LEPTON FLAVOUR VIOLATION

I. Masina (Saclay)

PLAN

① MOTIVATIONS for searches of → LFV: $\tau\to\mu\gamma$, $\mu\to e\gamma$, $\tau\to e$
↘ EDM: d_μ, d_e, d_τ

#1: $\not{L}_i$ IS there
#2: clean "smoking guns" for new phys.
#3: TEST for b.SM th's.

② Low En. SUSY : FLAVOUR PROBLEM !

→ review bounds on $\mathcal{L}_{soft}$: STRONG & INTERESTING informations

③ IMPLICATIONS for b.SM th + l.e. susy

E.g. → see-saw ← sort of "minimal" conte
↘ GUT (+ seesaw)

④ OUTLOOK

MOT. #1 : $\cancel{L}_i$ ($\gg \cancel{B}_i$)

The most **ELEGANT : SEE-SAW**

describe $\cancel{L}_i$ in **FUND. TH.**

$$\mathcal{L}_{h.e.} \ni \quad \nu^{cT}\, \hat{M}\, \nu^c + \nu^{cT} \underbrace{V_R\, \hat{y}_\nu\, V_L}_{y_\nu}\, \nu\, H_u^0$$

fl basis, $\hat{y}_e$

l.e. $\hookrightarrow$

$$y_\nu^T \frac{1}{\hat{M}}\, y_\nu v^2 = m_\nu^{eff} = U^*_{MNS}\, \hat{m}_\nu\, U^\dagger_{MNS} \leftarrow \begin{cases} \theta_{23}^{atm} \sim \pi/4 \\ \theta_{12}^{sol} \sim \pi/4 \\ \theta_{13}^{CHOOZ} \lesssim \theta_C \end{cases} \div \cancel{B}_i$$

***** $U_{MNS} \neq \mathbb{1}$ ⟿ $\cancel{L}_i$ at h.e. : $\boxed{V_L}$, V_R not BOTH $\mathbb{1}$

TRUE analog of V_{CKM}

Q: V_L small or large?

***** SEPARATE Knowledge from ignorance [Casas Ibarra '01]

$$\underbrace{\left(v V_L^T \hat{y}_\nu V_R^T \frac{1}{\sqrt{\hat{M}}}\right)}_{\text{disentangle need } R;} \left(\frac{1}{\sqrt{\hat{M}}} V_R \hat{y}_\nu V_L v\right) = \left(U^*_{MNS} \sqrt{\hat{m}_\nu}\, \overset{\mathbb{1}}{\overbrace{R\right)\left(R^T}} \sqrt{\hat{m}_\nu}\, U^\dagger_{MNS}\right)$$

further dis: need $\hat{M}$

R ← $O(3, \mathbb{C})$

$$\{\hat{y}_\nu, V_L, V_R, \hat{M} \;..\} \div \{\underbrace{\hat{m}_\nu, U_{MNS}}_{\text{"}\sim\text{known"}}, \underbrace{R, \hat{M}}_{\text{ignorance}}\}$$

fund. → V_L

	$\hat{y}_\nu$	V_L	V_R	$\hat{M}$	$\hat{m}_\nu$	U_{MNS}	R	$\hat{M}$
PAR. COUNT: $\mathbb{R}$	3	3	3	3	3	3	3	3
Φ_{CP}		1	5			3	3	

→ PHYS. MEANING OF R

[Lavignac, I.M., Savoy '02]

$$\sqrt{\hat{m}}\ R\ \sqrt{\hat{M}}$$

connects mass eig. of ν & ν^c

ROUGHLY encodes the **WEIGHTS** of the ν^c in determining m_i

$$\begin{array}{c|ccc|} & \overset{M_1}{\downarrow}\ \nu_1^c & \overset{M_2}{\downarrow}\ \nu_2^c & \overset{M_3}{\downarrow}\ \nu_3^c \\ \nu_1 & \times & \times & \times \\ \nu_2 & \times & \times & \times \\ m_3 \to \nu_3 & \otimes & \otimes & \otimes \end{array} = R$$

$$\text{E.g.: } \forall i \qquad m_{atm} \simeq m_3 = \overbrace{C_1\left(\frac{Y_{1i}^2}{M_1}\right)}^{=|R_{31}|^2 m_3} + \overbrace{C_2\left(\frac{Y_{2i}^2}{M_2}\right)}^{=|R_{32}|^2 m_3} + \overbrace{C_3\left(\frac{Y_{3i}^2}{M_3}\right)}^{=|R_{33}|^2 m_3}$$

BUT $R \in O(3, \mathbb{C})$! ⟹ MORE SUBTLE : $\cancel{CP}$ ph. ÷ boosts

→ PROCEEDE ?

Need OTHER OBSERVABLES

Y_L : [Fukugita Yanagida; Buchmuller Plumacher Pilaftsis Di Bari Barbieri, ...

