Understanding Neutrino Masses and Mixings

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Abstract

A brief overview of some of the ideas to understand small neutrino masses and large neutrino mixings are presented.

I. INTRODUCTION

There is now strong evidence for neutrino masses and mixings from the solar and atmospheric neutrino observations. The simplest way to understand the deficits in neutrino fluxes observed in the above experiments is to assume that the incident neutrinos oscillate into another species which cannot be detected. For neutrino oscillations to take place, they must have mass and mix among themselves, with appropriate mass differences and mixing angles. As far as the accelerator searches for such oscillation effects go, with the exception of the Los Alamos experiment (LSND), all others have yielded negative results. These are of course not in contradiction with the solar and atmospheric data since they probe different ranges of masses and mixings. In fact, the negative results from the two experiments, CHOOZ and PALO VERDE provide upper limits on one of the mixing angles that has interesting implications for theories of neutrino masses. In this brief overview, I wish to draw attention to some of the theoretical ideas for understanding neutrino mass and mixing patterns in extensions of the standard model. This article will focus specifically on the seesaw mechanism that seems to provide the simplest way to understand small neutrino masses[1] and some attempts to understand large neutrino mixings within these models.

A. Major theoretical issues in neutrino physics:

The major issues of interest in neutrino theory are driven by the following experimental results and conclusions derived from them. We will use the notation, where the flavor or weak eigen states ν_α (with $\alpha = e, \mu, \tau$) are expressed in terms of the mass eigenstates ν_i ($i = 1, 2, 3$) as $\nu_\alpha = \sum_i U_{\alpha i}\nu_i$. The $U_{\alpha i}$, the elements of the Pontecorvo-Maki-Nakagawa-Sakata matrix represent the observable mixing angles in the basis where the charged lepton masses are diagonal. In any other basis, one has $U = U_\ell^\dagger U_\nu$, where the matrices on the right hand side are the ones that diagonalize the charged lepton and neutrino mass matrices respectively.

1. Solar neutrinos:

Thanks to the SNO results on both charged and neutral currents, there now appears to be a winner among the various possible oscillation solutions to the solar neutrino puzzle[2]. It seems that the so called LMA MSW solution is preferred over the small angle as well as the low and pure vacuum solution, although the situation could change. The ongoing KAMLAND experiments will provide decisive insight in choosing between these solutions as well as to narrow down the value of $\Delta m^2_\odot$ as well as the mixing angles[2]. The present range of preferred values of these parameters are: $2 \times 10^{-5} \leq \Delta m^2_\odot/\mathrm{eV}^2 \leq 4 \times 10^{-4}$ and $0.62 \leq sin^2 2\theta_\odot \leq 0.99$ at 3σ confidence level. This range is expected to be considerably reduced by the KAMLAND data to appear later this year.

2. Atmospheric neutrinos:

Here evidence appears very convincing that the explanation of observed muon neutrino deficit in upward going muons as well as the azimuthal angle dependence of this spectrum involves oscillation of ν_μ to ν_τ, with $\Delta m^2_{\nu_\mu - \nu_\tau} \simeq 2.5 \times 10^{-3}$ eV^2 and maximal mixing $sin^2 2\theta_A \geq 0.84$ at 99% c.l.

3. LSND:

The evidence for ν_μ to ν_e oscillation from LSND needs to be confirmed by another experiment. The KARMEN experiment has eliminated part of the original LSND allowed domain. The MiniBOONE at Fermilab will either confirm or refute these observations more definitively. There has been no evidence for the presence of a sterile component in the solar neutrino spectrum in the SNO neutral current data, contrary to widely held expectations based on sterile neutrino models that explain LSND data, although sterile neutrino solutions to all oscillations including LSND are still possible. We will postpone any discussion of sterile neutrinos till Min-booNe experiment throws further light on this issue in two years.

4. Neutrinoless double beta decay:

Oscillation involve only mass differences and therefore do not give information on the over all scale of the neutrino masses. One may hope that neutrinoless double beta decay may provide this information. It however turns out that this hope is not completely justified even if the present limits on lifetime go up by two orders of magnitude as is contemplated in many experiments unless the neutrinos are quasi-degenerate with common mass in the range bigger than 0.05 eV.

