LEPTON FLAVOUR VIOLATION

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PLAN

① MOTIVATIONS for searches of
- LFV: $\tau\to\mu\gamma$, $\mu\to e\gamma$, $\tau\to e\gamma$
- EDM: d_μ, d_e, d_τ

#1: $\cancel{L_i}$ IS there
#2: clean "smoking guns" for new phys.
#3: TEST for b.SM th's.

② Low EN. SUSY : FLAVOUR PROBLEM!

→ review bounds on $\mathcal{L}_{soft}$: STRONG & INTERESTING informations

③ IMPLICATIONS for b.SM th + l.e. susy

E.g.
- → see-saw ← sort of "minimal" contr
- → GUT (+ seesaw)

④ OUTLOOK

MOT. #1 : $\cancel{L}_i$ ($\gg \cancel{B}_i$)

The most **ELEGANT** : **SEE-SAW**

describe $\cancel{L}_i$ in **FUND. TH.**

$$\mathcal{L}_{h.e.} \ni \nu^{cT} \hat{M} \nu^c + \nu^{cT} \underbrace{V_R \hat{y}_\nu V_L}_{y_\nu} \nu H_u^0$$

$f\ell$ basis, $\hat{y}_e$

$$\ell.e. \; \to \; y_\nu^T \frac{1}{\hat{M}} y_\nu v^2 = m_\nu^{eff} = U^*_{MNS} \hat{m}_\nu U^\dagger_{MNS}$$

$$\begin{cases} \theta_{23}^{atm} \sim \pi/4 \\ \theta_{12}^{sol} \sim \pi/4 \\ \theta_{13}^{CHOOZ} \lesssim \theta_C \quad \div \cancel{B}_i \end{cases}$$

* $U_{MNS} \neq \mathbb{1}$ $\rightsquigarrow$ $\cancel{L}_i$ at h.e. : V_L, V_R not BOTH $\mathbb{1}$

TRUE analog of V_{CKM}

Q: V_L small or large?

* SEPARATE Knowledge from ignorance [Casas Ibarra '01]

$$\underbrace{\left(v V_L^T \hat{y}_\nu V_R^T \frac{1}{\sqrt{\hat{M}}}\right)}_{\text{disentangle: need } R;} \left(\frac{1}{\sqrt{\hat{M}}} V_R \hat{y}_\nu V_L v\right) = \overbrace{(U^*_{MNS} \sqrt{\hat{m}_\nu} R)(R^T}^{\mathbb{1}} \sqrt{\hat{m}_\nu} U^\dagger_{MNS})$$

further dis: need $\hat{M}$

R: $O(3, \mathbb{C})$

$$\{\hat{y}_\nu, V_L, V_R, \hat{M}\} \div \{\underbrace{\hat{m}_\nu, U_{MNS}}_{\text{"}\sim\text{known"}}, \underbrace{R, \hat{M}}_{\text{ignorance}}\}$$

fund.

	$\hat{y}_\nu$	V_L	V_R	$\hat{M}$	$\hat{m}_\nu$	U_{MNS}	R	$\hat{M}$
PAR. COUNT: $\mathbb{R}$	3	3	3	3	3	3	3	3
Φ_{CP}		1	5			3	3	

→ PHYS. MEANING OF R [Lavignac, I.M., Savoy '02]

$\sqrt{\hat{m}}\ R\ \sqrt{\hat{M}}$

connects mass eig. of ν & ν^c

ROUGHLY encodes the **WEIGHTS** of the ν^c in determining m_i

$$\begin{array}{c|ccc|}
 & \overset{M_1}{\downarrow}\,\nu_1^c & \overset{M_2}{\downarrow}\,\nu_2^c & \overset{M_3}{\downarrow}\,\nu_3^c \\
\nu_1 & \times & \times & \times \\
\nu_2 & \times & \times & \times \\
m_3 \to \nu_3 & \times & \times & \times
\end{array} = R$$

E.g.: Hi

$$m_{atm} \simeq m_3 = \overbrace{C_1\left(\frac{Y_{1i}^2}{M_1}\right)}^{=|R_{31}|^2 m_3} + \overbrace{C_2\left(\frac{Y_{2i}^2}{M_2}\right)}^{=|R_{32}|^2 m_3} + \overbrace{C_3\left(\frac{Y_{3i}^2}{M_3}\right)}^{=|R_{33}|^2 m_3}$$

BUT $R \in O(3,\mathbb{C})$! ⟹ MORE SUBTLE : $\not{CP}$ ph. ÷ boosts

→ PROCEEDE? Need OTHER OBSERVABLES

Y_L : [Fukugita Yanagida; Buchmuller Plumacher Pilaftsis Di Bari Barbieri, ...