Take $M_1 < M_2 < M_3$ $\qquad R_{ij} = |R_{ij}| e^{i\phi_{ij}/2}$

[Lisbon-Saclay Coll., in preparation]

$$\epsilon = \# \frac{-|R_{11}|^2 m_{sol}^2 \sin\phi_{11} + |R_{31}|^2 m_{atm}^2 \sin\phi_{31}}{|R_{11}|^2 m_1 + |R_{21}|^2 m_2 + |R_{31}|^2 m_3} \frac{M}{v}$$

comp. of ν_1^c in terms of m_i

N.B. : 1) $\underset{R_{i1}, \hat{m}, M_1}{\epsilon} \perp \underset{U_{MNS}, \hat{m}}{m_\nu^{eff}}$ $\qquad$ they have totally indep. $\cancel{CP}$ ph.

2) $Y_L \neq U_{MNS}^\dagger$: if it were → $R = \mathbb{1}$ → $\epsilon = 0$

BASIC MOT. #2 : new physics "smoking guns"

$$\mathcal{L}_{eff} = \Lambda^2 \phi^2$$
$$+ (D\phi)^2 + \bar\psi \not{D} \psi + F^2 + \bar\psi\psi\phi + \phi^4 \quad \Big\} \text{ SM } \; d \leq 4$$
$$+ \boxed{\frac{1}{\Lambda} \bar\psi\bar\psi\phi\phi} + \cdots \quad \rightsquigarrow m_\nu^{eff}$$
$$+ \frac{1}{\Lambda^2} \bar\psi\psi\bar\psi\psi + \boxed{\frac{1}{\Lambda^2} \bar\psi \sigma^{\mu\nu} \gamma_5 \psi F_{\mu\nu} \phi} + \cdots \quad \Big\} \text{ NEW PHYS } \; d > 4$$
$$+ \cdots$$

LFV	Δa_f	EDM
$\tau \to \mu\gamma$	Δa_μ	d_μ
$\mu \to e\gamma$	$\vdots$	d_e
$\tau \to e\gamma$		$\vdots$

$d>4$ op. could be computed if we knew the fund. th!

E.g. : seesaw : link between these observables ?

NONE (but m_ν^{eff}) **HAS BEEN SEEN** → UPPER LIMITS

BASIC MOT. #

TEST

for th. beyond S

→ non susy seesaw :

effects $\propto 1/M_R$ too small

→ low energy susy :

effects $\propto 1/m_{susy}$ SUSY FLAVOUR PROBLEM :

STRONG Constraints on $\mathcal{L}_{soft}$

EXP. STATUS & PROSPECTS

	present	future	SM
BR($\tau \to \mu\gamma$)	$1.1\ 10^{-6}$	10^{-9} (?)	–
BR($\mu \to e\gamma$)	$1.2\ 10^{-11}$	10^{-14} (PSI)	–
BR($\tau \to e\gamma$)	$2.7\ 10^{-6}$		–
d_μ (e cm)	10^{-18}	10^{-24} (BNL) 10^{-26} (KEK)	$< 10^{-35}$
d_e (e cm)	$1.5\ 10^{-27}$	10^{-29}	$< 10^{-38}$
d_τ	$3.1\ 10^{-16}$		$< 10^{-33}$

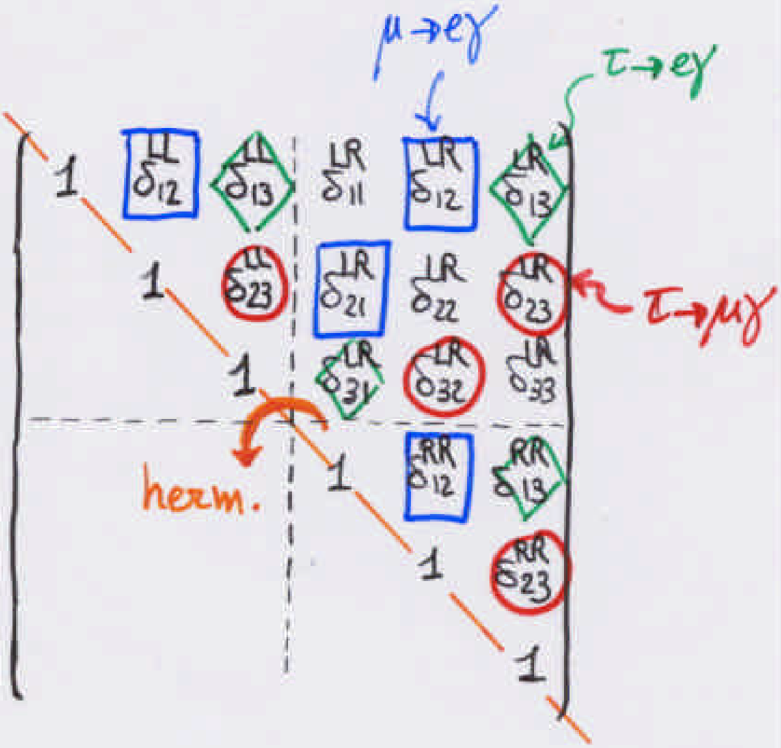
smoking guns for new phys.,
... if seen ...