Nevertheless, neutrinoless double beta decay is an experimental of fundamental significance since its observation will for the first time give evidence for the breakdown of B-L quantum number in the laboratory and will confirm the general belief based on theory and cosmology that this indeed might be the case. Searches for $\beta\beta_{0\nu}$ decay has been going on for several years and a new round of higher precision experiments are on the verge of being lunched. The most stringent limits on this decay are from the enriched ^{76}Ge experiment by the Heidelberg-Moscow as well as the IGEX collaborations and can be converted to a constraint on masses and mixing angles as: $\sum_i U^2_{ei} m_i \leq 0.3$ eV, with an uncertainty of a factor of 2 to 3 due to nuclear matrix elements. Presently planned experiments such as GENIUS, MAJORANA, CUORE, EXO and MOON are expected to push this limit down by one order of magnitude.

5. U_{e3}:

The reactor experiments CHOOZ and PALO VERDE experiments imply that $U_{e3} \leq 0.16$.

All this information can be summarized in the following form for the **U** matrix (ignoring CP violation):

$$\mathbf{U} \simeq \begin{pmatrix} c & s & \epsilon \\ -\frac{s+c\epsilon}{\sqrt{2}} & \frac{c-s\epsilon}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s-c\epsilon}{\sqrt{2}} & \frac{-c-s\epsilon}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{1}$$

where $\epsilon \equiv U_{e3}$.

As far as the mass pattern goes however, there are three possibilities: (i) normal hierarchy: $m_1 \ll m_2 \ll m_3$; (ii) inverted hierarchy : $m_1 \simeq -m_2 \gg m_3$ and (iii) approximately degenerate pattern $m_1 \simeq m_2 \simeq m_3$, where m_i are the eigenvalues of the neutrino mass matrix. In the first case, the atmospheric and the solar neutrino data give direct information on m_3 and m_2 respectively. On the other hand, in the last case, the mass differences between the first and the second eigenvalues will be chosen to fit the solar neutrino data and the second and the third to fit the atmospheric neutrino data.

Three of the major theoretical challenges in neutrino physics now are:

- How does one understand the extreme smallness of the neutrino masses ?
- How does one understand two large mixing angles among neutrinos given that there is so much similarity between quarks and leptons at the level of interactions and that the quark mixings are small?
- What is the mass pattern among the neutrinos and how does one understand them from a theoretical point of view simultaneously with the near bimaximal mixng pattern ? In particular, why is $\Delta m^2_{\odot}/\Delta m^2_A \ll 1$.

II. SEESAW MECHANISM FOR SMALL NEUTRINO MASSES

It is well known that in the standard model the neutrino is massless due to a combination of two reasons: (i) one, its righthanded partner (ν_R) is absent and (ii) the model has exact global $B - L$ symmetry. Clearly, to understand a nonzero neutrino mass, one must give up one of the above assumptions. If one blindly included a ν_R to the standard model as a singlet, the status of neutrino would be parallel to all other fermions in the model and one would be hard put to understand why its mass is so much smaller than that of other fermions. Clearly there must be some other new ingredient that must be added.

A first hint of this new ingredient came from the observation of Weinberg that if B-L symmetry is broken by some high scale physics, in the effective low energy theory, one can have operators of the form $(LH)^2/M$, where M denotes the scale of new physics[3]. This after electroweak symmetry breaking would lead to a neutrino mass $\sim \frac{v_{wk}^2}{M}$. The key question now is what is the value of M ?

In the absence of any B-L violating physics all the way upto the Planck scale and assuming that nonperturbative Planck scale physics breaks all global symmetries such as the global B-L symmetry present in the standard model, the above higher dimensional operators takes the form[4] $LHLH/M_{P\ell}$ (where L is a lepton doublet and H is the Higgs doublet). This operator leads to masses for neutrinos of order 10^{-5} eV or less and are therefore not adequate for understanding observations. Thus a nontrivial extension of the standard model is called for wherein, the requisite value for M to explain the atmospheric neutrino data (of order 10^{14} GeV or so) must be the scale of B-L breaking. One then faces a "naturalness" question similar to the Higgs mass problem of the standard model i.e. why the radiative corrections do not send the mass M upto the Planck scale.