Take $M_1 < M_2 < M_3$ $\qquad R_{ij} = |R_{ij}| e^{i\phi_{ij}/2}$

[Lisbon-Saclay Coll., in preparation]

$$\epsilon = \# \frac{-|R_{11}|^2 m_{sol}^2 \sin\phi_{11} + |R_{31}|^2 m_{atm}^2 \sin\phi_{31}}{|R_{11}|^2 m_1 + |R_{21}|^2 m_2 + |R_{31}|^2 m_3} \frac{M}{v^2}$$

comp. of ν_1^c in terms of m_i

N.B. : 1) $\underset{R_{i1},\hat{m},M_1}{\epsilon} \perp \underset{U_{MNS},\hat{m}}{m_\nu^{eff}}$ they have totally indep. $\not{CP}$ ph.

2) $Y_L \neq U_{MNS}^\dagger$: if it were → $R = \mathbb{1}$ → $\epsilon = 0$

BASIC MOT. #2 : new physics "smoking guns"

$$\mathcal{L}_{eff} = \Lambda^2 \phi^2$$

$$+ (D\phi)^2 + \bar\psi \not{D} \psi + F^2 + \bar\psi\psi\phi + \phi^4 \quad \Big\} \text{ SM } d \le 4$$

$$+ \frac{1}{\Lambda} \bar\psi\bar\psi\phi\phi + \cdots$$

m_ν^{eff}

$$+ \frac{1}{\Lambda^2} \bar\psi\psi\bar\psi\psi + \frac{1}{\Lambda^2} \bar\psi \sigma^{\mu\nu} \gamma_5 \psi F_{\mu\nu} \phi + \cdots \quad \Big\} \text{ NEW PHYS } d > 4$$

$$+ \cdots$$

LFV	Δa_f	EDM
$\tau \to \mu\gamma$ $\mu \to e\gamma$ $\tau \to e\gamma$	Δa_μ $\vdots$	d_μ d_e $\vdots$

$d > 4$ op. could be computed if we knew the fund. th !

E.g. : seesaw : link between these observables ?

NONE (but m_ν^{eff}) **HAS BEEN SEEN** $\to$ UPPER LIMITS

BASIC MOT. #3

TEST

for th. beyond SM

$\to$ non susy seesaw :

effects $\propto 1/M_R$ too small

$\to$ low energy susy :

effects $\propto 1/m_{susy}$ SUSY FLAVOUR PROBLEM : STRONG Contraints on $\mathcal{L}_{soft}$

EXP. STATUS & PROSPECTS

	present	future	SM
$BR(\tau\to\mu\gamma)$	$1.1\ 10^{-6}$	10^{-9} (?)	–
$BR(\mu\to e\gamma)$	$1.2\ 10^{-11}$	10^{-14} (PSI)	–
$BR(\tau\to e\gamma)$	$2.7\ 10^{-6}$		–
d_μ (e cm)	10^{-18}	10^{-24} (BNL) 10^{-26} (KEK)	$< 10^{-35}$
d_e (e cm)	$1.5\ 10^{-27}$	10^{-29}	$<10^{-38}$
d_τ	$3.1\ 10^{-16}$		$<10^{-33}$

smoking guns for new phys.,
... if seen ...

CONSTRAINTS ON $\mathcal{M}^2_{\tilde{e}}$

$\hat{y}_e$ basis

6×6 herm. matrix
expr. $\tilde{f}$-f misalign.

$$\mathcal{M}^2_{\tilde{e}} = \begin{array}{c|cc} & \tilde{L}_i & \tilde{e}^{c\,*}_i \\ \hline \tilde{L}^*_i & m^2_{LL} & m^2_{LR} \\ \tilde{e}^c_i & m^2_{RL} & m^2_{RR} \end{array} = \overline{m}^2_{\tilde{f}} \left(\begin{array}{ccc|ccc} 1 & \boxed{\delta^{LL}_{12}} & \delta^{LL}_{13} & \delta^{LR}_{11} & \boxed{\delta^{LR}_{12}} & \delta^{LR}_{13} \\ & 1 & \delta^{LL}_{23} & \boxed{\delta^{LR}_{21}} & \delta^{LR}_{22} & \delta^{LR}_{23} \\ & & 1 & \delta^{LR}_{31} & \delta^{LR}_{32} & \delta^{LR}_{33} \\ \hline & & & 1 & \boxed{\delta^{RR}_{12}} & \delta^{RR}_{13} \\ & & & & 1 & \delta^{RR}_{23} \\ & & & & & 1 \end{array} \right)$$

$\mu \to e\gamma$

$\tau \to e\gamma$

$\tau \to \mu\gamma$

herm.

