q_i(\mu).$$

Information about New Physics and heavy particles is encoded in the short-distance (Wilson) coefficient functions  $C_i$ , while the hadronic matrix elements of the operators  $Q_i$  contain all long-distance strong-interaction effects. The evaluation of these matrix elements constitutes the principal theoretical challenge in obtaining precise predictions for the decay rate. A consistent calculation of the Wilson coefficients at next-to-leading order (NLO) requires 3-loop anomalous dimensions [1], electroweak radiative corrections [2, 3, 4], and 2-loop matching conditions at the weak scale. These matching conditions depend on the underlying high-energy theory and so are sensitive to physics beyond the SM. They are known for the SM [5, 6], two-Higgs-doublet models (2HDMs) [7, 8], left-right symmetric models [9], the so-called “constrained minimal supersymmetric SM” (CMSSM) [9, 10], and the CMSSM with large  $\tan\beta$  [11, 12]. The relevant hadronic

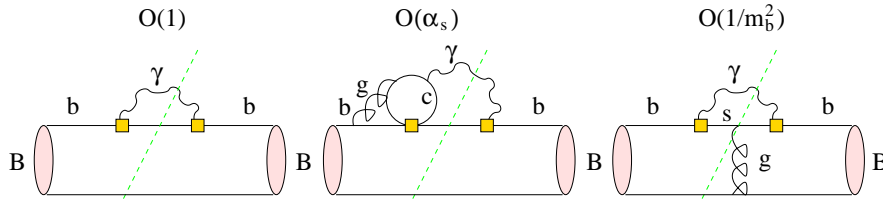


Figure 2: Application of the operator product expansion to the calculation of inclusive  $B$ -meson decay rates.

matrix elements required for the total inclusive  $B \rightarrow X_s \gamma$  decay rate are calculated using the operator product expansion, as illustrated in Figure 2. At NLO one needs 2-loop matrix elements of four-quark operators [13, 14]. It is state of the art to include power corrections of order  $(\Lambda_{\text{QCD}}/m_b)^2$  and  $(\Lambda_{\text{QCD}}/m_c)^2$  in the heavy-quark expansion, using the techniques of heavy-quark effective theory [15, 16, 17, 18, 19].

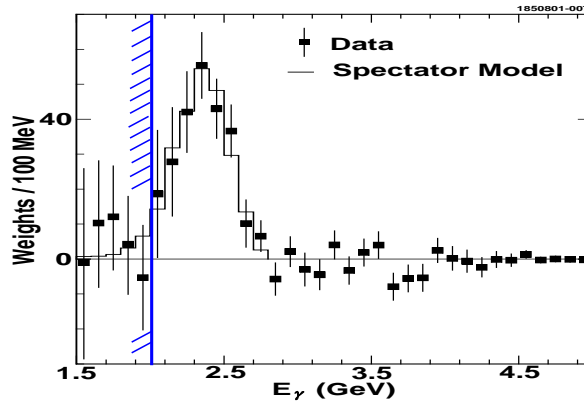


Figure 3: Cut on the photon energy applied in the CLEO analysis of the  $B \rightarrow X_s \gamma$  branching ratio [20].

Measurements of the inclusive  $B \rightarrow X_s \gamma$  decay rate rely on a cut on the photon energy in the  $B$ -meson rest frame, because only highly energetic photons can be distinguished from the background. Typically, only events with  $E_\gamma > 2.0$  GeV or so are recorded, as shown in Figure 3. Accounting for such a cut theoretically is difficult and introduces sensitivity to the shape of the photon spectrum (“Fermi motion”), which can be analyzed using the twist expansion [21, 22, 23].