We will see below that there are at least two candidate symmetries which are compelling from other arguments and provide a reason for the stability of the new scale mass M. Both these symmetries are local symmetries and are connected with adding right handed neutrinos to the standard model: (i) local $B - L$ and/or (ii) $SU(2)_H$ horizontal symmetry. The most widely discussed example is the first one but there also very interesting arguments for the second or perhaps a combination of both. The mass M in these examples is the Majorana mass of the right handed neutrinos that break either or both these symmetries (i.e. in the exact symmetry limit the RH neutrinos have zero mass). In both cases we get what is known in the literature as the seesaw mechanism.

A. Right handed neutrino, seesaw mechanism and neutrino mass

As the first example of a model with right handed neutrino N_R, consider making the standard model completely quark lepton symmetric by adding one N_R per generation. This expands the gauge symmetry of the electroweak interactions to $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$ or to its full left-right symmetric extension $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry. In the latter case, the fermion doublets $(u,d)_{L,R}$ and $(\nu, e)_{L,R}$ are assigned to the left-right gauge group in a parity symmetric manner. The electric charge formula for the model takes a very interesting form[5]: $Q = I_{3L} + I_{3R} + \frac{B-L}{2}$. It can be concluded from this that below the scale v_R where the $SU(2)_R \times U(1)_{B-L}$ symmetry breaks down to the standard model and above the scale of M_W, one has the relation $\Delta I_{3R} \;=\; -\Delta\frac{B-L}{2}$. This relation has the profound consequence that neutrino must be a Majorana particle and that there must be lepton number violating interactions in nature. Furthermore it explains why the right handed neutrino mass is so much smaller than the Planck scale- it is connected with the breaking of local $B-L$ symmetry. To see how small neutrino masses are explained by this, note that the $\nu_L - \nu_R$ mass matrix for three generations takes the form:

$$M \;=\; \begin{pmatrix} M_{LL} & M_{LR} \\ M^T_{LR} & M_{RR} \end{pmatrix} \tag{2}$$

where $M_{RR} = \mathbf{f} v_R$ is the Majorana mass matrix of the right handed neutrinos, ($\mathbf{f}$ is the new Yukawa coupling matrix that determines the right handed neutrino masses). The first term $M_{LL} \;\simeq\; \mathbf{f}\frac{v^2_{wk}}{v_R}$ is the induced Majorana mass matrix for the left handed neutrinos and is characteristic of the existence of asymptotic parity symmetry. (It would for example be absent if the local symmetry is $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$.) The contribution $M_{LR} \equiv M_D = \mathbf{Y} v_{wk}$ is the Dirac mass matrix connecting the left and the right handed neutrinos. The diagonalization of this mass matrix leads to following form for the light neutrino masses:

$$M_\nu \simeq \; \mathbf{f}\frac{\lambda v^2_{wk}}{v_R} - \frac{1}{v_R} M^T_D \mathbf{f}^{-1} M_D; \tag{3}$$

$\mathbf{f}$, the Yukawa coupling matrix that is responsible for the masses of the heavy right handed neutrinos characterizes the high scale physics, whereas all other parameters denote physics at the weak scale. We have called this generalized formula for neutrino masses, the type II seesaw formula to distinguish it from the one that is commonly used in literature where the first term of Eq. (2) is absent. Important feature of this formula is that both terms vanish as $v_R \to \infty$ and since $v_R \gg v_{wk}$, the the neutrino masses are much smaller than the charged fermion masses. As was particularly emphasized in the third paper of ref.[1], the dominance of V-A interaction in the low energy weak processes is now connected to smallness of neutrino masses.

If in the above seesaw formula, the second term dominates, this leads to the canonical type I seesaw formula and leads to the often discussed hierarchical neutrino masses, which in the approximation of small mixings lead to $m_{\nu_i} \simeq m^2_{f_i}/v_R$, where f_i is either a charged lepton or a quark depending on the kind of model for neutrinos.

On the other hand, in models where the first term dominates, the neutrino masses are almost generation independent. Therefore, if there is indication for neutrinos being degenerate in mass from observations, one will have to resort to type II seesaw mechanism for its understanding.

A further advantage of the right handed neutrino and seesaw mechanism is that it fits in very nicely into grand unified frameworks based on SO(10) models. The coupling constant unification then provides a theoretical justification for the high seesaw scale and hence the small neutrino masses. Furthermore, the **16**-dim. spinor representation of SO(10) has just the right quantum numbers to fit the ν_R in addition to the standard model particles of each generation.