$\sim$deg $\tilde{f}$

[some mass.ins. ref: Moroi Hisano Nomura Tobe Yamaguchi Feng Matchev Shadmi ....]

NON OBS. OF $\longrightarrow$ upper BOUNDS ON

MDM: $\Delta a_{i_{e,\mu,\tau}} \longrightarrow \overline{m}^2_{\tilde{f}},\ \mathrm{Re}(\delta^{LR}_{ii})$

LFV: $\ell_i \to \ell_j \gamma \longrightarrow |\delta^{LL}_{ij}|, |\delta^{RR}_{ij}|, |\delta^{LR}_{ij}|, |\delta^{LR}_{ji}|$

EDM: $d_i \xrightarrow{FC} \mathrm{Im}(\delta^{LR}_{ii})$

$ \xrightarrow[\exists j > i]{FV} \mathrm{Im}(\delta^{LL}_{ij}\delta^{LR}_{jj}\delta^{RR}_{ji}),\ \mathrm{Im}(\delta^{LR}_{ij}\delta^{RL}_{jj}\delta^{LR}_{ji}),\ \mathrm{Im}(\delta^{LL}_{ij}\delta^{LR}_{ij}),\ \mathrm{Im}(\delta^{LR}_{ij}\delta^{RR}_{ji})$

$\to$ CATALOGUE of bounds on δ's : $\{M_1, m_{\tilde{e}^c}\}$ plane for fix $tg\beta$
[I.M. Savoy, in preparation] (+ gaug. univ and μ from rad ew)

NOT ENCYCLOPEDIA : INSIGHTS

$BR(\tau \to \mu\gamma) \lesssim 10^{-6}$ (pr) (-9) (fut) $\longrightarrow d_\mu^{FV_{max}} \approx 10^{-21}$ (-24) ecm $= 10^{-3}\, d_\mu^{pr}$

$BR(\mu \to e\gamma) \lesssim 10^{-11}$ (pr) $\longrightarrow d_e^{FV(12)max} \approx 10^{-28} = 10^{-1}\, d_e^{pr}$

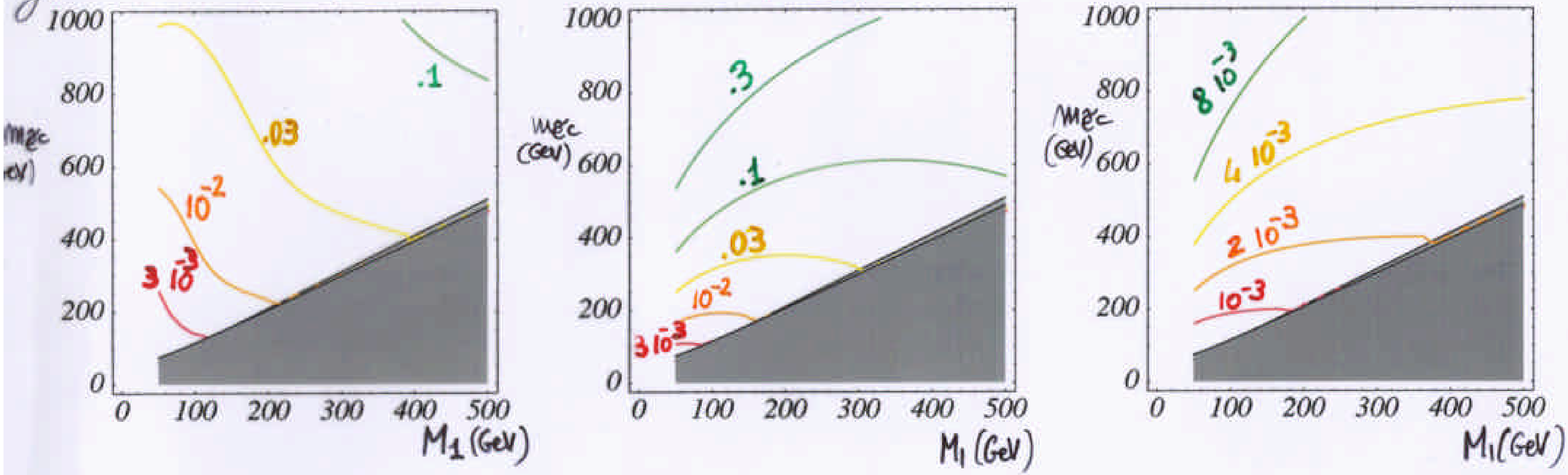
obs. $d_\mu > d_\mu^{\dots}$
$\Downarrow$
meas. of FC