CONSTRAINTS ON $\mathcal{M}^2_{\tilde{e}}$

$\hat{y}_e$ basis

6x6 herm. matrix
expr. f-$\tilde{f}$ misalign.

$$\mathcal{M}^2_{\tilde{e}} = \begin{array}{c|c:c|} & \tilde{L}_i & \tilde{e}^{c\,*}_i \\ \tilde{L}^*_i & m^2_{LL} & m^2_{LR} \\ \hdashline \tilde{e}^c_i & m^2_{RL} & m^2_{RR} \end{array} = \overline{m}^2_{\tilde{f}} \left[\begin{array}{ccc:ccc} 1 & \delta^{LL}_{12} & \delta^{LL}_{13} & \delta^{LR}_{11} & \delta^{LR}_{12} & \delta^{LR}_{13} \\ & 1 & \delta^{LL}_{23} & \delta^{LR}_{21} & \delta^{LR}_{22} & \delta^{LR}_{23} \\ & & 1 & \delta^{LR}_{31} & \delta^{LR}_{32} & \delta^{LR}_{33} \\ \hdashline & & & 1 & \delta^{RR}_{12} & \delta^{RR}_{13} \\ & & & & 1 & \delta^{RR}_{23} \\ & & & & & 1 \end{array} \right]$$



$\sim$deg $\tilde{f}$

[some mass.ins. ref:
Moroi Hisano Nomura Tobe
Yamaguchi Feng Matchev
Shadmi ....]

NON OBS. OF ⟶ **upper BOUNDS ON**

MDM: $\Delta a_{i_{e,\mu,\tau}} \longrightarrow \overline{m}^2_{\tilde{f}},\ \mathrm{Re}(\delta^{LR}_{ii})$

LFV: $l_i \to l_j \gamma \longrightarrow |\delta^{LL}_{ij}|, |\delta^{RR}_{ij}|, |\delta^{LR}_{ij}|, |\delta^{LR}_{ji}|$

EDM: $d_i \xrightarrow{FC} \mathrm{Im}(\delta^{LR}_{ii})$

$\xrightarrow[\exists j>i]{FV} \mathrm{Im}(\delta^{LL}_{ij}\delta^{LR}_{jj}\delta^{RR}_{ji}),\ \mathrm{Im}(\delta^{LR}_{ij}\delta^{RL}_{jj}\delta^{LR}_{ji}),\ \mathrm{Im}(\delta^{LL}_{ij}\delta^{LR}_{ij}),\ \mathrm{Im}(\delta^{LR}_{ij}\delta^{RR}_{ji})$

→ CATALOGUE of bounds on δ's : $\{M_1, m_{\tilde{e}^c}\}$ plane for fix $tg\beta$
[I.M. Savoy, in preparation]
(+ gaug. univ and μ from rad ew)