1. *Double seesaw with a low scale for $B - L$ symmetry*

As we saw from the previous discussion, the conventional seesaw mechanism requires rather high scale for the B-L symmetry breaking and the corresponding right handed neutrino mass (of order $\geq 10^{14}$ GeV). There is however no way at present to know what the scale of B-L symmetry breaking is. There are for example models bases on string compactification[6] where the $B - L$ is quite possibly is in the TeV range. In this case small neutrino mass can be implemented by a double seesaw mechanism suggested in Ref.[7]. The idea is to take a right handed neutrino N and a singlet neutrino S which has extra quantum numbers which prevent it from coupling to the left handed neutrino. One can then write a three by three neutrino mass matrix in the basis (ν, N, S) of the form:

$$M = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M \\ 0 & M & \mu \end{pmatrix} \tag{4}$$

For the case $\mu \ll M \approx M_{B-L}$, (where M_{B-L} is the $B-L$ breaking scale) this matrix has one light and two heavy states. A generalization of this mechanism to the case of three generations is straightforward. One important point here is that to keep $\mu \sim m_D$, one also needs some additional gauge symmetries, which often are a part of the string models.

B. $SU(2)_H$ local symmetry and 3×2 seesaw with two N_R's

A symmetry among the different generation has often been suspected as a possible way to understand the different properties of the quarks and leptons of different generations. This symmetry for the three generation case could be either a $U(1)$, $SU(2)$ or an $SU(3)$ symmetry and a local symmetry. Of these three possibilities, the third one requires that we include additional fermions to cancel anomalies. Of the remaining two, we choose $SU(2)_H$ since it has the following interesting property i.e. if it operate on right handed charged leptons, cancellation of global Witten anomaly requires that we must introduce at least two right handed neutrinos $(N_{eR}, N_{\mu R})$ transforming as a doublet under the group. Thus two right handed neutrinos is the minimal set required theoretically. Clearly, the mass of the right handed neutrinos are connected to the breaking of the $SU(2)_H$ symmetry[8]. An additional feature of these matrices is that they lead to a 3×2 seesaw as compared to the 3×3 seesaw in the case of the left-right symmetric (or SO(10)) models. This could of course be a part of the latter class of models if $v_H \ll M_{B-L}$. A distinct feature of the models with $3{\times}2$ seesaw is that one of the light neutrinos is massless. In this sense, in these models all parameters of a real neutrino mass matrix are determinable by only oscillation experiments.

C. Does m_ν necessarily imply a right handed neutrino ?

An interesting class of models were proposed in the 80's where one extends the standard model without adding right handed neutrinos but extra Higgs bosons to understand small m_ν. In one class of models, one adds a Higgs triplet with $B-L=2$ with a large mass $M \gg M_W$[9]. The triplet field acquires small vev of order $v_T \sim \frac{M_W^2}{M}$[9]. Thus, modulo the unknown origin of $M \gg M_W$, this provides an understanding of the small m_ν.

There are however several problems with this mechanism as well as all mechanisms that populate the weak scale with extra Higgs bosons: (i) there is no simple symmetry that can guarantee that the heavy triplet mass M (or the mass of any intermediate scale Higgs) is stable under radiative corrections; (ii) to generate the baryon asymmetry of the universe, one has to go beyond the minimal model version of the triplet model and add more triplets to the theory; otherwise there will be no CP violating phase that one can use to generate lepton asymmetry; (iii) furthermore, the triplet decay and out of equilibrium decay condition of Sakharov implies that $M \geq 10^{13}$ GeV. This scale is much higher than the conventional reheating temperature in inflation models with supersymmetry for TeV scale gravitinos. For all these reasons, below we will focus on the most promising scenario for light neutrino masses i.e. the seesaw models with heavy right handed neutrino to understand large neutrino mixings.