Constr. on ϕ_{ij} ? Measure BR $(\ell_i \to \ell_j\gamma)$ & its $d_i^{FV_{max}} > d_i^{exp.\,bound}$

$BR(\tau \to \mu\gamma) < 10^{-9}$

$\left(BR \propto \delta^2 \Rightarrow 10^{-6} = \text{present} : \times 30 \right)$

$tg\beta = 10$ $\quad |\delta^{LL}_{23}| \quad |\delta^{RR}_{23}| \quad |\delta^{LR}_{23}|, |\delta^{LR}_{32}|$



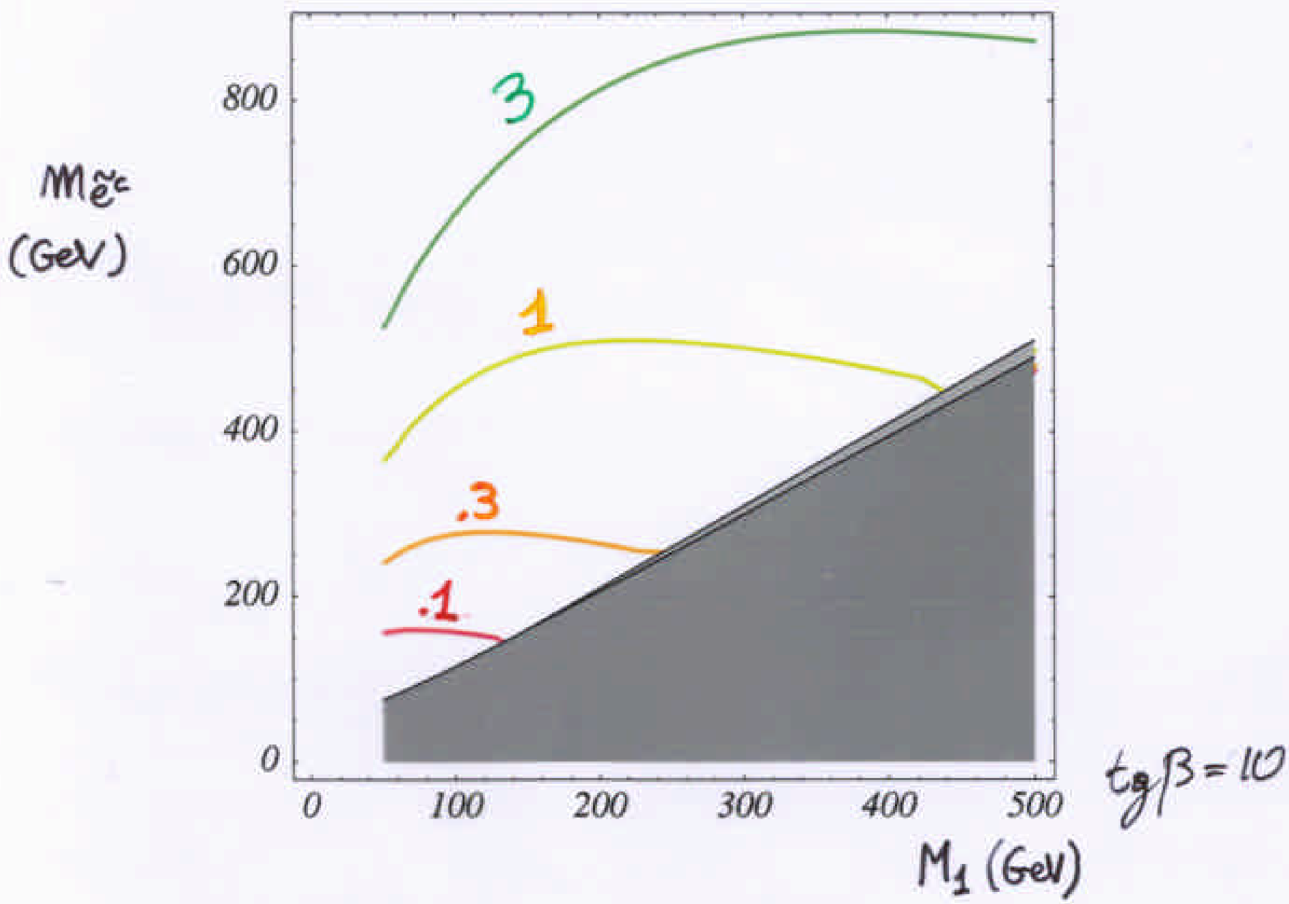
N.B. : $\dfrac{BR^{\tau\to\mu\gamma}}{BR^{\mu\to e\gamma}} = 17\%\ \dfrac{\delta_{23}^2}{\delta_{12}^2} \longrightarrow BR(\mu\to e\gamma) < 10^{-11} \quad : \times\ 3\ 10^{-2}$