Recently, there have been several improvements in the theoretical understanding of the inclusive  $B \rightarrow X_s \gamma$  branching ratio. It has been pointed out that, because the photon-energy cut ensures that no open charm can be produced in the final state, it is appropriate to use a running charm-quark mass rather than the pole mass in the calculation of diagrams containing charm-quark loops [24]. This leads to an enhancement of the rate by about 10%. The calculation of the 2-loop matrix elements for penguin operators has been completed, which however has a negligible effect on the rate [25]. Finally, it has been pointed out that one can avoid the common practice of normalizing the radiative rate to the semileptonic  $B \rightarrow X_c l \nu$  rate (which introduces additional uncertainties) by

using a physical  $b$ -quark mass definition instead of the pole mass [26]. The final results obtained with an (unrealistically low) energy cut  $E_\gamma > 1.6$  GeV are

$$\text{Br}(B \rightarrow X_s \gamma) = \begin{cases} (3.57 \pm 0.30) \cdot 10^{-4} & [25] , \\ (3.54 \pm 0.30) \cdot 10^{-4} & [26] . \end{cases}$$

Extrapolation of these values to  $E_\gamma > 2.0$  GeV yields [26]

$$\text{Br}(B \rightarrow X_s \gamma) = (3.26 \pm 0.27_{-0.18}^{+0.09}) \cdot 10^{-4} .$$

Here the second error accounts for the uncertainty in the treatment of Fermi motion [3]. The theoretical value compares well with the CLEO measurement  $\text{Br}(B \rightarrow X_s \gamma) = (2.94 \pm 0.39 \pm 0.25) \cdot 10^{-4}$  [20] obtained with  $E_\gamma > 2.0$  GeV. One should refrain from extrapolating the theoretical predictions down to much lower energies, where in any case no measurement can be done. This extrapolation is plagued by large theoretical uncertainties in the treatment of soft photons and  $c\bar{c}$  resonance production. Unfortunately, the Belle collaboration has chosen to extrapolate their measurement to lower energies using a theoretical model. Their result  $(3.36 \pm 0.53 \pm 0.42_{-0.54}^{+0.50}) \cdot 10^{-4}$  [27] may be compared with the theoretical extrapolation  $\text{Br}(B \rightarrow X_s \gamma) = (3.64 \pm 0.31) \cdot 10^{-4}$  obtained from the above results using the same model.

The most important conclusion to be drawn from the excellent agreement between SM theory and experimental data is that there appears to be no room left for drastic New-Physics effects in FCNC processes based on  $b \rightarrow s\gamma$  transitions.

### 3 Generic Implications for New Physics

A large portion of the New-Physics literature focuses on models with “minimal flavor violation”, whose primary motivation is to avoid disasters in the flavor sector (see [28] for an elegant effective field-theory approach to this class of models). In such models the CKM matrix is assumed to be the only source of quark-flavor mixing. Prominent examples are the type-II 2HDM and the CMSSM. In these scenarios only moderate FCNC effects are allowed after the constraints from electroweak precision data are taken into account. Models with minimal flavor violation are phenomenologically “preferred”, since data show no evidence for non-standard flavor or CP violation. However, these models are theoretically somewhat ad hoc.

More generic models of New Physics contain new sources of flavor violation. Examples are general SUSY extensions of the SM, models with new quark generations, etc. These models are more “natural”, since after all we expect some physics beyond the SM to explain the origin of flavor. Generically, however, they can have drastic effects on FCNC processes, such as  $K-\bar{K}$  mixing,  $B \rightarrow X_s \gamma$ ,  $K \rightarrow \pi \nu \bar{\nu}$ , etc. In particular, generic SUSY models can naturally lead to huge FCNC effects. (Or those effects were predicted before their existence was excluded experimentally.) The fact that such large effects are not realized in Nature is sometimes called the SUSY flavor problem.

At this point a caveat is in order. Most extensions of the SM come with a plethora of new parameters, most of which are related to the flavor sector (e.g., there are 43

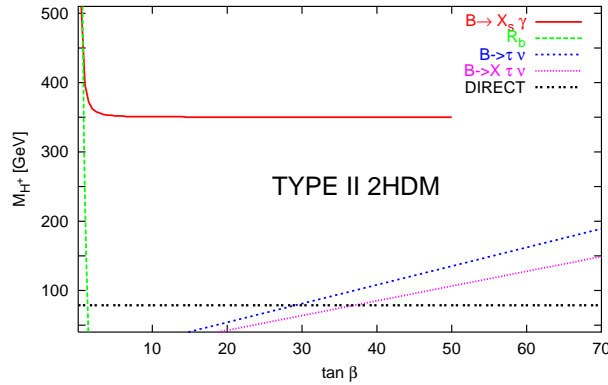


Figure 4: Bound on the charged-Higgs mass in the type-II 2HDM derived from the analysis of the inclusive  $B \rightarrow X_s \gamma$  branching ratio (solid line) [24].

