The cosmological constant

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Cosmological constant

The cosmological constant appears as a constant in the Einstein equations :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G_N T_{\mu\nu} + \lambda g_{\mu\nu},$$

G_N Newton's constant, $T_{\mu\nu}$ energy-momentum tensor.

$$[\lambda] = L^{-2}$$

Constraint on λ: more convenient to work with a Robertson-Walker metric $\leftrightarrow$ homogeneity and isotropy

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

$$\boxed{H^2 \equiv \frac{\dot{a}^2(t)}{a^2(t)} = \frac{1}{3}(\lambda + 8\pi G_N \rho) - \frac{k}{a^2}}$$

Hubble parameter ρ energy density spatial curvature term

$\dot{a} = da/dt$

Presently

$$|\lambda| \leq H_0^2 = h_0^2 . 10^{-52}\ \text{m}^{-2}$$

Not a problem as long as one remains classical.

But combine gravity ($8\pi G_N$) with quantum theory ($\hbar$):

$$\text{mass scale} \quad m_P = \sqrt{\frac{\hbar c}{8\pi G_N}} = 2.4 \times 10^{18}\ \text{GeV/c}^2$$

$$\text{length scale} \quad \ell_P = \frac{\hbar}{m_P c} = 8.1 \times 10^{-35}\ \text{m}$$

$$\Longrightarrow \qquad |\lambda| \leq H_0^2 \sim 10^{-120}\ m_P^2$$

obvious solution: $\lambda = 0$

Vacuum energy or "cosmological constant" problem

Set $\lambda = 0$ but assume that there is a nonzero vacuum (*i.e.* groundstate) energy:

$$< T_{\mu\nu} >= -< \rho > g_{\mu\nu}$$

then

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G_N T_{\mu\nu} + 8\pi G_N < \rho > g_{\mu\nu}$$

effective cosmological constant:

$$\lambda_{eff} = 8\pi G_N < \rho > \equiv \Lambda^4 / m_P^2$$

$$\frac{\Lambda^4}{m_P^2} = |\lambda_{eff}| \leq H_0^2 = 10^{-120} m_P^2$$

$$\Lambda \leq 10^{-30}\, m_P \sim 10^{-3} \text{ eV}$$

Observational results

Increasing number of indications that the Universe is presently undergoing accelerated expansion.

standard equation for the conservation of energy:

$$\dot{\rho} = -3(p+\rho)H$$

allows to derive from the Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p)$$

ρ energy density and p pressure

Matter-dominated universe ($p \sim 0$) is decelerating.

Need for a component with negative pressure

e.g. a cosmological constant is associated with a contribution to the energy-momentum tensor

$$T^\mu_\nu = -\Lambda^4 \delta^\mu_\nu = (-\rho, p, p, p)$$

Hence equation of state: $p = -\rho$ $\quad \rightarrow$ acceleration

Discussion of data often expressed in terms of the energy density Λ^4 stored in the vacuum versus the energy density ρ_M in matter fields (baryons, neutrinos, hidden matter,...)

Normalize with the critical density (corresponding to a flat Universe):

$$\Omega_\Lambda = \frac{\Lambda^4}{\rho_c}, \;\; \Omega_M = \frac{\rho_M}{\rho_c}, \;\; \Omega_k = -\frac{k}{3a^2H^2}.$$

$$\rho_c = \frac{3H^2}{8\pi G_N}$$

Then

$$\Omega_M + \Omega_\Lambda + \Omega_k = 1$$

Note: $\Omega_M + \Omega_\Lambda = 1$ is a prediction of many inflation models ($\leftrightarrow$ $k = 0$ flat space)

CMB data: the first acoustic peak is expected at an "angular" scale $\ell \sim 200/\sqrt{\Omega_M + \Omega_\Lambda}$

Supernova $SnIa$ data

Relation between the flux f received on earth and the absolute luminosity $\mathcal{L}$ of the supernova depends on its redshift z, but also on the geometry of spacetime. In terms of apparent m_B and absolute M magnitudes *