III. SOME ATTEMPTS TO UNDERSTAND LARGE MIXINGS

One of the major mysteries of neutrino physics is the large mixing mixing angles both for solar and the atmospheric neutrino oscillation. This is because of the simple fact that there is so much similarity in the interactions between the quarks and leptons and quarks mixings between different generations are of course well known to be very small. In the seesaw framework one may attribute this to the fact that a central ingredient in understanding the neutrino mass matrix is the mass matrix of the right handed neutrino which reflects high scale physics whereas quark physics is low scale physics and it can dictate only the pattern of the Dirac mass of the neutrinos. While this is qualitatively a reasonable argument, it is not much help in providing a quantitative understanding. The general strategy to make any headway towards a quantitative understanding is to search for mass matrices that fit observations and then search for symmetry or dynamical reasons for their origin.

To get useful mass patterns, one must first note that in the absence of CP violation, the symmetric Majorana mass matrix for the light neutrinos $\mathcal{M}_\nu$ contains six parameters, whereas observations give only five pieces of observation i.e. $\Delta m^2_{A,\cdot}$, $\theta_{12} \equiv \theta_\odot$, $\theta_{23} \equiv \theta_A$ and $U_{e3} \equiv \theta_{13}$. The absence of the sixth piece of information is essentially reflected in the fact that the precise mass pattern (normal, inverted or degenerate) of neutrinos is not known. So to make any progress, one may try to make ansatzes that reduce the number of parameters in a mass matrix either (A) by making different elements equal or (B) putting them to zero in a basis where the charged leptons are diagonal.

An example of the first strategy is the zeroth order mass matrix discussed in[10]:

$$M_\nu = \begin{pmatrix} A+D & F & F \\ F & A & D \\ F & D & A \end{pmatrix} \tag{5}$$

This leads to an exact bimaximal pattern but allows for all different mass patterns. Since the present data implies that there are deviations from the exact bimaximal form, this mass matrix must have additional small corrections.

Three different mass patterns can emerge from this mass matrix in various limits: e.g. (i) for $F \ll A \simeq -D$, one gets the normal hierarchy; (ii) for $F \gg A, D$, one has the inverted pattern for masses and (iii) the parameter region $F, D \ll A$ leads to the degenerate case. An interesting symmetry of this mass matrix is the $\nu_\mu \leftrightarrow \nu_\tau$ interchange symmetry, which is obvious from the matrix; but in the limit where $A = D = 0$, there appears a much more interesting symmetry i.e. the continuous symmetry $L_e - L_\mu - L_\tau$[11]. If the inverted mass matrix is confirmed by future experiments, this symmetry will provide an important clue to new neutrino related physics beyond the standard model. Inverted mass pattern is the only case where such an interesting leptonic symmetry appears. Let us therefore discuss the implications of this symmetry further.

A. Approximate $L_e - L_\mu - L_\tau$ symmetry and neutrino mixings

In the exact $L_e - L_\mu - L_\tau$ symmetry limit, the model not only leads naturally to large solar and atmospheric mixing angles but it also leads to vanishing U_{e3} as well as $\Delta m^2_{odot}/\Delta m^2_A = 0$. Therefore the model raises the hope that a small U_{e3} as well as the smallness of $\Delta m^2_{odot}/\Delta m^2_A$ can be understood in a natural manner. One must therefore add small symmetry breaking terms to this model and examine the consequences.

This question was studied in two papers[12]. In the second paper of [12], the following mass matrix for neutrinos was considered that includes small $L_e - L_\mu - L_\tau$ violating terms.

$$\mathcal{M}_\nu = m \begin{pmatrix} z & c & s \\ c & y & d \\ s & d & x \end{pmatrix}. \tag{6}$$

where $c = cos\theta$ and $s = sin\theta$. The charged lepton mass matrix is chosen to have a diagonal form in this basis and $L_e - L_\mu - L_\tau$ symmetric.

In the presence of the small symmetry breaking terms, we find the following sumrules involving the neutrino observables and the elements of the neutrino mass matrix. The nontrivial ones are:

$$\begin{aligned} \sin^2 2\theta_\odot &= 1 - (\frac{\triangle m^2_\odot}{4\triangle m^2_A} - z)^2 + O(\delta^3) \\ \frac{\triangle m^2_\odot}{\triangle m^2_A} &= 2(z + \vec{v}\cdot\vec{x}) + O(\delta^2) \\ U_{e3} &= \vec{A}\cdot(\vec{v}\times\vec{x}) + O(\delta^3) \end{aligned} \tag{7}$$

where $\vec{v} = (\cos^2\theta, \sin^2\theta, \sqrt{2}\sin\theta\cos\theta)$, $\vec{x} = (x, y, \sqrt{2}d)$ and $\vec{A} = \frac{1}{\sqrt{2}}(1,1,0)$. δ in the preceding equations represents the small parameters in the mass matrix.

