The cosmological constant

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Abstract

We discuss issues related with the cosmological constant problem in the light of recent observations which tend to indicate that the expansion of the universe is presently undergoing an acceleration. Models of fundamental physics which address these issues are discussed.

1 Introduction

Despite the tremendous successes of fundamental physics, as well of general relativity applied to cosmology, in the XX$^{\mathrm{th}}$ century, we are left with the feeling that the story is not complete yet because of the cosmological constant problem. As we will discuss below, vacuum energy provides a source of cosmological constant and it is not understood what is the mechanism which cancels the vacuum energy through the different phase transitions that the universe undergoes. Not much progress has been made on this issue from the theoretical side. On the other hand, some recent observations tend to give a specific role to vacuum energy. First, the latest results on the cosmic microwave background, in particular the fact that the energy density of the universe seems to coincide with the critical density, tend to favor an inflation era where vacuum energy was driving the evolution of the universe. Second, the observation that the expansion of the universe is presently accelerating is easily understood in a context where vacuum energy, or a more general form of dark energy, makes the majority of the present energy content of the universe.

2 The cosmological constant problem

As is well known, the cosmological constant appears as a constant in Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_{_N} T_{\mu\nu} + \lambda g_{\mu\nu}, \tag{1}$$

where $G_{_N}$ is Newton's constant, $T_{\mu\nu}$ is the energy-momentum tensor and $R_{\mu\nu}$ the Ricci tensor, which is obtained from the Riemann tensor measuring the curvature of spacetime.

The cosmological constant λ is thus of the dimension of an inverse length squared. It was introduced by Einstein [1] in order to build a static universe model, its repulsive effect compensating the gravitational attraction, but, as we will now see, constraints on the expansion of the Universe impose for it a very small upper value.

It is more convenient to work in the specific context of a homogeneous and isotropic Friedmann-Lemaître universe, with a Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \tag{2}$$

where $a(t)$ is the cosmic scale factor, which is time-dependent in an expanding or contracting universe. Implementing energy conservation into the Einstein equations then leads to the Friedmann equation, which gives an expression for the Hubble parameter H measuring the rate of the expansion of the Universe:

$$H^2 \equiv \frac{\dot{a}^2(t)}{a^2(t)} = \frac{1}{3}\left(\lambda + 8\pi G_N \rho\right) - \frac{k}{a^2}, \tag{3}$$

In this equation, we use standard notations: $\dot{a}$ is the time derivative of the cosmic scale factor, $\rho = T^0{}_0$ is the energy density and the term proportional to k is a spatial curvature term. Note that the cosmological constant appears as a constant contribution to the Hubble parameter.

Evaluating each term of the Friedmann equation at present time t_0 allows for a rapid estimation of an upper limit on λ. Indeed, we have for the Hubble constant H_0 i.e. the present value of the Hubble parameter, $H_0 = h_0 \times 100$ km.s^{-1}Mpc^{-1} with h_0 of order one, whereas the present energy density ρ_0 is certainly within one order of magnitude of the critical energy density $\rho_c = 3H_0^2/(8\pi G_N) = h_0^2\ 2.10^{-26}$ kg.m^{-3}; moreover the spatial curvature term certainly does not represent presently a dominant contribution to the expansion of the Universe. Thus, (3) considered at present time implies the following constraint on λ:

$$|\lambda| \leq H_0^2\ . \tag{4}$$

In other words, the length scale $\ell_\Lambda \equiv |\lambda|^{-1/2}$ associated with the cosmological constant must be larger than the Hubble length $\ell_{H_0} \equiv cH_0^{-1} = h_0^{-1}.10^{26}$ m, and thus be a cosmological distance.

This is not a problem as long as one remains classical: ℓ_{H_0} provides a natural cosmological scale for our present Universe. The problem arises when one tries to combine gravity with quantum theory. Indeed, from Newton's constant *and* the Planck constant $\hbar$, one can construct the reduced Planck mass scale $m_P = \sqrt{\hbar c/(8\pi G_N)} = 2.4 \times 10^{18}$ GeV/c^2. The corresponding length scale is the Planck length

$$\ell_P = \frac{\hbar}{m_P c} = 8.1 \times 10^{-35}\ \text{m}\ . \tag{5}$$

The above constraint now reads:

$$\ell_\Lambda \equiv |\lambda|^{-1/2} \geq \ell_{H_0} = \frac{c}{H_0} \sim 10^{60}\ \ell_P. \tag{6}$$

In other words, there are more than sixty orders of magnitude between the scale associated with the cosmological constant and the scale of quantum gravity.