$$m_B = 5\log(H_0 d_L) + M - 5\log H_0 + 25$$

If $SnIa$ standard candles, only z dependent term is

$$H_0 d_L = cz\left[1 + \frac{1-q_0}{2}z + \cdots\right]$$

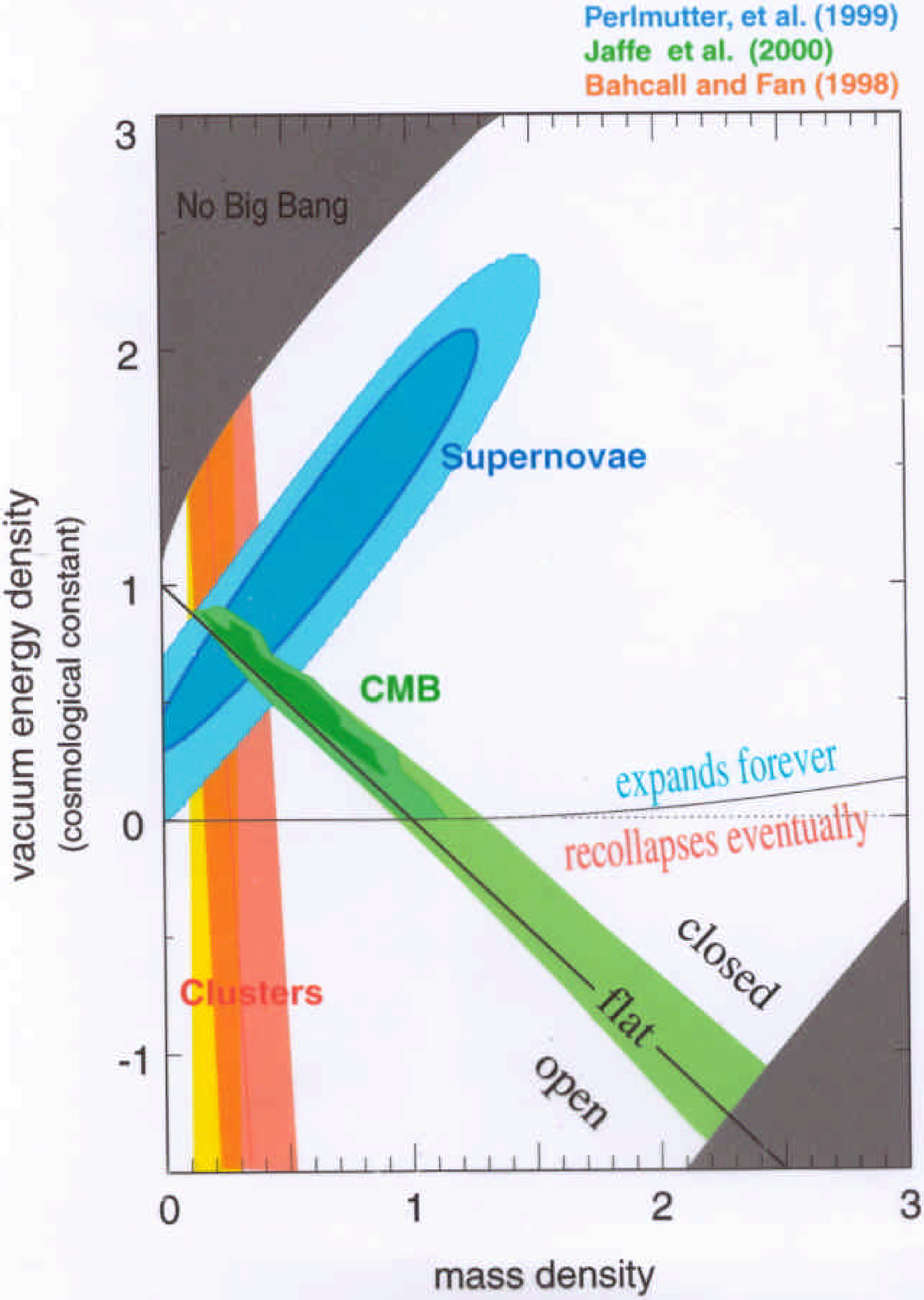
with q_0 deceleration parameter:

$$q_0 \equiv -a\ddot{a}/\dot{a}^2 = \Omega_M/2 - \Omega_\Lambda$$

Other results come from gravitational lensing. Several methods are used:

- abundance of multiply-imaged quasar sources
- strong lensing by massive clusters of galaxies
- weak lensing

* *magnitude is* $-2.5\ \log_{10}$ *luminosity* $+$ *constant*



Perlmutter, et al. (1999)
Jaffe et al. (2000)
Bahcall and Fan (1998)
No Big Bang
Supernovae
CMB
expands forever
recollapses eventually
closed
flat
open
Clusters
vacuum energy density
(cosmological constant)
mass density
3
2
1
0
-1
0
1
2
3

Cosmic coincidence problem

Why is $\rho_\Lambda \equiv \Lambda^4 \sim \rho_H$ now?