One of the major consequences of these relations is that (i) there is a close connection between the measured value of the solar mixing angle and the neutrino mass measured in neutrinoless double beta decay i.e. z; (ii) the present values for the solar mixing angle can be used to predict the $m_{\beta\beta}$ for a value of the $\Delta m^2_\odot$. For instance, for $sin^2 2\theta_\odot = 0.9$, we would predict $(\frac{\triangle m^2_\odot}{4\triangle m^2_A} - z) = 0.3$. For small $\Delta m^2_\odot$, this implies $m_{\beta\beta} \simeq 0.01$ eV. The second relation involving the $\Delta m^2_\odot/\Delta m^2_A$ in terms of x, y, z, d tells us that for this to be the case, we must have strong cancellation between the various small parameters. Given this, the above $m_{\beta\beta}$ value is expected to be within the reach of new double beta decay experiments contemplated. Note however that the $sin^2 2\theta_\odot$ cannot be larger than 0.9 in the case of approximate $L_e - L_\mu - L_\tau$ symmetry.

If the value of $sin^2 2\theta_\odot$ is ultimately determined to be less than 0.9, the question one may ask is whether the idea of $L_e - L_\mu - L_\tau$ symmetry is dead. The answer is in the negative since so far we have explored the breaking of $L_e - L_\mu - L_\tau$ symmetry only in the neutrino mass matrix. It was shown in the first paper of [12] that if the symmetry is broken in the charged lepton mass, one can lower the $sin^2\theta_{odot}$ to 0.85 or so.

B. Approximate $L_e - L_\mu - L_\tau$ symmetry from $SU(2)_H$ horizontal symmetry

It can be shown that an $SU(2)_H$ model for leptons leads quite generally to an approximate $L_e - L_\mu - L_\tau$ symmetry for neutrinos. As already noted, a distinct feature of $SU(2)_H$ symmetry is that there are two right handed neutrinos instead of three and therefore one has a 3×2 seesaw rather than the usual 3×3 one.

Furthermore the $SU(2)$ horizontal symmetry restricts both the Dirac mass of the neutrino as well as the righthanded neutrino mass matrix to the forms[8] leading to 5×5 mass matrix for heavy and light neutrinos of the form:

$$M_{\nu_L,\nu_R} = \begin{pmatrix} 0 & 0 & 0 & h_0\kappa_0 & 0 \\ 0 & 0 & 0 & 0 & h_0\kappa_0 \\ 0 & 0 & 0 & h_1\kappa_1 & h_1\kappa_2 \\ h_0\kappa_0 & 0 & h_1\kappa_1 & 0 & fv'_H \\ 0 & h_0\kappa_0 & h_1\kappa_2 & fv'_H & 0 \end{pmatrix} \quad (8)$$

After seesaw diagonalization, it leads to the light neutrino mass matrix of the form:

$$\mathcal{M}_\nu = -M_D M_R^{-1} M_D^T \quad (9)$$

where $M_D = \begin{pmatrix} h_0\kappa_0 & 0 \\ 0 & h_0\kappa_0 \\ h_1\kappa_1 & h_1\kappa_2 \end{pmatrix}$; $M_R^{-1} = \frac{1}{fv'_H}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. The resulting light Majorana neutrino mass matrix $\mathcal{M}_\nu$ is given by:

$$\mathcal{M}_\nu \;=\; -\frac{1}{f v'_H}\begin{pmatrix} 0 & (h_0\kappa_0)^2 & h_0 h_1 \kappa_0 \kappa_2 \\ (h_0\kappa_0)^2 & 0 & h_0 h_1 \kappa_0 \kappa_1 \\ h_0 h_1 \kappa_0 \kappa_2 & h_0 h_1 \kappa_0 \kappa_1 & 2h_1^2 \kappa_1 \kappa_2 \end{pmatrix} \tag{10}$$

First of all as discussed before, this leads to one neutrino which is massless. To get the physical neutrino mixings, we also need the charged lepton mass matrix defined by $\bar{\psi}_L \mathcal{M}_\ell \psi_R$. This is given in our model by:

$$\mathcal{M}_\ell \;=\; \begin{pmatrix} h'_2\kappa_0 & 0 & -h'_1\kappa_2 \\ 0 & h'_2\kappa_0 & h'_1\kappa_1 \\ h'_4\kappa_1 & h'_4\kappa_2 & h'_3\kappa_0 \end{pmatrix} \tag{11}$$

Note that in the limit of $\kappa_1 = 0$, the neutrino mass matrix has the $L_e - L_\mu - L_\tau$ symmetry and also there are mixing effects coming from the charged lepton sector so that one can get a lower value for $sin^2 2\theta_\odot$.