NOT ENCYCLOPEDIA : INSIGHTS

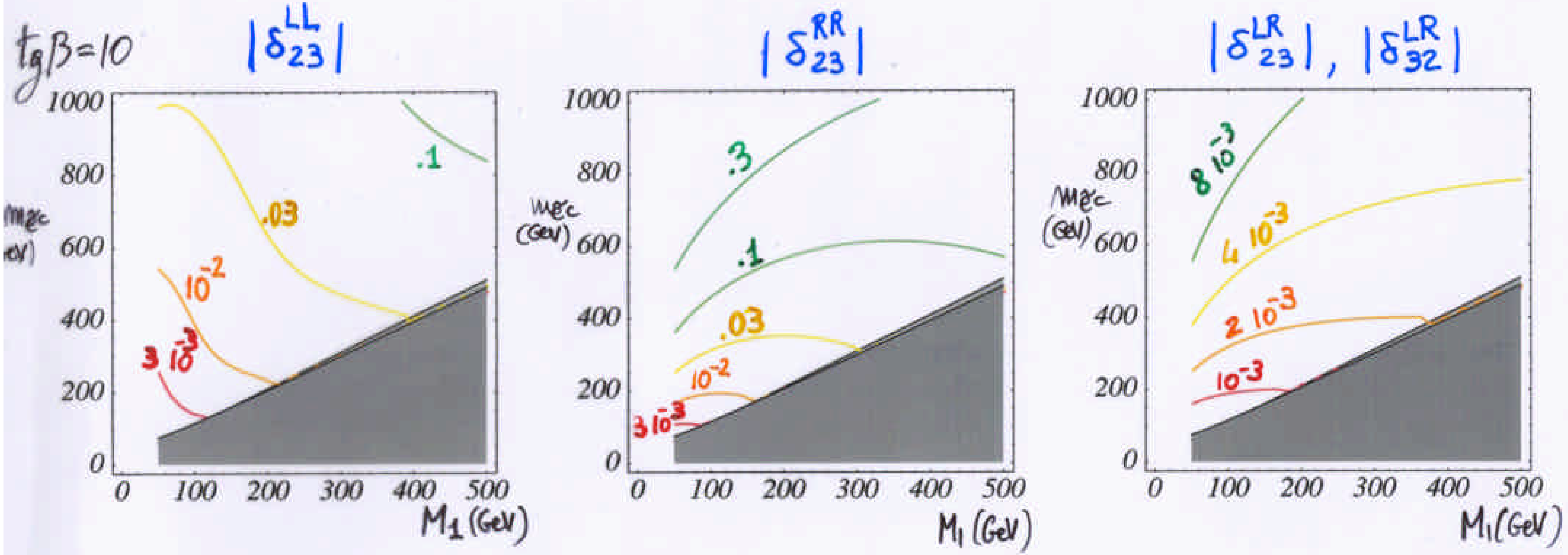
$BR(\tau\to\mu\gamma) \leq 10^{-6\ (-9)\,(fut)}$ (pr) $\longrightarrow d_\mu^{FV_{max}} \approx 10^{-21\,(-24)}$ ecm $= 10^{-3} d_\mu^{pr}$

$BR(\mu\to e\gamma) \leq 10^{-11}$ (pr) . $\longrightarrow d_e^{FV(12)max} \approx 10^{-28} = 10^{-1} d_e^{pr}$

} obs. $d_\mu > d$
⇓
meas. of FC

Constr. on ϕ_{ij} ? Measure BR $(l_i \to l_j\gamma)$ & its $d_i^{FV_{max}} > d_i^{exp.\,bound}$

$BR(\tau \to \mu\gamma) < 10^{-9}$ $\left(BR \propto \delta^2 \Rightarrow 10^{-6} = \text{present} : \times 30\right)$



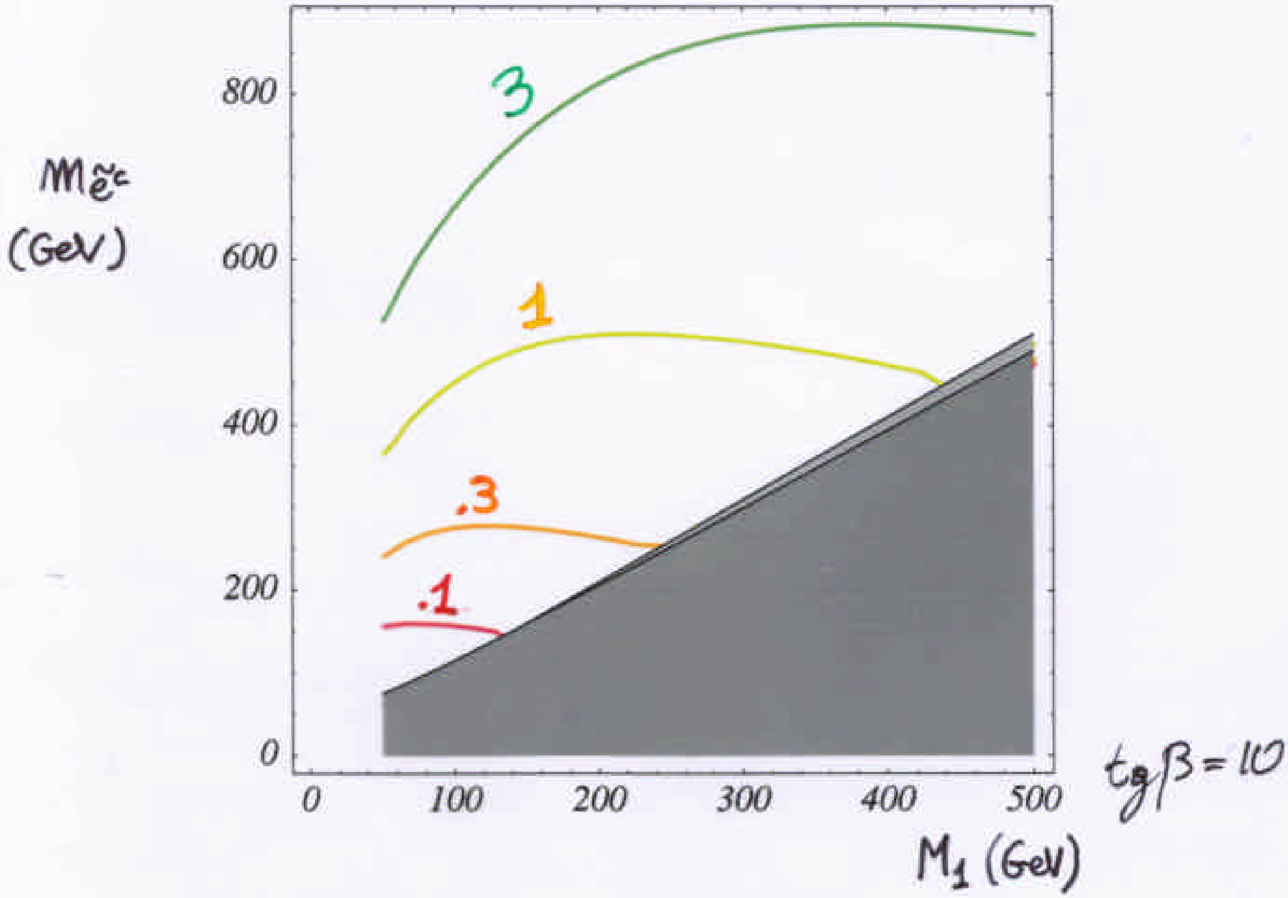
N.B. : $\dfrac{BR^{\tau\to\mu\gamma}}{BR^{\mu\to e\gamma}} = 17\%\ \dfrac{\delta_{23}^2}{\delta_{12}^2} \longrightarrow BR(\mu\to e\gamma) < 10^{-11}$: $\times\ 3\ 10^{-2}$