$$\dot{\rho} = -3H(p+\rho) = -3\frac{\dot{a}}{a}(p+\rho)$$

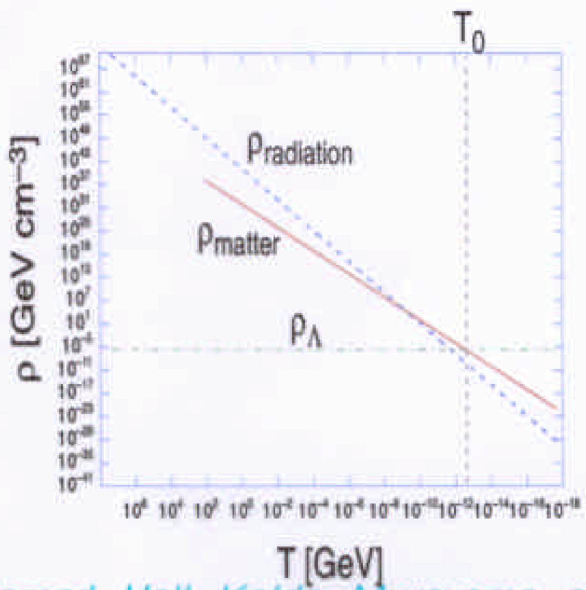
$\rightarrow$ Cosmological constant $\qquad \rho_\lambda = \text{cst}$

Matter $(p \sim 0)$ $\qquad \rho_m \propto a^{-3}$

Or, to avoid any reference to us:

Why does the vacuum energy starts to dominate at a time t_Λ $(z_\Lambda \sim 1)$ which almost coincides with the epoch t_G $(z_G \sim 1$ to $3)$ of galaxy formation?

COSMIC COINCIDENCE PROBLEM



Arkani-Hamed, Hall, Kolda, Murayama, astro-ph/0005111

Tends to lead to an anthropic approach

Vilenkin, Weinberg...

Consider regions with different values of t_G and t_Λ.

- i) when ρ_Λ starts to dominate (at t_Λ), the Universe enters a de Sitter phase of exponential expansion.

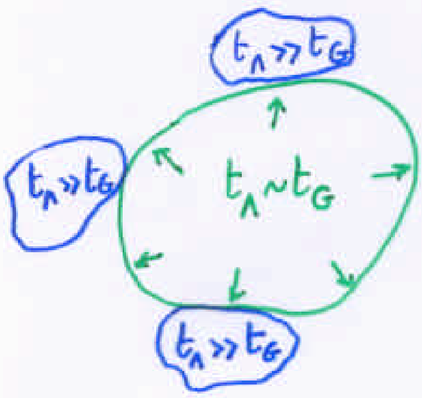
  ii) galaxy formation (at t_G) must precede this phase (otherwise no observer available)

$$t_G \leq t_\Lambda$$

- Regions with $t_\Lambda \gg t_G$ have not undergone yet any de Sitter phase of reacceleration and are thus "phase-space suppressed" compared with regions with $t_\Lambda \sim t_G$:

$$t_\Lambda \gtrsim t_G$$

$$\rho_\Lambda \sim \rho_M$$



Inspired by the physics of quantum Fermi liquids?

Volovik

Assume that $|\text{vac}>$ belongs to the class of ph. states

$$\rho_\Lambda = \frac{1}{V} < \text{vac}|\mathcal{H} - \mu\mathcal{N}|\text{vac} > \equiv \ \epsilon - \mu n$$

Then external pressure acting on the liquid

$$p_\Lambda = -\frac{d\left(V\epsilon\left(n = \frac{N}{V}\right)\right)}{dV} = -\epsilon + n\frac{d\epsilon}{dn} = -\rho_\Lambda$$

At equilibrium, in the absence of external force,

$$p = 0 \quad \Rightarrow \quad \rho_\Lambda = 0$$

Compensation between low energy ("Planckian") and high energy ("trans-Planckian") degrees of freedom.

Does this imply that the cosmological constant always vanishes?

- not at high temperature: liquid $\rightarrow$ gas; vacuum cannot exist without an external pressure

- other contributions induce a "small" cosmological constant e.g. presence of phonons with equation of state $P_M = \frac{1}{3}\rho_M$

$$P = 0 = p_\Lambda + p_M \quad \rightarrow \quad \rho_\Lambda = -p_\Lambda = p_M = \frac{1}{3}\rho_M$$

Solutions to the problems?