$< 10^{-14} \quad : \times\ 10^{-3}$

$\dfrac{BR^{\tau\to\mu\gamma}}{BR^{\tau\to e\gamma}} = \dfrac{\delta_{23}^2}{\delta_{13}^2} \longrightarrow BR(\tau\to e\gamma) < 10^{-6} \quad : \times 30$

$BR(\tau \to \mu\gamma) < 10^{-6}$

$\hookrightarrow \quad d_\mu^{FV\,max} \times 10^{-21}$ e cm



$\delta = ?$ TEST for b.SM th. with l.e. susy

$$\delta_{ij} = \delta_{ij}^{(0)} + \delta_{ij}^{rad}$$

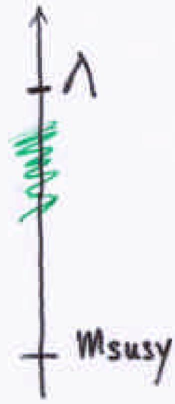
at Λ — from Λ to m_{susy}

NO CONSPIRACY
$\downarrow$
CONSTRAIN BOTH

EVEN IF $\delta^{(0)} = 0$ (univ & $\mathbb{R}$ b.c. at Λ)

δ^{rad} can be GENERATED FROM RGE

IF below Λ there are NEW INT. b.SM



E.g.

GUT [Barbieri Hall '94]

$Y_t \sim 1$ induces f-$\tilde{f}$ mis. but $\to$ SU(5) too small; $\to$ SO(10) maybe [Romanino Strumia '01]

SEE-SAW [Borzumati Masiero '86]

$Y_{\nu_3} \sim 1$ can give f-$\tilde{f}$ mis. :

$$\bar{m}_{\tilde{f}}^2 \, \delta_{ij}^{LL} = \tilde{m}_{ij}^{2\,LL} = \frac{1}{8\pi^2}(3m_0^2 + A_0^2)\underbrace{\left(Y_\nu^\dagger \ln\frac{\Lambda}{\hat{M}} Y_\nu\right)_{ij}}_{\equiv C_{ij}}$$

$\to$ BOUND ON δ_{ij}^{LL} $\to$ BOUND ON C_{ij}

$\to$ IT HAS BEEN SHOWN [Ellis Lola Sato Tobe Yanagida Casas Ibarra

SOME SIMPLE seesaw realiz. $\to$ $\mu \to e\gamma$, $\tau \to \mu\gamma$ AT HAND

e.g.: $U(1)_{q \geq 0}$; $R = \mathbb{1}$; ...

kills already some model

Q: IS IT A GENERAL FEATURE OF THE SEE-SAW?

[Lavignac, I.M., Savoy '01]

→ NO, **BUT** LINK $\tau\to\mu\gamma$ ($\mu\to e\gamma$) rate to PHYS. CHARAC. of seesaw reali

e.g.: from $\tau\to\mu\gamma$ LEARN if $V_{L_{23}}$ large / small

IN FACT

$$C = Y_\nu^\dagger \ln\frac{\Lambda}{\hat{M}} Y_\nu \underset{\substack{\uparrow \\ \text{forget log}}}{\approx} Y_\nu^\dagger Y_\nu \longrightarrow \mathcal{M}^2_{\tilde{\ell}} = \overline{m}_{\tilde{f}}^2 \begin{pmatrix} \begin{matrix} 1 & \delta & \delta_3 \\ & 1 & \delta_2 \\ & & 1 \end{matrix} & \vdots \\ \cdots & \cdots \end{pmatrix}$$

$\sim V_L^\dagger \hat{d} V_L$

$$= U_{MNS}\sqrt{\hat{m}}\, R^* \ln\frac{\Lambda}{\hat{M}}\, \hat{M}\, R^T \sqrt{\hat{m}}\, U^\dagger_{MNS}$$

LFV gives inf (constr) ON V_L:

$\theta^{(L)} \nearrow \Rightarrow C \nearrow$

?