$< 10^{-14}$: $\times\ 10^{-3}$

$\dfrac{BR^{\tau\to\mu\gamma}}{BR^{\tau\to e\gamma}} = \dfrac{\delta_{23}^2}{\delta_{13}^2} \longrightarrow BR(\tau\to e\gamma) < 10^{-6}$: $\times 30$

$BR(\tau \to \mu\gamma) < 10^{-6}$

↳ $d_\mu^{FV\,max} \times 10^{-21}$ e cm



$\delta = ?$ TEST for b.SM th. with l.e. susy

$$\underset{\text{at }\Lambda}{\delta_{ij}} = \delta_{ij}^{(0)} + \delta_{ij}^{rad}$$

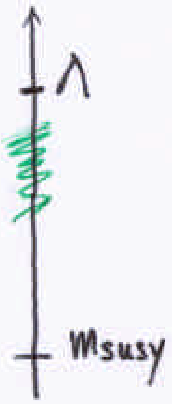
at Λ from Λ to m_{susy}

NO CONSPIRACY
↓
CONSTRAIN BOTH

EVEN IF $\delta^{(0)} = 0$ (univ & $\mathbb{R}$ b.c. at Λ)

δ^{rad} can be GENERATED FROM RGE

IF below Λ there are NEW INT. b.SM



E.g.

GUT [Barbieri Hall '94]

$Y_t \sim 1$ induces f-$\tilde f$ mis. but
- SU(5) too small
- SO(10) maybe [Romanino Strumia '01]

SEE-SAW [Borzumati Masiero '86]

$Y_{\nu_3} \sim 1$ can give f-$\tilde f$ mis.:

$$\overline{m}_{\tilde f}^{2}\, \delta_{ij}^{LL} = \tilde m_{ij}^{2\,LL} = \frac{1}{8\pi^2}\left(3m_0^2 + 2A_0^2\right)\underbrace{\left(Y_\nu^{+} \ln\frac{\Lambda}{\hat M}\, Y_\nu\right)_{ij}}_{\equiv C_{ij}}$$

→ BOUND ON δ_{ij}^{LL} → BOUND ON C_{ij}

→ IT HAS BEEN SHOWN [Ellis Lola Sato Tobe Yanagida Casas Ibarra

SOME SIMPLE seesaw realiz. → $\mu \to e\gamma$, $\tau \to \mu\gamma$ AT HAND

e.g.: $U(1)_{q \geq 0}$; $R = \mathbb{1}$; ...

kills already some model

Q: IS IT A GENERAL FEATURE OF THE SEE-SAW?