- Vacuum energy: Models of relaxation of the cosmological constant

→ violations of Lorentz invariance

- Cosmic coincidence: Models of quintessence

→ time variation of constants

→ violations of equivalence principle

A no-go theorem

Weinberg

Not possible to have a vanishing λ as a consequence of the equation of motion of some fields

Consider constant fields ϕ_n: eq. of motion $\delta\mathcal{L}/\delta\phi_n = 0$

Remember that $\delta\left(\sqrt{g}\lambda\right) = \sqrt{g}g^{\mu\nu}\delta g_{\mu\nu}\lambda/2$

Hence would need

$$g_{\mu\nu}\frac{\delta\mathcal{L}}{\delta g_{\mu\nu}} = \sum_n \left(\frac{\delta\mathcal{L}}{\delta\phi_n}\right) f_n(\phi)$$

Amounts to a symmetry cond: invariance of $\mathcal{L}$ under

$$\delta g_{\mu\nu} = 2\epsilon g_{\mu\nu},\ \ \delta\phi_n = -\epsilon f_n(\phi)$$

Possible to redefine the fields $\phi_n \to \phi, \sigma_a$ such that

$$\delta g_{\mu\nu} = 2\epsilon g_{\mu\nu},\ \ \delta\phi = -\epsilon,\ \ \delta\sigma_a = 0$$

Then $\quad \mathcal{L} = e^{4\phi}\sqrt{g}\mathcal{L}(\sigma).$

Key assumption: Lorentz invariance

- Dolgov compensation mechanism

involves free massless vector or tensor (non-gauge) fields e.g. $\mathcal{L} \sim \eta D_\mu A_\nu D^\mu A^\nu$

⊖ leads to strong breaking of Lorentz invariance

Rubakov, Tinyakov

- Brown-Teitelboim mechanism

generalizes particle creation by electric field in 1+1 dimensions to higher dimensions (and a 3-form field coupled to a membrane in 3+1 dimensions)

⊕ generalizable to string set up

⊖ requires very small membrane charges:

non-perturbative effects

Feng, March-Russell, Sethi, Wilczek

multiple 4-forms

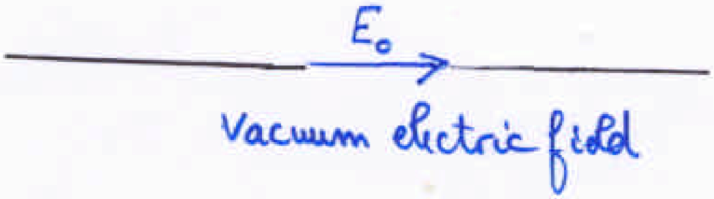
Bousso, Polchinski

- Rubakov relaxation of cosmological cst at inflation

⊕ couples the problem to inflation

RELAXATION MODELS : dynamical neutralisation of λ

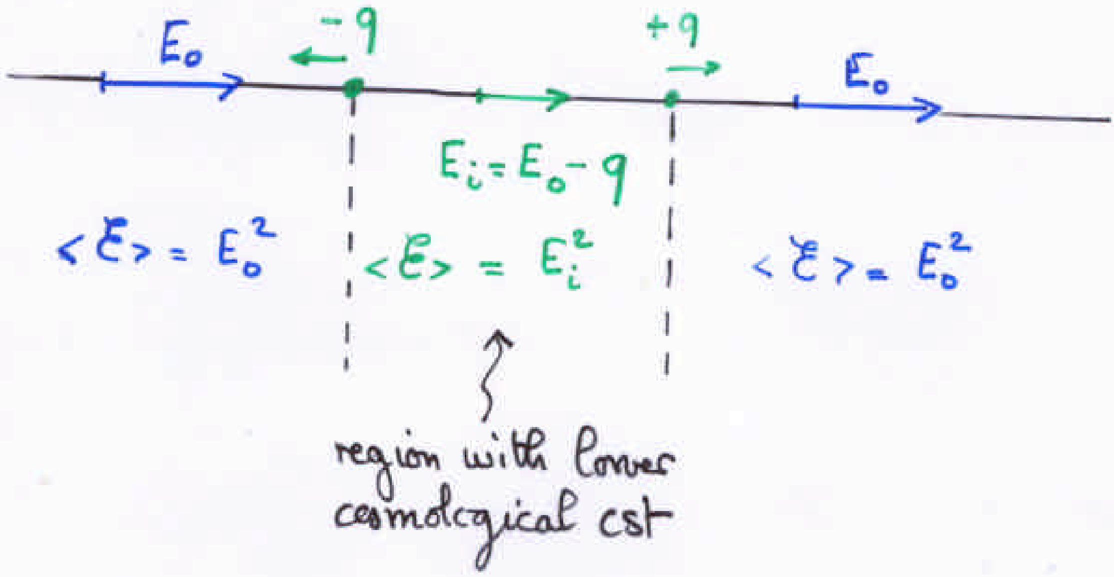
- A (1+1)-dimensional model.