CLASSIFICATION CRITERION FOR SEE-SAW MOD. (Hi case*)

$$\begin{array}{c c} & \begin{matrix} M_1 & M_2 & M_3 \end{matrix} \\ R = & \begin{pmatrix} & & \\ & & \\ \times & \times & \times \end{pmatrix} \\ & \leftarrow m_{atm} \end{array}$$

Who DOMINATES m_{atm}?

* for ill bounds are important

NONE	HEAVIEST: M_3	ONE OF THE LIGHTEST: M_i
(O(1) O(1) O(1))	(s, s, O(1))	(s, O(1), s) or (O(1), s, s)

$\theta_{atm} \approx \theta^{(L)}_{23} \sim \pi/4$

$M_3 \lesssim 5\,10^{14}$ GeV

Y_ν asymm → SO(5)?

"RAIN" U(1) need f.t. for $m_2 < m_3$

no f.t. but need much richer fl. symm. than plain U(1): hol. or U(2), ...

$C_{\tau\mu} \sim 7$

$C_{\mu e} \sim \frac{m_2}{m_3} 7$ ← dep on sol. solution

$\theta_{atm} \neq \theta^{(L)}_{23} \lesssim \pi/4$

$M_3 \gtrsim 10^{16}$ GeV

Y_ν can have SMALL θ : SO(10)