new CP-violating phases in the MSSM!). In the context of particle searches it is a common (perhaps legitimate) practice to make vastly simplifying assumptions about these parameters, typically reducing their number from over a hundred (in “minimal” SUSY models) to less than about 5. These simplifications are dangerous in the context of flavor physics. In a generic model, basically each flavor-changing process receives its own New-Physics contributions. Adjusting flavor parameters in an ad hoc way may lead to correlations between observables that are strongly model dependent (such as, e.g., correlations between  $K-\bar{K}$  mixing  $\leftrightarrow B \rightarrow X_s \gamma \leftrightarrow K \rightarrow \pi \nu \bar{\nu}$ ).

## 4 Specific New-Physics Models

After these remarks, let me now discuss some particular New-Physics scenarios in more detail.

### 4.1 Type-II 2HDM

In this model there is a charged-Higgs contribution to the  $b \rightarrow s \gamma$  transition amplitude, which adds constructively to the SM contribution. As a result, one obtains a strong bound on the charged-Higgs mass, which is in fact stronger than the bounds obtained from direct searches. The complete NLO analysis of this bound has been presented in [7, 8]. The most recent evaluation yields  $m_{H^+} > 350$  GeV at 99% CL [24], as illustrated in Figure 4.

More generally, one should expect constructive or destructive interference of New-Physics effects with the SM contribution. A useful general formula is [3]

$$10^4 \text{Br}(B \rightarrow X_s \gamma) \approx 3.26 + 1.40 \text{Re} \xi_7 + 0.14 \text{Re} \xi_8 \\ + 0.37 \left( |\xi_7|^2 + |\xi_7^R|^2 \right) + 0.08 \text{Re} \left( \xi_7 \xi_8^* + \xi_7^R \xi_8^{R*} \right),$$

where the New-Physics contributions are parameterized as

$$\xi_{7,8} = \frac{C_{7,8}^{\text{NP}}(m_W)}{C_{7,8}^{\text{SM}}(m_W)}, \quad \xi_{7,8}^R = \frac{C_{7,8}^{R,\text{NP}}(m_W)}{C_{7,8}^{\text{SM}}(m_W)}.$$

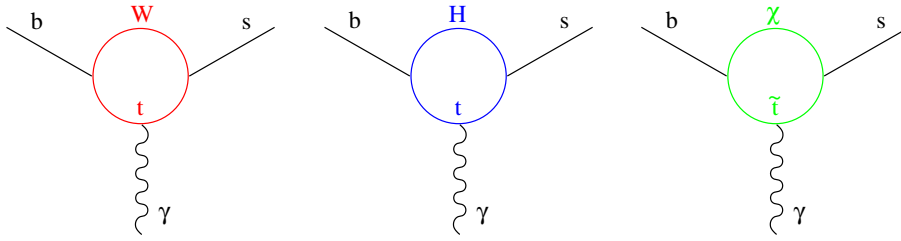


Figure 5: Examples of SUSY penguin diagrams present in the CMSSM.

Here  $C_7$  and  $C_8$  are the Wilson coefficients of the electro-magnetic and chromo-magnetic dipole operators, respectively, and  $C_{7,8}^R$  are the corresponding coefficients of non-standard operators with the opposite chirality of the quark fields. Note that despite the apparent agreement between data and SM theory (corresponding to  $\xi_i = 0$ ) it is still possible to have significant New-Physics contributions in  $B \rightarrow X_s \gamma$  decays, provided that  $\text{Re } \xi_{7,8} < 0$  (destructive interference) and one is willing to accept some moderate fine-tuning.

## 4.2 CMSSM with minimal flavor violation

In this highly constrained SUSY model there are three types of contributions to the dipole coefficients:

$$C_{7,8}(m_W) = \underbrace{C_{7,8}^{\text{SM}}(m_W)}_{\text{SM}} + \underbrace{C_{7,8}^{\text{H}}(m_W)}_{\text{type-II 2HDM}} + \underbrace{C_{7,8}^{\text{X}}(m_W)}_{\text{chargino-stop}}$$

They are illustrated in Figure 5. Several recent analyses of these contributions exist, some including novel higher-order terms that are enhanced for large  $\tan \beta$  [11, 12, 29, 30]. As shown in Figure 6, agreement with the data strongly favors negative values of  $\mu A_t$  (with positive  $\mu$ ). An important finding is that large- $\tan \beta$  corrections can weaken the bound on the charged-Higgs mass significantly, even in the decoupling limit.