$\langle \mathcal{E} \rangle = E_0^2$

$\sim$ cosmo. cst

Particle creation:



- A (3+1)-dimensional model (Brown-Teitelboim '87)

Consider a 3-form $A_{\mu\nu\rho}$ coupled to a membrane (2-b

$$\mathcal{S} = \int d^4x \sqrt{|g|} \left(\frac{m_P^2}{2} R - m_P^2 \lambda_b - \frac{1}{2 \times 4!} F^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} \right.$$

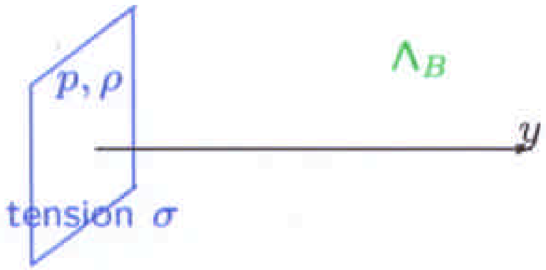
$$+ \frac{1}{3!} \int d^4x \; \partial_\mu \left(\sqrt{|g|} \, F^{\mu\nu\rho\sigma} A_{\nu\rho\sigma} \right) + \dots$$

$$F = dA$$

Brane universes and self-tuning

Arkani-Hamed, Dimopoulos, Kaloper, Sundrum; Kachru, Silverstein

Braneworld set up



Cosmological constant on the brane:

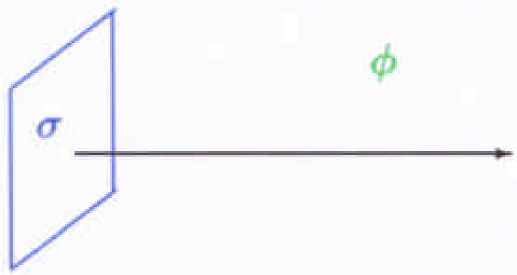
$$\lambda = \frac{1}{2M_5^3}\,\Lambda_B + \frac{1}{12M_5^6}\,\sigma^2$$

M_5 5-dimensional Planck scale

⊖ requires fine-tuning between string tension and bulk vacuum energy (Randall-Sundrum condition)

⊕ decouples the cosmological constant from the brane vacuum energy i.e. the tension σ

SELF TUNING: introduce a bulk scalar field ϕ which couples conformally to matter on the brane



$$\begin{aligned} \mathcal{S} &= \mathcal{S}_{bk} + \mathcal{S}_{br} = M_5^3 \int d^5x \sqrt{|g_5|} \left[\frac{1}{2} R^{(5)} - \frac{3}{2} \partial^m \phi \, \partial_m \phi - 3\mathcal{V}(\phi) \right] \\ & + \int_{\text{brane}} d^4x \sqrt{|g_4|} \, f^2(\phi) \, (-\sigma), \end{aligned}$$

One finds static spatially flat solutions to the classical equations of motion for any value of the brane tension

e.g. $\mathcal{V}(\Phi) = 0$, $f^2(\phi) = Ce^{\mp\phi}$

Warped background $ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$

$$\begin{aligned} A(y) &= \frac{1}{2} \ln \left(1 - \frac{|y|}{y_c} \right) \\ \phi(y) &= \phi_0 \pm \ln \left(1 - \frac{|y|}{y_c} \right) \end{aligned}$$

Naked singularity at $|y| = y_c > 0$

- Fine tuning associated with the presence of the singularity: one may cure the singularity by adding a second brane but the content of the second brane is then fine-tuned.

Forste, Lalak, Lavignac, Nilles

- Include the one-loop corrections to gravity in the bulk

P.B., Charmousis, Davis, Dufaux

$$\begin{aligned} S_{\text{bulk}} &= \frac{M_5^3}{2}\int d^4x\,dy\sqrt{-g}\,\{R-\zeta(\nabla\phi)^2 \\ &\qquad +\alpha e^{-\zeta\phi}\left[\mathcal{L}_{GB}+c_2(\nabla\phi)^4\right]-2\Lambda_B e^{\zeta\phi}\} \\ S_{\text{brane}} &= -\int d^4x\sqrt{|g_4|}\,\sigma e^{\chi(\phi)} \end{aligned}$$