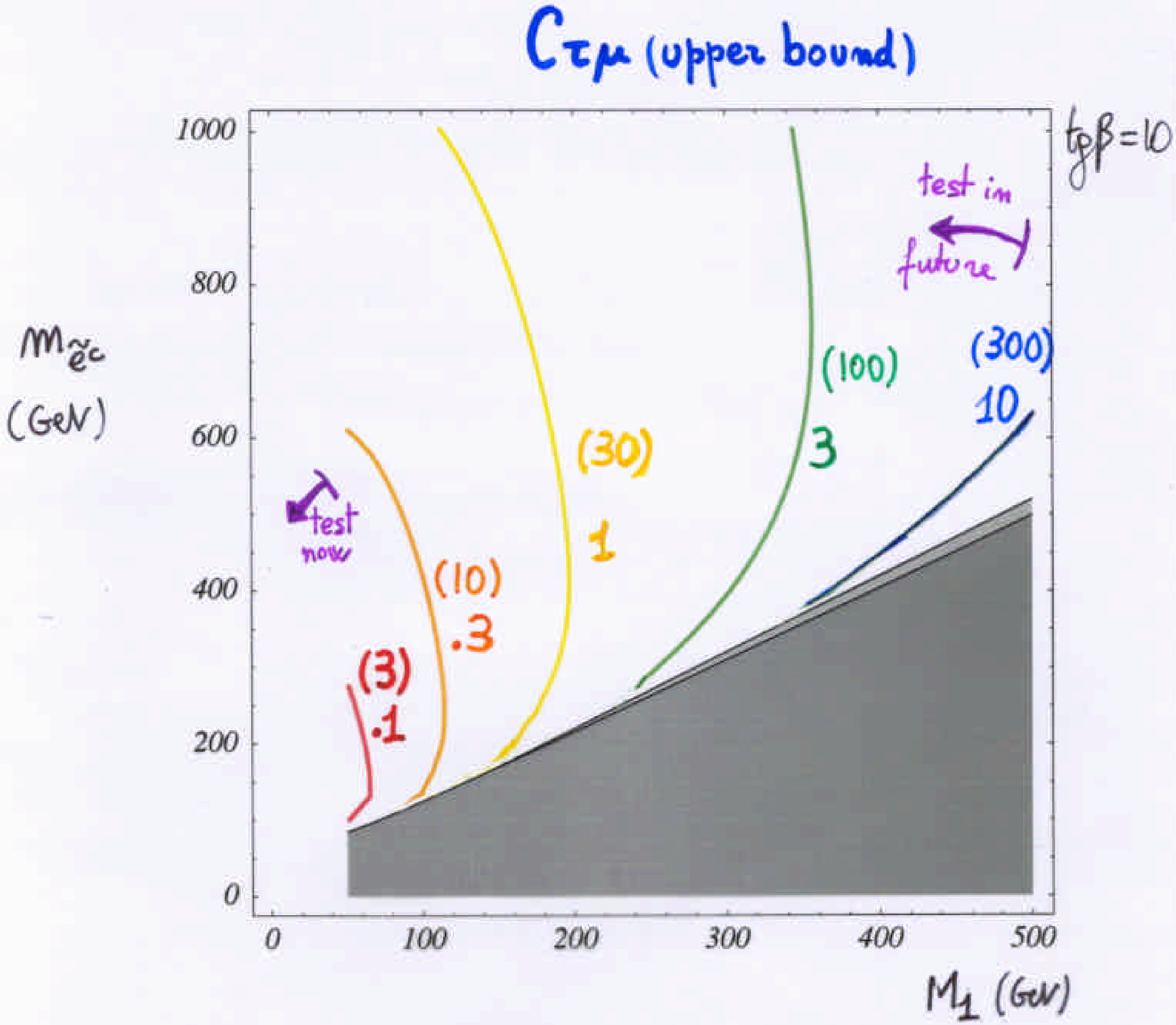
$C_{\tau\mu} \lesssim 7$

$C_{\mu e}$ a lot mod. dep.: no general connection with sol. solution

$$BR(\tau \to \mu\gamma) \leq 10^{-9} \overset{(-6)}{} $$

(−6) ← present

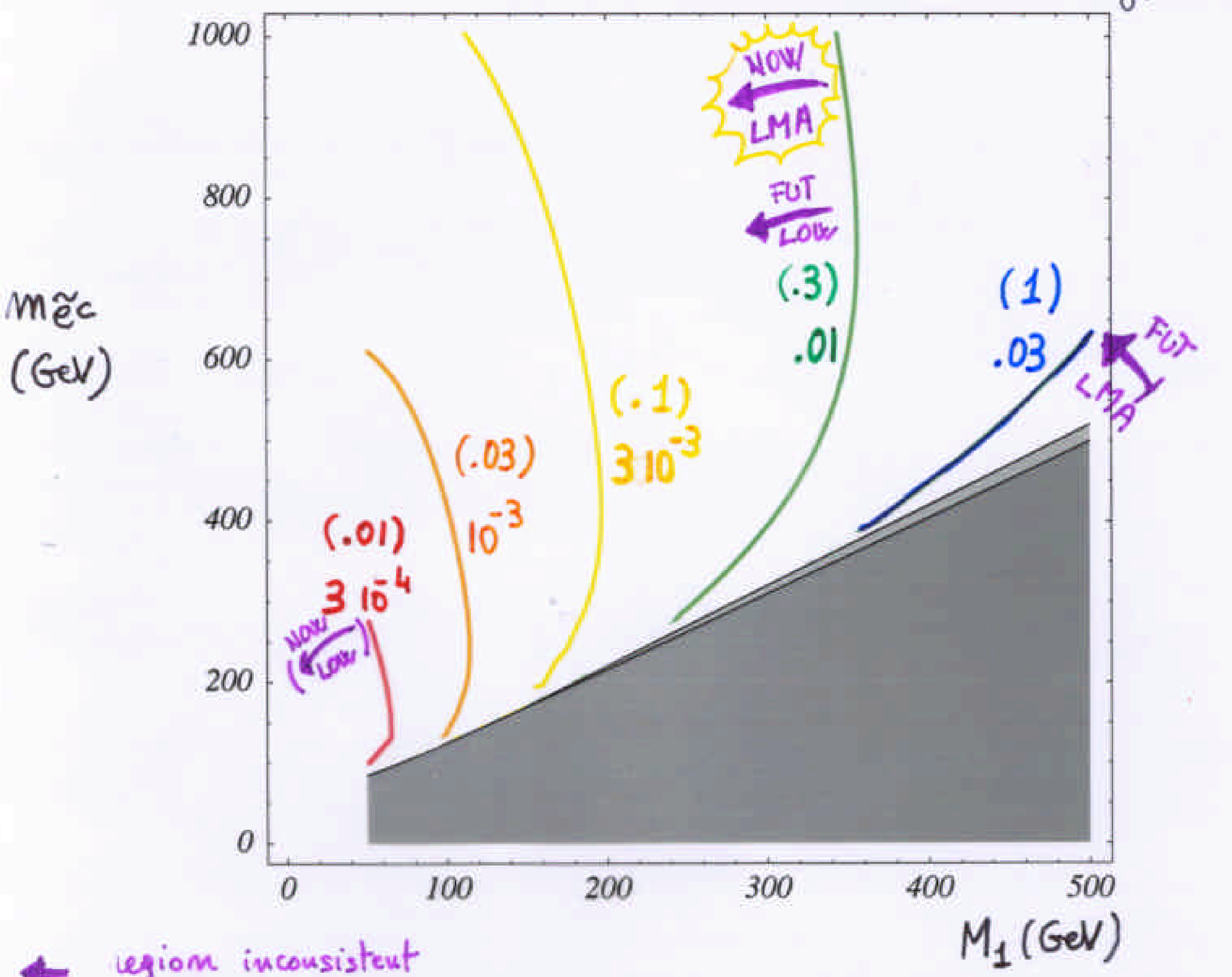
10^{-9} ← future?