[Lavignac, I.M, Savoy '01]

→ NO, **BUT** LINK $\tau\to\mu\gamma$ ($\mu\to e\gamma$) rate to PHYS. CHARAC. of seesaw reali

e.g.: from $\tau\to\mu\gamma$ LEARN if V^L_{23} $\begin{smallmatrix}\text{large}\\ \text{small}\end{smallmatrix}$

IN FACT

$$C = Y_\nu^\dagger \ln\frac{\Lambda}{\hat M} Y_\nu \underset{\uparrow}{\approx} Y_\nu^\dagger Y_\nu \longrightarrow \mathcal{M}^2_{\tilde\ell} = \bar m^2_{\tilde f}\left(\begin{array}{ccc|c} \multicolumn{3}{c|}{\begin{smallmatrix}1 & \delta & \delta\\ & 1 & \delta\\ & & 1\end{smallmatrix}} & \\ \hline & & & \cdots \end{array}\right)$$

forget log

$\sim V_L^\dagger \hat d V_L$

$$= U_{MNS}\sqrt{\hat m}\, R^* \ln\frac{\Lambda}{\hat M}\,\hat M\, R^T \sqrt{\hat m}\, U^\dagger_{MNS}$$

LFV gives inf(constr) ON V_L:

$$\underset{\underset{?}{\uparrow}}{\theta^{(L)}}\nearrow \Rightarrow C\nearrow$$

CLASSIFICATION CRITERION FOR SEE-SAW MOD. (Hi case*)

*for ill bounds are important

$$R = \begin{array}{c|ccc|} & M_1 & M_2 & M_3 \\ & & & \\ m_{atm}\rightarrow & \times & \times & \times \end{array}$$

Who DOMINATES m_{atm}?

NONE
$(O(1)\ \ O(1)\ \ O(1))$

HEAVIEST: M_3
$(s,\ s,\ O(1))$

ONE OF THE LIGHTEST: M_1
$(s, O(1), s)$ or $(O(1), s, s)$

NONE / HEAVIEST:

$$\theta_{atm} \approx \theta^{(L)}_{23} \sim \pi/4$$

$M_3 \lesssim 5\ 10^{14}$ GeV

Y_ν asymm → SO(5)?

"RAIN" U(1) need f.t. for $m_2 < m_3$

no f.t. but need much richer fl. symm. than plain U(1): hol. or U(2),...

$$C_{\tau\mu} \sim 7$$

$$C_{\mu e} \sim \frac{m_2}{m_3}\,7$$

← dep on sol. solution

ONE OF THE LIGHTEST:

$$\theta_{atm} \neq \theta^{(L)}_{23} \lesssim \pi/4$$

$M_3 \gtrsim 10^{16}$ GeV

Y_ν can have SMALL θ : SO(10

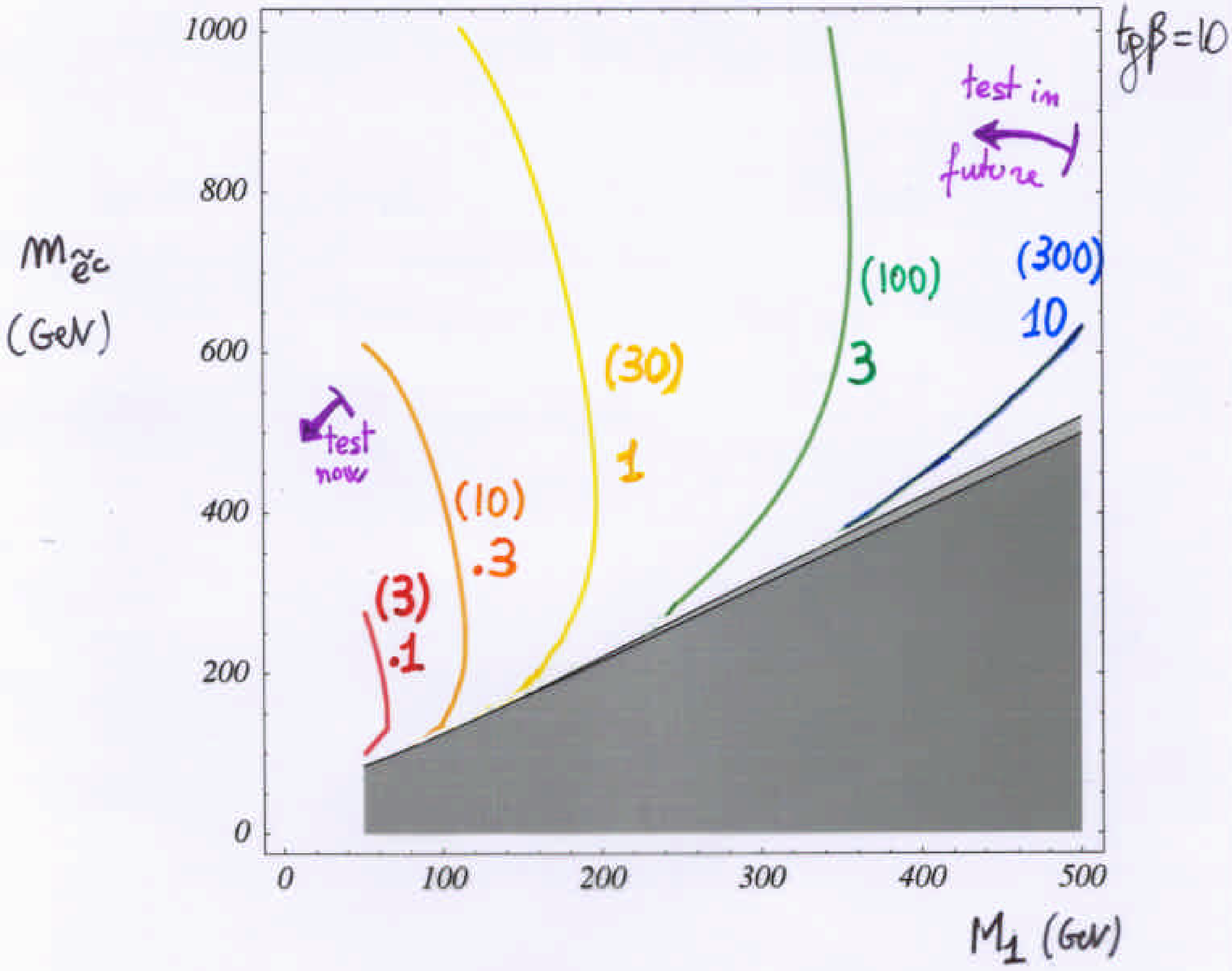
$$C_{\tau\mu} \lesssim 7$$

$C_{\mu e}$ a lot mod. dep.: no general connection with sol. solution

(-6) ← present

$BR(\tau \to \mu\gamma) \leq 10^{-9}$ ← future?

$C_{\tau\mu}$ (upper bound)