with the Gauss-Bonnet combination

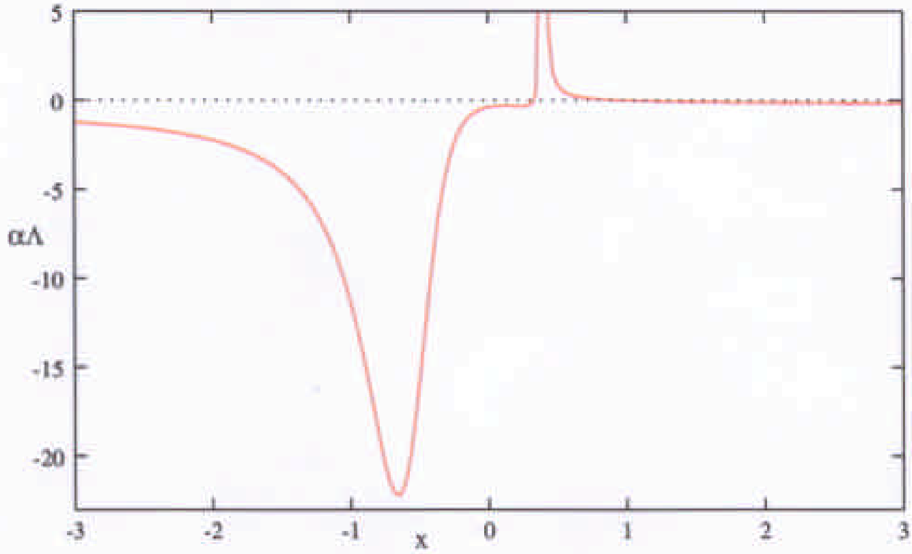
$$\mathcal{L}_{GB}=R^2-4R_{ab}R^{ab}+R^{abcd}R_{abcd}$$

There exist solutions with no naked singularity at finite distance from the brane

cf. singularity-free cosmological solutions of Antoniadis, Rizos, Tamvakis

$$
\begin{aligned}
A(y) &= A_0 + x \ln\left(1 + \frac{|y|}{y_c}\right) \\
\phi(y) &= \phi_0 - \frac{2}{\zeta} \ln\left(1 + \frac{|y|}{y_c}\right)
\end{aligned}
$$

$\alpha\Lambda_B$ as a function of x:



$$
\begin{aligned}
M_P^2 &= M_5^3 \int_0^\infty dy e^{2A(y)} \left(1 + 4\alpha e^{-\zeta\phi(y)}(3A'^2(y) + 2A''(y)]\right) \\
&\propto \left[\frac{y_c}{2x+1}\left(1 + \frac{y}{y_c}\right)^{2x+1}\right]_0^\infty
\end{aligned}
$$

Planck mass finite for $x < -1/2$

• Replace the bulk scalar field by a black hole configuration in the bulk

→ Flat branes in black hole backgrounds

Csaki, Erlich, Grojean

Possible to find flat brane solutions without naked singularities, without enough parameters to avoid fine tuning of the theory

In the case of AdS-Reissner-Nordstrom black hole, enough parameters (M, Q) so that they can be tuned to take into account variations of the brane vacuum energy

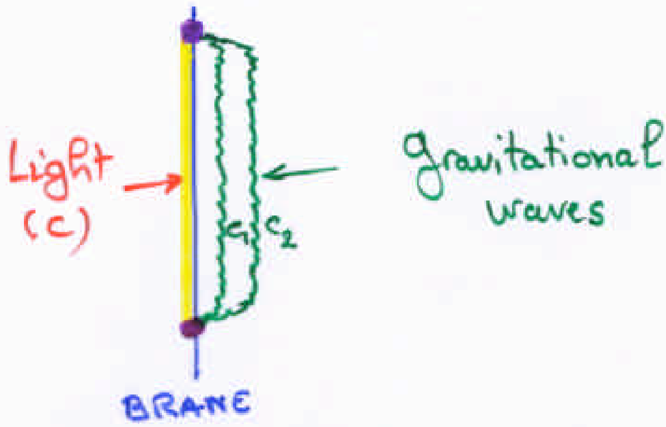
Leads to violations of Lorentz invariance

Kalbermann, Halevi; Chung, Freese; Ishihara; Chung, Kolb, Riotto; Csaki, Erlich, Grojean

Bulk metric:

$$ds^2 = -\ h(y)\ dt^2 + y^2\ d\mathrm{x}^2 + \frac{1}{h(y)} dy^2$$

$h(y)$ describes the black hole configuration



Speed of light (confined to the brane)

$\neq$

Speed of gravity waves

Quintessence

Why not replace λ by a dynamical (i.e. time-dependent) component with negative pressure (to have acceleration $\rho + 3p < 0$)

Hence equation of state:

$$p = w\rho \qquad (-1 \leq) w \leq 0$$

Several candidates:

- network of light, nonintercommuting topological defects

  Vilenkin:Spergel,Pen

$w = -n/3$ $\qquad$ n dimension of the defect

- very light pseudo-Goldstone bosons

  Frieman,Hill,Stebbins,Waga

- quintessence

  Wetterich...