$(-11) \leftarrow$ present

$BR(\mu \to e\gamma) \leq 10^{-14} \leftarrow$ future

$C_{\mu e}$ (upper bound) $\qquad tg\beta = 10$



← region inconsistent with plain U(1)

Anarchical see saw: $C_{\mu e} \sim O(7)\, y^2$ already crisis for $y \sim O(1)$

EDM & SEE-SAW : $d_\mu \to 10^{-24(26)}$ e cm USEFUL for th ?

EVEN IF $\delta^{(0)} = 0$ (ν&R b.c.), $\delta^{rad} \neq 0$ for EDM

BUT $\hat{M} \propto \mathbb{1} \xrightarrow{FC\,\&\,FV} \frac{d_e}{d_\mu} \sim \frac{m_e}{m_\mu} \sim \frac{1}{200}$ $d_\mu < 10^{-25}$ e cm

$< 10^{-27}$ e cm   Too low ....

[Ellis Hisano Lola Raidal Shimizu Romanino Strumia, ...]
& NATH'S TALK

IF $\hat{M}$ hierarchical → avoid naive scaling in FC & FV

FC : $d_\mu \propto \mathrm{Im}\left[Y_\nu^\dagger \ln\frac{\Lambda}{\hat{M}} Y_\nu \, , \, Y_\nu^\dagger Y_\nu \right]_{22}$ [Ellis et al]

Phases of ϵ ? Take $M_1 < M_2 < M_3$ and

$$Y_\nu = \begin{pmatrix} s & s & s \\ s & |Y_{22}| e^{i\phi_{22}} & |Y_{23}| e^{i\phi_{23}} \\ s & Y_{32} & Y_{33} \end{pmatrix} \to \begin{cases} d_\mu \propto |Y_{32} Y_{33} Y_{22} Y_{23}| \sin(\phi_{22} - \phi_{23}) + \cdots \\ \epsilon \propto f(s, Y) \end{cases}$$

sub leading effects

R   R

→ NO direct relation

$\phi_{22} = \phi_{23} \not\Rightarrow \epsilon = 0$

FV : 4 contrib. of type $\mathrm{Im}\left(\delta_{ij}^{\cdot\cdot} \, \delta_{ji}^{\cdot\cdot}\right) < |\delta_{ij}^{\cdot\cdot} \delta_{ji}^{\cdot\cdot}|$ seesaw

L,R

Correlation of ph.

$$\boxed{d_\mu^{FV} < d_\mu^{FV\,max}}$$

calculated from $\tau \to \mu\gamma$

[J.M. Savoy, in preparation]

CONCLUSIONS & OUTLOOK

→ SUSY at l.e., even if f-$\tilde{f}$ are perfectly aligned at Λ,

$\not{L}_i$ of the seesaw (V_L, V_R) = SOURCE of f-$\tilde{f}$ mis

↓

LFV, EDM

COMPETITIVE & COMPLEMENTARY w.r.to DIRECT SEARCHES → Could be measured

→ LFV $\xleftrightarrow[\text{see saw}]{}$ V_L test models:

$\tau \to \mu\gamma$ test CLASSES with diff. phys. char

EVEN A NON-MEAS. GIVES INF. ON SEE-SAW →

$\mu \to e\gamma$ more MOD. DEP. but STRONG INF.

EDM : in seesaw need $\hat{M}$ very hier. to measure > 10^{-25} e cm

→ EFFECT OF SEESAW ← "sort of minimal effect"

... AND IF YOU MEASURE SOMETHING?

↑ WHICH SOURCE?

seesaw...?
GUT...?
else...?

NEED TWO MEASUREMENTS :

$BR(\tau \to \mu\gamma)$ ↪ $d_\mu^{FV_{max}}$ ↔ ? d_μ

> d_μ from FC : ?

< small ph in FV → seesaw

~ O(1) ph in FV → seesaw + SOM. ELSE

e.g. SU(5)