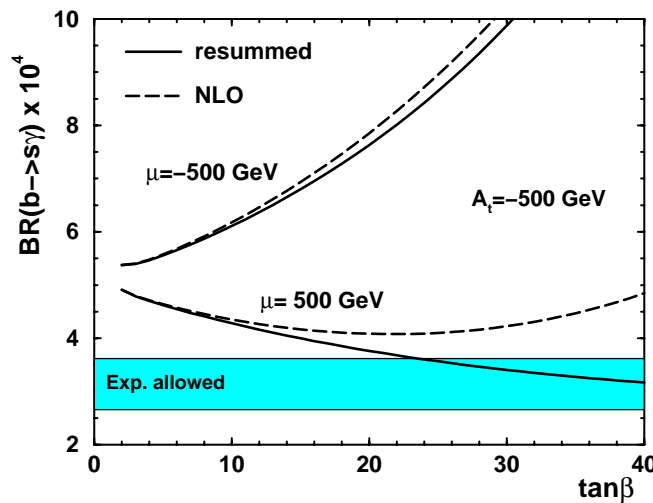


Figure 6:  $B \rightarrow X_s \gamma$  constraints on the parameters of the CMSSM [12].

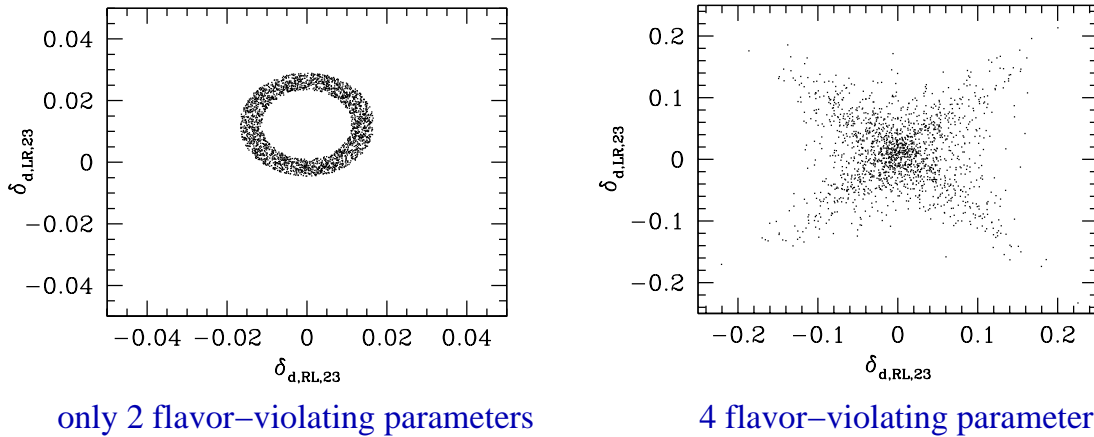


Figure 7: Constraints on SUSY flavor violation in a general scenario without minimal flavor violation [33].

### 4.3 Unconstrained MSSM

In a more general (but still “minimal”) SUSY scenario there are new flavor-changing quark-squark-gluino couplings, which can be parameterized in terms of off-diagonal entries in the squark mass matrix, e.g.  $\delta_{23}^{LR} = (m_{LR}^2)_{23}/m_q^2$ , which by naive power counting are expected to be of  $O(1)$ . Many analyses of such couplings have adopted the mass-insertion approximation (see, e.g., [31, 32]), accompanied by the simplifying assumption that a single flavor-changing coupling is responsible for the dominant New-Physics effects. Recently, a more complete analysis of SUSY flavor violation taking into account the interplay of contributions from gluinos, neutralinos, charginos, and charged Higgs has been presented [33]. As illustrated in Figure 7, the authors find that the resulting constraints on  $\delta_{23}^{LR,RL}$  can be significantly relaxed (typically by an order of magnitude) due to interference effects.