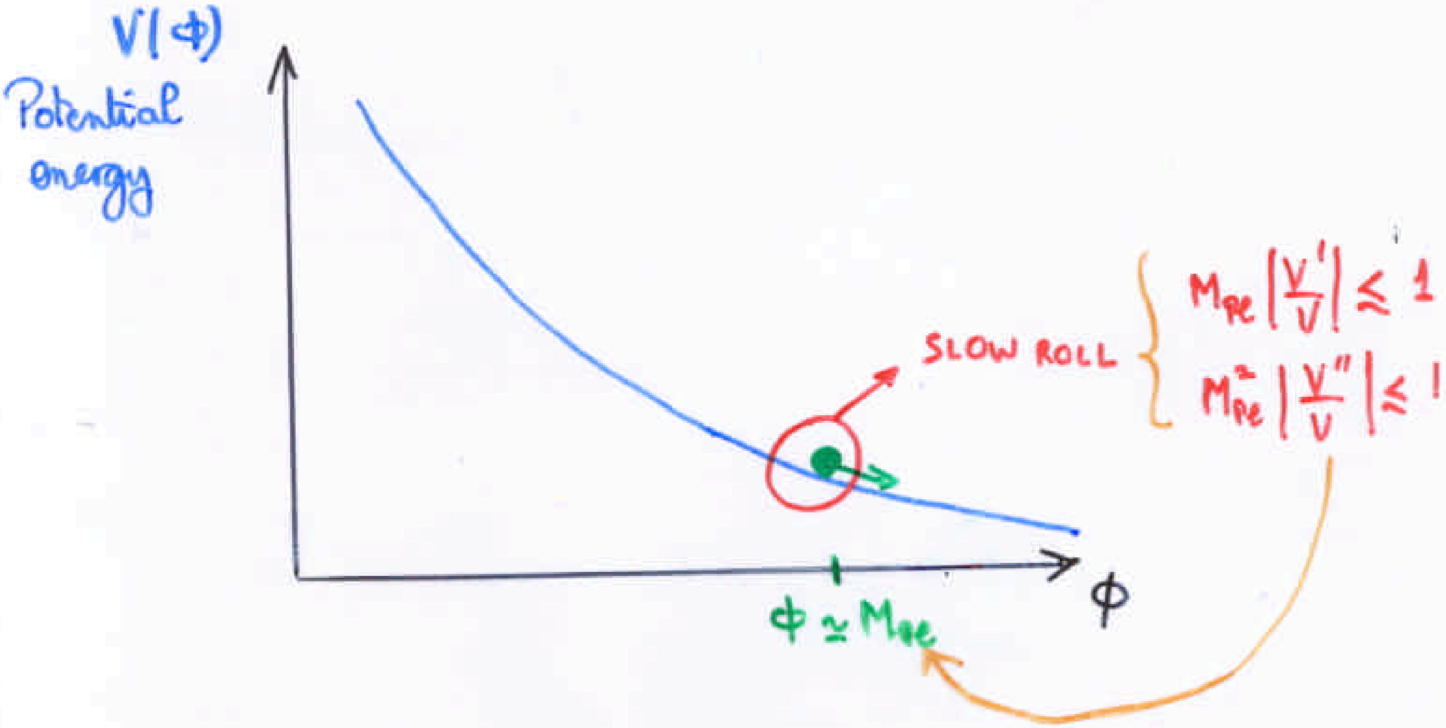
QUINTESSENCE

Scalar field ϕ slowly evolving in a potential which decreases monotonically to 0 as $\phi \to \infty$.

Wetterich

Ratra, Peebles

Caldwell, Dave, Steinhardt



Simplest model (motivated by a dilatation anomaly)

Wetterich

$$\mathcal{S} = \sqrt{g}\left[-\frac{M_P^2}{2}\,R - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V_0 e^{-\alpha\phi/M_P}\right]$$

Scaling solution:

$$\phi \quad = \quad \phi_0 + \frac{2}{\alpha}\ln\ (t/t_0)$$

$$p_\phi = w_\phi\ \rho_\phi \quad \text{with} \quad w_\phi = \frac{\alpha^2}{3} - 1$$

Solution is an attractor if $\alpha^2 < 3(1 + w_M)$

Otherwise, global attractor solution has

$$\Omega_\phi \equiv \frac{\rho_\phi}{\rho_\phi + \rho_M} = \frac{3}{\alpha^2}(1 + w_M)$$

w_ϕ is constrained by observation

Example of models

- k-essence

Armendariz-Picon, Mukhanov, Steinhardt

Addresses the vquestion as to why the quintessence component has started to dominate only recently i.e. during radiation domination

Model uses non-linear kinetic terms for the scalar field:

$$\mathcal{L}_{\mathrm{kin}} = \phi^{-2}\mathcal{K}\left[(\nabla\phi)^2\right]$$

Radiation-dominated era: $\rho_\phi/\rho_{\mathrm{rad}} \sim$ cst

Matter-dominated era: $\rho_\phi < 0$

$\rho_\phi/\rho_{\mathrm{mat}} \nearrow$ until ρ_ϕ dominates.