IV. LARGE MIXINGS IN MODELS WITH QUARK-LEPTON UNIFICATION

The $L_e - L_\mu - L_\tau$ model discussed above treats the quarks and leptons on a fundamentally different footing. On the other hand it could be that at very short distances there is quark lepton unification[13]. I give below two of a number of ideas, where models with quark lepton symmetry can lead to large neutrino mixings. In the models discussed below large mixings arise dynamically and without need for any extra symmetries starting with small mixings at very short distances as would be dictated by quark lepton symmetry.

A. Radiative magnification of mixing angles

In this class of models dynamics of radiative corrections plays an essential role in understanding the maximal mixings. The basic idea is that at the seesaw scale, all mixings angles are small, a situation quite natural if the pattern of **f** Yukawa coupling is similar to the quark sector. Since the observed neutrino mixings are weak scale observables, one must extrapolate[14] the seesaw scale mass matrices to the weak scale and recalculate the mixing angles.

The extrapolation formula is

$$\mathcal{M}_\nu(M_Z) = \mathbf{I}\mathcal{M}_\nu(v_R)\mathbf{I} \tag{12}$$

$$where \qquad \mathbf{I}_{\alpha\alpha} = \left(1 - \frac{h_\alpha^2}{16\pi^2}\right) \tag{13}$$

Note that since $h_\alpha = \sqrt{2}m_\alpha/v_{wk}$ (α being the charged lepton index), in the extrapolation only the τ-lepton makes a difference. In the MSSM, this increases the $\mathcal{M}_{\tau\tau}$ entry of the neutrino mass matrix and essentially leaves the others unchanged. It was shown in ref.[15] that if the muon and the tau neutrinos are nearly degenerate in mass at the seesaw scale, and in supersymmetric theories, the $tan\beta \geq 5$, the radiative corrections can become large enough so that at the weak scale the two diagonal elements of $\mathcal{M}_\nu$ which were nearly equal but different at the seesaw scale become extremely degenerate. This leads to an enhancement of the mixing angle to become almost maximal and a solution to the atmospheric neutrino deficit emerges even though at the seesaw scale, the mixing angles were small. This happens only if the experimental observable $\Delta m^2_{23} \leq 0$ a possibility can be tested in contemplated long base line experiments. Also for this mechanism to work, the overall scale of neutrino masses must be in the range of 0.1 eV or so making the idea testable in forthcoming double beta decay experiments.

Several comment are in order: (i) to get a near degenerate mass spectrum without additional assumptions, one must use the type II seesaw mechanism as in Eq. (3); (ii) An interesting question is whether this mechanism can be extended to the case of three generations and whether it can explain the bimaximal pattern also. This question was investigated in the ref.[15] and it was found that the answer to the first question is yes and to the second, it is "no".

There gave been several recent works on the effect of RGE's on neutrino mixings, which make one optimistic that the idea of ref.[15] can be extended to a realistic three generation situation case. For instance, a recent work by the Munich group[16] claims that if the two lighter neutrinos in a normal hierarchy model are quasi-degenerate, then seesaw scale RG corrections can enhance the solar mixing angle without affecting the atmospheric mixing angle. For this to work, one needs a strong hierarchy among the right handed neutrino masses. So perhaps one could combine the idea of [15] and [16] to generate two large mixing angles starting from both small angles at the seesaw scale. A second recent work[17] has used the techniques of ref. [15] to study radiative magnification of solar angle in texture zero neutrino mass matrices.