### 4.4 Flavor violation from light $\tilde{b}$ squarks

The presence of a light  $\tilde{b}$  squark with mass  $\sim 2\text{--}4$  GeV, accompanied by a light gluino with mass  $\sim 15$  GeV, could explain the observed excess of  $b$ -production at the Tevatron [34]. This would naturally give rise to new sources of  $b \rightarrow s$  FCNC transitions. The resulting flavor violations can be parameterized in terms of parameters  $\epsilon_{sb}^{LR}$  etc., which naively could be of  $O(1)$ . However, a complete NLO analysis of the inclusive  $B \rightarrow X_s \gamma$  branching ratio in this scenario yields extremely tight constraints on these couplings [26], as illustrated in Figure 8. This imposes severe constraints on model building.

## 5 CP Asymmetry in $B \rightarrow X_s \gamma$ Decays

Searching for direct CP violation in radiative decays provides an additional, powerful probe for physics beyond the SM [35, 36, 37, 38]. This is basically a null effect in the

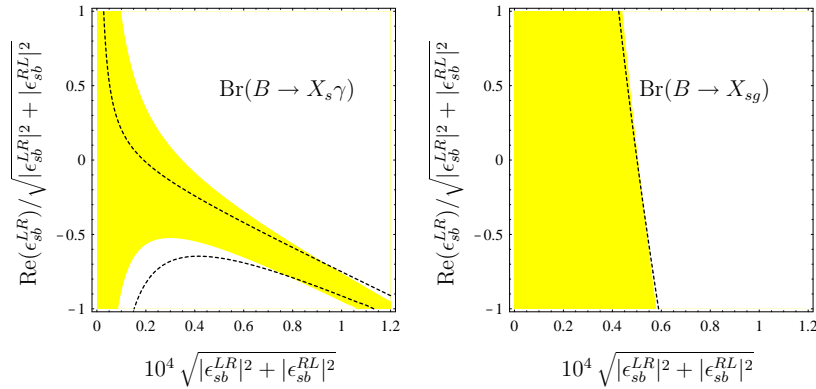


Figure 8: Constraints on the flavor-changing couplings describing SUSY flavor violations mediated by a light  $\tilde{b}$  squark [26].

SM, since

$$A_{\text{CP}}^{\text{SM}}(B \rightarrow X_s \gamma) \sim \underbrace{\alpha_s(m_b)}_{\text{strong phase}} \times \underbrace{\frac{V_{ub}}{V_{cb}}}_{\text{CKM suppr.}} \times \underbrace{\frac{m_c^2}{m_b^2}}_{\text{GIM suppr.}} \approx 0.5\% .$$

Moreover, in the SM the asymmetry vanishes (due to unitarity of the CKM matrix) if no distinction between  $s$  and  $d$  quarks in the final state is made.

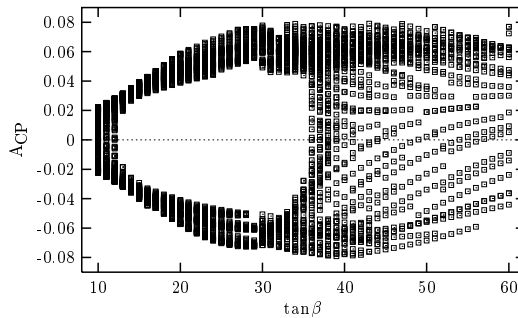


Figure 9: Direct CP violation in a SUSY model with minimal flavor violation but explicit CP violation [30].

Large CP asymmetries are possible in many extensions of the SM with new CP-violating couplings entering the Wilson coefficients. A useful approximate expression for the asymmetry (assuming that there are no new operators present) is [38]

$$A_{\text{CP}} \approx 1.3\% \text{Im}(C_2/C_7) - 9.5\% \text{Im}(C_8/C_7) .$$

The first term is important for models with  $|C_7| \approx |C_7^{\text{SM}}|$  but with a non-standard phase ( $\arg(C_7) \neq 0$ ), and can lead to CP asymmetries of about 5%. The second term is important for models with enhanced chromo-magnetic dipole transitions ( $C_8$ ) and new CP-violating couplings, and can lead to CP asymmetries exceeding 10–20% without conflicting with the total  $B \rightarrow X_s \gamma$  branching ratio. Figure 9 illustrates this fact in the context of the MSSM with minimal flavor violation but explicit CP violation ( $\phi_\mu, \phi_A \neq 0$ )



[29, 30]. Including large- $\tan\beta$  enhanced contributions beyond leading order, one finds significant complex contributions to  $C_{7,8}$ , which can lead to  $A_{\text{CP}}(B \rightarrow X_s\gamma)$  of order 10% without spoiling the SM prediction for the branching ratio.