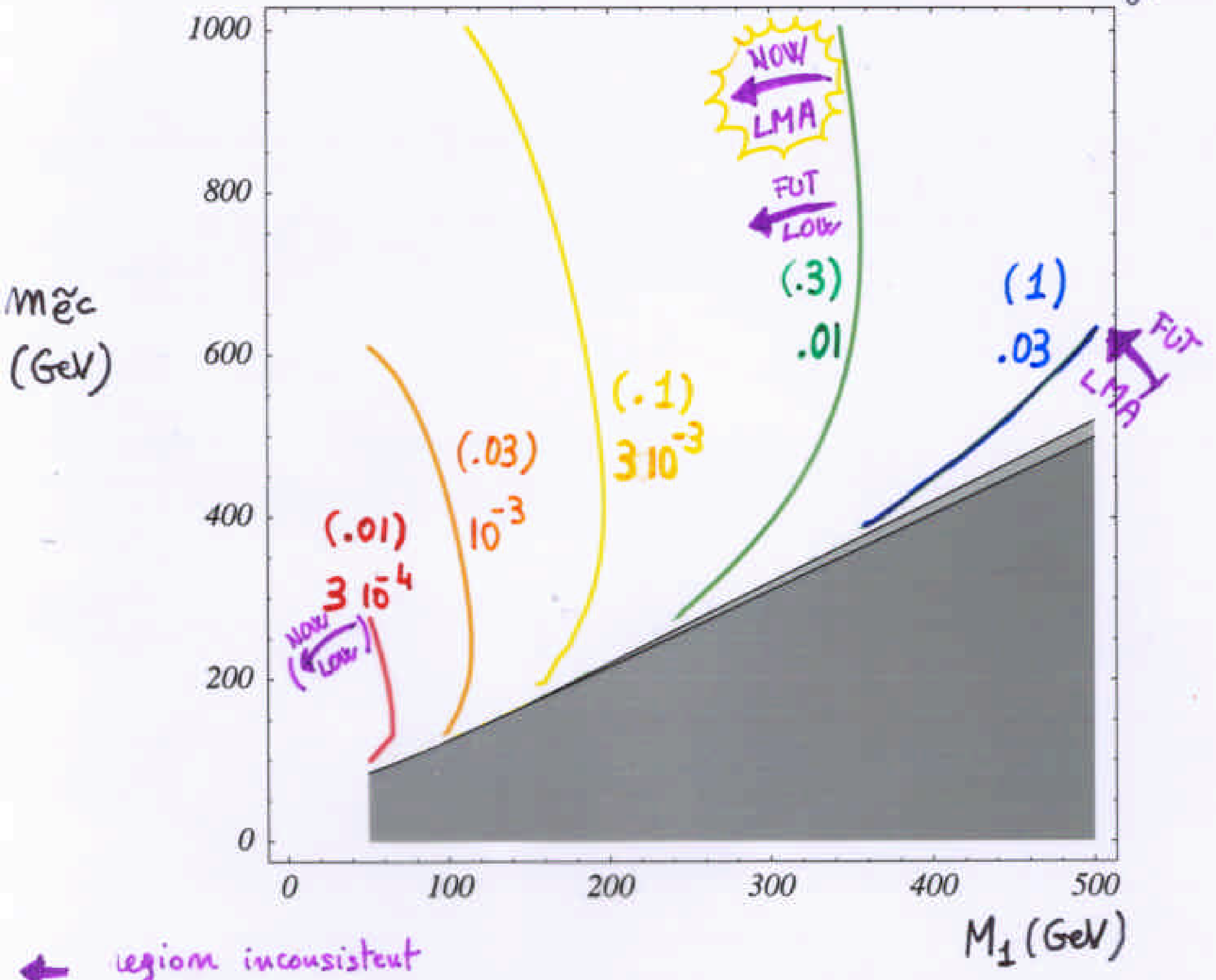


(−11) ← present

$BR(\mu \to e\gamma) \leq 10^{-14}$ ← future

$C_{\mu e}$ (upper bound)

$tg\beta = 10$



← region inconsistent with plain U(1)

Anarchical see saw : $C_{\mu e} \sim O(7)\, y^2$ already crisis for $y \sim O(1)$

EDM & SEE-SAW : $d_\mu \to 10^{-24(26)}$ e cm USEFUL for th ?

EVEN IF $\delta^{(0)} = 0$ (U&R b.c.) , $\delta^{rad} \neq 0$ for EDM

BUT $\hat{M} \propto \mathbb{1} \xrightarrow{FC\&FV} \dfrac{d_e}{d_\mu} \sim \dfrac{m_e}{m_\mu} \sim \dfrac{1}{200}$ $\qquad d_\mu < 10^{-25}$ e cm

$< 10^{-27}$ e cm $\qquad$ Too low ....

[Ellis Hisano Lola
Raidal Shimizu
Romanino Strumia, ...]
& NATH'S TALK

IF $\hat{M}$ hierarchical → avoid naive scaling in FC & FV

FC : $d_\mu \propto \mathrm{Im}\left[Y_\nu^\dagger \ln\dfrac{\Lambda}{\hat{M}} Y_\nu \,,\, Y_\nu^\dagger Y_\nu \right]_{22}$ [Ellis et al]

Phases of ϵ ? Take $M_1 < M_2 < M_3$ and

$$Y_\nu = \begin{pmatrix} s & s & s \\ s & |Y_{22}| e^{i\phi_{22}} & |Y_{23}| e^{i\phi_{23}} \\ s & \underset{\nearrow\,\mathbb{R}}{\textcircled{y_{32}}} & \underset{\nearrow\,\mathbb{R}}{\textcircled{y_{33}}} \end{pmatrix} \Rightarrow \begin{cases} d_\mu \propto |Y_{32} Y_{33} Y_{22} Y_{23}| \sin(\phi_{22} - \phi_{23}) + \cdots \\ \epsilon \propto f(s, y) \end{cases}$$

sub leading effects

→ NO direct relation

$\phi_{22} = \phi_{23} \not\Rightarrow \epsilon = 0$

FV : 4 contrib. of type $\mathrm{Im}\left(\overset{L,R}{\delta_{ij}^{\cdot\cdot}}\, \delta_{ji}^{\cdot\cdot}\right) < |\delta_{ij} \delta_{ji}|$ $\xrightarrow{\text{seesaw}}$ $\boxed{d_\mu^{FV} < d_\mu^{FV max}}$

Correlation of ph.

calculated fr
$\tau \to \mu\gamma$

[I.M. Savoy, in preparation]

CONCLUSIONS & OUTLOOK

→ Susy at l.e., even if f-$\tilde{f}$ are perfectly aligned at Λ,

$\not{K}_i$ of the seesaw (V_L, V_R) = SOURCE of f-$\tilde{f}$ mis

↓

LFV, EDM

Could be measured

COMPETITIVE &
COMPLEMENTARY
w.r.to DIRECT SEARCHES

→ LFV $\xleftrightarrow[\text{see saw}]{}$ V_L test models:

$\tau \to \mu \gamma$ test CLASSES with diff. phys. char

$\mu \to e \gamma$ more MOD. DEP. but STRONG INF.

EVEN A
NON-MEAS.
GIVES INF.
ON SEE-SAW

EDM : in seesaw need $\hat{M}$ very hier. to measure $> 10^{-25}$ e cm

→ EFFECT OF SEESAW ← "sort of minimal effect"

... AND IF YOU MEASURE SOMETHING?

↑ WHICH SOURCE?

seesaw ...?
GUT...?
else ...?

NEED TWO MEASUREMENTS :

$BR(\tau \to \mu \gamma)$

$\hookrightarrow d_\mu^{FV_{max}} \overset{?}{\longleftrightarrow} d_\mu$

$>$ d_μ from FC : ?

$<$ small ph in FV → seesaw

$\sim$ $O(1)$ ph in FV → seesaw + som. ELSE

↗ e.g. SU(5)