B. A minimal SO(10) model

Another suggestion for understanding large atmospheric mixing has been made within a class of SO(10) models, which are strongly suggested by local B-L symmetry, large seesaw scale and grand unification ideas. The basic ingredients of this idea are the following properties of the SO(10) model: (i) that one can construct a minimal SO(10) model with only two multiplets that couple to fermions i.e. **10** and **126** and another that breaks SO(10) down to the left-right model. The second breaks the B-L symmetry and the first the electroweak symmetry. (ii) A second property of SO(10) models [18] is that **126** contains submultiplets that not only contribute to charged fermion but also to the left and right handed Majorana masses (M_{LL}, M_{RR} respectively in Eq. (2)) for the neutrinos. This leads to a tremendous reduction of the number of arbitrary parameters in the model, as we will see below.

There are only two Yukawa coupling matrices in this model: (i) h for the **10** Higgs and (ii) f for the **126** Higgs. SO(10) has the property that the Yukawa couplings involving the **10** and **126** Higgs representations are symmetric. Therefore if we ignore CP violation and work in a basis where one of these two sets of Yukawa coupling matrices is diagonal, then it will have only nine parameters. Noting the fact that the (2,2,15) submultiplet of **126** has a standard model doublet that contributes to charged fermion masses, one can write the quark and lepton mass matrices as follows[18]:

$$\begin{aligned} M_u &= h\kappa_u + f v_u \qquad (14)\\ M_d &= h\kappa_d + f v_d \\ M_\ell &= h\kappa_d - 3f v_d \\ M_{\nu_D} &= h\kappa_u - 3f v_u \end{aligned} \qquad (15)$$

where $\kappa_{u,d}$ are the vev's of the up and down Higgs vevs of the standard model doublets in **10** Higgs and $v_{u,d}$ are the corresponding vevs for the same doublets in **126**. Note that there are 13 parameters in the above equations and there are 13 inputs (six quark masses, three lepton masses and three quark mixing angles and weak scale). Thus all parameters of the model that go into fermion masses are determined.

To determine the light neutrino masses, we use the seesaw formula in Eq. (3), where the **f** is nothing but the **126** Yukawa coupling. Thus all parameters that give neutrino mixings except an overall scale are determined. These models were extensively discussed in the last decade[19]. Initially CP phases were ignored and more recently CP phases have been included in the analysis.

A very interesting point regarding these models has been noted in Ref.[20], where it is pointed out that if the direct triplet term in type II seesaw dominates, then it provides a very natural understanding of the large atmospheric mixing angle without invoking any symmetries. A simple way to see this is to note that when the triplet term dominates the seesaw formula, we have the neutrino mass matrix $\mathcal{M}_\nu \propto f$, where f matrix is the **126** coupling to fermions discussed earlier. Using the above equations, one can derive the following sumrule (sumrule was already noted in the second reference of [19]):

$$\mathcal{M}_\nu = c(M_d - M_\ell) \tag{16}$$

Now quark lepton symmetry implies that for the second and third generation, the $M_{d,\ell}$ have the following general form:

$$M_d = \begin{pmatrix} \epsilon_1 & \epsilon_2 \\ \epsilon_2 & m_b \end{pmatrix} \tag{17}$$

and

$$M_\ell = \begin{pmatrix} \epsilon'_1 & \epsilon'_2 \\ \epsilon'_2 & m_\tau \end{pmatrix} \tag{18}$$

where $\epsilon_i \ll m_{b,\tau}$ as is required by low energy observations. It is well known that in supersymmetric theories, when low energy quark and lepton masses are extrapolated to the GUT scale, one gets approximately that $m_b \simeq m_\tau$. One then sees from the above sumrule for neutrino masses that all entries for the neutrino mass matrix are of the same order leading very naturally to the atmospheric mixing angle to be large. Thus one has a natural understanding of the large atmospheric neutrino mixing angle. No extra symmetries are assumed for this purpose.

V. CONCLUSION

In conclusion, the seesaw mechanism appears by far to be the simplest way to understand the small neutrino masses. The large right handed neutrino mass implied by this also helps in understanding origin of matter in the universe. Our understanding of mixings on the other hand, is at a very preliminary level. A particular challenge to theorists is to understand the so called bimaximal mixing pattern, which is emerging as the favorite, if the solar neutrino deficit is to be solved via the large angle MSW. Several symmetry and dynamical approaches to understanding large mixings are noted. This idea however needs to be extended to include the solar mixing angle to see its viability.

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