- Run-away dilaton

Gasperini, Piazza, Veneziano

Quintesssence field is the dilaton in the strong (string) coupling limite $\phi \to \infty$

Saturation assumed in this limit:

$$\alpha_{GUT} \sim C_A + \mathcal{O}(e^{-\phi}) \quad , \quad (M_P/s)^2 \sim N + \mathcal{O}(e^{-\phi})$$

$\ominus$ remains to explain why $\lambda = 0$ in this limit

How to test experimentally quintessence?

→ time i.e. redshift dependence of the equation of state parameter $w_\phi(z)$ over the range $0 < z < 2$

Huterer, Turner...

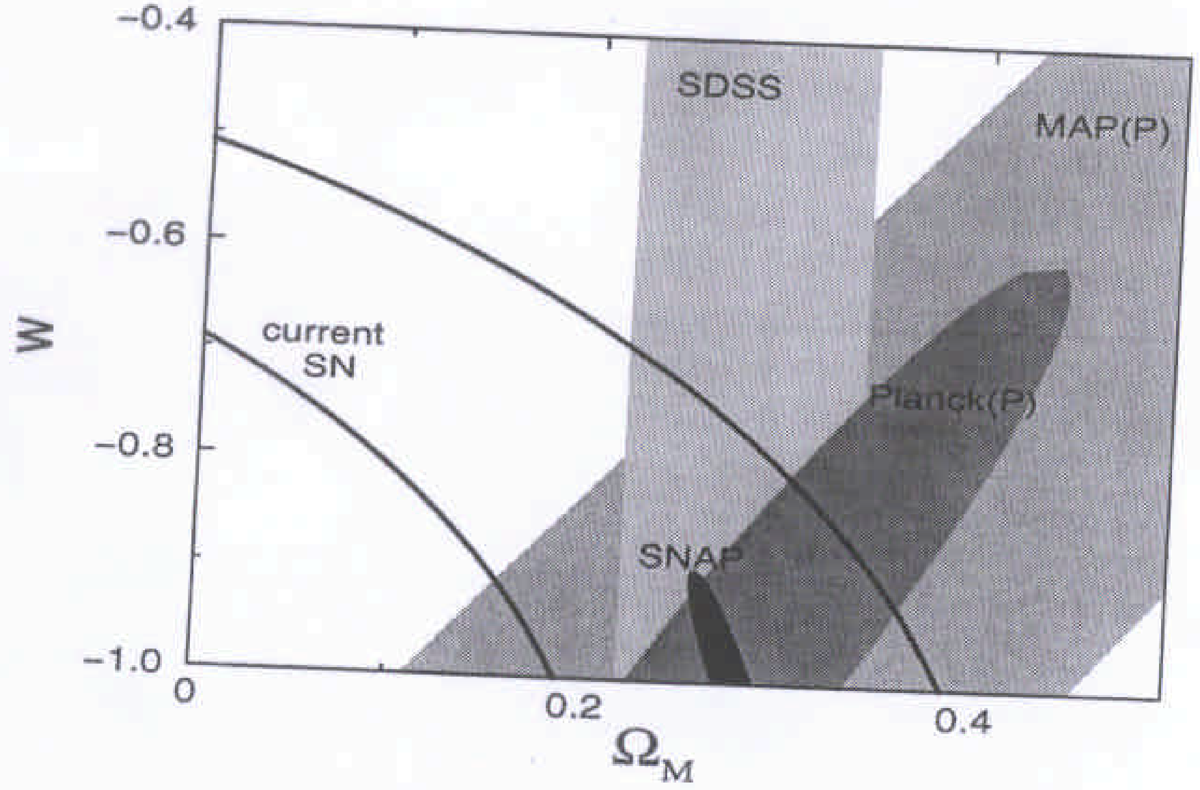
→ time variation of couplings

Wetterich...

→ violations of the equivalence principle

Damour, Piazza, Veneziano

A. SNe Ia and CMB



Huterer, Turner

Conclusion

Increasing list of problems associated with the cosmological constant

$\oplus$

Increasing list of observational data that may discriminate between the theorists' wildest ideas

$\downarrow$

Window open to some physics that we have missed until now?