## 6 Photon Spectrum as a QCD Tool

The  $B \rightarrow X_s\gamma$  photon-energy spectrum is insensitive to New-Physics effects and therefore a great QCD laboratory. It is useful for measuring with good precision some hadronic parameters that are important elsewhere in  $B$  physics and, in particular, for the determination of the CKM matrix. The moments  $\langle E_\gamma \rangle$  and  $(\langle E_\gamma^2 \rangle - \langle E_\gamma \rangle^2)$  of the photon spectrum provide a precise determination of the  $b$ -quark mass and other heavy-quark effective theory parameters, which helps in the determination of  $|V_{cb}|$ . Combining information from the  $B \rightarrow X_s\gamma$  photon spectrum and the  $B \rightarrow X_u l\nu$  charged-lepton spectrum provides for the currently best route to measuring  $|V_{ub}|$ , which is immensely important for unitarity-triangle physics at the  $B$  factories.

Let me illustrate this connection in a bit more detail. It has been shown long ago that the leading non-perturbative effects in the endpoint regions of the  $B \rightarrow X_s\gamma$  photon spectrum and the  $B \rightarrow X_u l\nu$  charged-lepton spectrum can be related to a universal shape function (up to  $\Lambda_{\text{QCD}}/m_b$  corrections) [23]. Using a measurement of the  $B \rightarrow X_s\gamma$  photon spectrum  $S(E_\gamma)$ , one can predict the fraction of  $B \rightarrow X_u l\nu$  events with charged-lepton energy  $E_l > E_0$  via

$$F_u(E_0) = \int_{E_0}^{m_B/2} dE_\gamma w(E_\gamma, E_0) S(E_\gamma),$$

where the weight function  $w(E_\gamma, E_0)$  is known including perturbative and  $\Lambda_{\text{QCD}}/m_b$  corrections [23, 39, 40, 41, 42]. One can then extract  $|V_{ub}|$  from a measurement of the  $B \rightarrow X_u l\nu$  decay rate in the region above 2.2 GeV. The resulting theoretical uncertainty on  $|V_{ub}|$  is of order 10% or less. The first experimental analysis using this strategy has been presented by CLEO [43]. It gives the rather precise value

$$|V_{ub}| = (4.08 \pm 0.56_{\text{exp}} \pm 0.29_{\text{th}}) \cdot 10^{-3}.$$

## 7 Exclusive Radiative Decays

Folklore says that the exclusive decays  $B \rightarrow K^*\gamma$  and  $B \rightarrow \rho\gamma$  are affected by large hadronic uncertainties and so are not very useful as far as searches for New Physics are concerned. This is a misconception.

### 7.1 QCD factorization

There has been significant recent progress in the theory of exclusive hadronic  $B$  decays based on QCD factorization theorems [44]. In particular, a factorization formula for  $B \rightarrow V\gamma$  decays (with  $V = K^*$  or  $\rho$ ) has been established [45, 46], which reads

$$\langle V\gamma(\epsilon)|Q_i|B \rangle = \left[ F_{B \rightarrow V}(0) T_i^I + \int_0^1 d\xi dx T_i^{II}(\xi, x) \Phi_B(\xi) \Phi_V(x) \right] \cdot \epsilon^*.$$

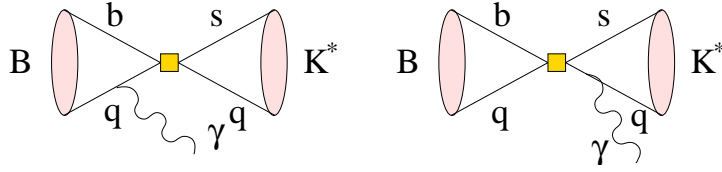


Figure 10: Dominant SM source of isospin violation in  $B \rightarrow K^* \gamma$  decays.

This formula is believed to be true to all orders in perturbation theory, and up to corrections of order  $\Lambda_{\text{QCD}}/m_b$ , which can be expected to be small.

The establishment of QCD factorization as the leading term in a rigorous heavy-quark expansion opens up novel strategies for New-Physics searches, since e.g. the CP asymmetries in exclusive modes can be enhanced with respect to those in inclusive decays. A particularly important application of this formalism concerns the CKM-suppressed  $b \rightarrow d\gamma$  transitions, where inclusive measurements are hindered by the large  $b \rightarrow s\gamma$  background. The SM prediction is that  $b \rightarrow d\gamma$  decays are about 20 times smaller than the corresponding  $b \rightarrow s\gamma$  decays,<sup>1</sup> but CP asymmetries are predicted to be 20 times larger!

## 7.2 Photon polarization

Radiative  $B$  decays in the SM predominantly have helicity structure  $b_R \rightarrow s_L \gamma_L$ ; however, in many extensions of the SM (left-right symmetric models, some SUSY models, etc.) there can be couplings with opposite helicity. It has been suggested that the photon polarization could be measured in exclusive decays of the type  $B \rightarrow K_{\text{res}} \gamma$  followed by  $K_{\text{res}} \rightarrow K^* \pi \rightarrow K \pi \pi$ , by studying the up-down asymmetry of the photon direction relative to the  $K \pi \pi$  decay plane [48]. The resulting asymmetry has been calculated to be  $(34 \pm 5)\%$  for  $K_1(1400)$ . Gross deviations from this prediction could signal the presence of opposite-chirality transitions induced by physics beyond the SM.

## 7.3 Isospin violation in $B \rightarrow K^* \gamma$ decays

In the SM, the theoretical prediction for the isospin asymmetry [49]

$$\Delta_{0-} = \frac{\Gamma(B^0 \rightarrow K^{*0} \gamma) - \Gamma(B^- \rightarrow K^{*-} \gamma)}{\Gamma(B^0 \rightarrow K^{*0} \gamma) + \Gamma(B^- \rightarrow K^{*-} \gamma)} = (8 \pm 3)\%$$

is dominated by a contribution due to the penguin operator  $Q_6 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}$ , as illustrated in Figure 10. As a result, this asymmetry is a direct probe of the sign and magnitude of the ratio  $\text{Re}(C_6/C_7)$  of Wilson coefficients, thus providing a completely new window to New Physics (in the sense of probing a new operator). If future precise measurements could establish a positive value for the asymmetry, as predicted by the SM, this would exclude a large portion of MSSM parameter space at large  $\tan \beta$  [49].

<sup>1</sup>This expectation is supported by the tight experimental bounds  $\text{Br}(B^- \rightarrow \rho^- \gamma) < 2.3 \cdot 10^{-6}$  and  $\text{Br}(B^0 \rightarrow \rho^0 \gamma) < 1.4 \cdot 10^{-6}$  reported by BaBar [47], which imply  $\text{Br}(B^- \rightarrow \rho^- \gamma)/\text{Br}(B^- \rightarrow K^{*-} \gamma) < 0.06$  and  $2\text{Br}(B^0 \rightarrow \rho^0 \gamma)/\text{Br}(B^0 \rightarrow K^{*0} \gamma) < 0.07$ .

## 8 Conclusions

Rare radiative decays based on the quark transition  $b \rightarrow s\gamma$  are the “mother” of all FCNC processes. They benefit from good theoretical control and experimental accessibility, large sensitivity to New Physics, and the availability of many observables (rates, CP asymmetries, photon polarization, isospin violation). The present data already place tight constraints on several extensions of the SM (including SUSY models), but more detailed analyses exploring many observables are needed to thoroughly probe for New Physics.

In this talk I had no time to discuss other, related processes such as  $b \rightarrow sl^+l^-$ ,  $b \rightarrow s\nu\bar{\nu}$ ,  $K \rightarrow \pi\nu\bar{\nu}$ , which are equally rich in their phenomenology and their reach for physics beyond the SM. Although analyses of flavor-changing processes in radiative and other rare  $B$  decays have so far not shown any evidence (within present errors) for physics beyond the SM, only a pessimist would use this fact as an argument against supersymmetry. An optimist would instead look forward to SUSY 2003!

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