A rather obvious solution is to take $\lambda = 0$. This is as valid a choice as any other in a pure gravity theory. Unfortunately, it is an unnatural one when one introduces any kind of matter. Indeed, set λ to zero but assume that there is a non-vanishing vacuum (*i.e.* ground state) energy: $< T_{\mu\nu} >= \rho_{\rm vac} g_{\mu\nu}$; then the Einstein equations (1) read

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_{_N} T_{\mu\nu} + 8\pi G_{_N} \rho_{\rm vac} g_{\mu\nu} \ . \tag{7}$$

The last term is interpreted as an effective cosmological constant (from now on, we set $\hbar = c = 1$):

$$\lambda_{\rm eff} = 8\pi G_{_N} \rho_{\rm vac} \equiv \frac{\Lambda^4}{m_{_P}^2} \ . \tag{8}$$

Generically, $\rho_{\rm vac}$ receives a non-zero contribution from symmetry breaking: for instance, the scale Λ would be typically of the order of 100 GeV in the case of the electroweak gauge symmetry breaking or 1 TeV in the case of supersymmetry breaking. But the constraint (6) now reads:

$$\Lambda \leq 10^{-30} \ m_{_P} \sim 10^{-3} \ {\rm eV}. \tag{9}$$

It is this very unnatural fine-tuning of parameters (in explicit cases $\rho_{\rm vac}$ and thus Λ are functions of the parameters of the theory) that is referred to as the *cosmological constant problem*, or more accurately the vacuum energy problem.

The most natural reason why vacuum energy would be vanishing is a symmetry argument. Global supersymmetry indeed provides such a rationale. The problem is that, at the same time, supersymmetry predicts equal boson and fermion masses and therefore needs to be broken. The amount of breaking necessary to push the supersymmetric partners high enough not to have been observed yet, is incompatible with the limit (9).

Moreover, in the context of cosmology, we should consider supersymmetry in a gravity context and thus work with its local version, supergravity. The criterion of vanishing vacuum energy is traded for one of vanishing gravitino mass. Local supersymmetry is then absolutely compatible with a non-vanishing vacuum energy, preferably a negative one (although possibly a positive one). This is both a blessing and a problem: supersymmetry may be broken while the cosmological constant remains small, but we have lost our rationale for a vanishing, or very small, cosmological constant and fine-tuning raises again its ugly head.

In the context of brane theories, one may imagine that supersymmetry is only broken on the brane where quarks, leptons and gauge interactions are localized. In the rest of the higher-dimensional spacetime, supersymmetry would be a valid symmetry. It should be noted however that the cosmological constant measured on the brane (the one we observe) receives contributions both from the bulk vacuum energy Λ_B and from the brane tension σ (brane vacuum energy), e.g. in a 5-dimensional set up:

$$\lambda = \frac{1}{2M_5^3} \Lambda_B + \frac{1}{12 M_5^6} \sigma^2 \ , \tag{10}$$

M_5 being the 5-dimensional Planck scale. It remains to explain dynamically why the two contributions cancel.

Over the last years, there has been an increasing number of indications that the Universe is presently undergoing accelerated expansion. This appears to be a strong departure from the standard picture of a matter-dominated Universe. Indeed, the standard equation for the conservation of energy,

$$\dot{\rho} = -3(p+\rho)H, \tag{11}$$

allows to derive from the Friedmann equation (3), written in the case of a universe dominated by a component with energy density ρ and pressure p:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p) \ . \tag{12}$$

Obviously, a matter-dominated ($p \sim 0$) universe is decelerating. One needs instead a component with a negative pressure.

A cosmological constant is associated with a contribution to the energy-momentum tensor as in (7)(8):

$$T^\mu_\nu = -\Lambda^4 \delta^\mu_\nu = (-\rho, p, p, p) \tag{13}$$

The associated equation of motion is therefore $p = -\rho$. It follows from (12) that a cosmological constant tends to accelerate expansion.

Recent cosmological observation is usually expressed in the plane (Ω_M, Ω_Λ). It is certainly remarkable that a very diverse set of data singles out the same region in this parameter space: $\Omega_M \sim 0.2$ to 0.3 and $\Omega_\Lambda \sim 0.7$ to 0.8.

This raises a new problem. Since matter and a cosmological constant evolve very differently, why should they be of the same order at present times? Indeed, for a component of equation of state $p = w\rho$, we may rewrite (11) as

$$\frac{\dot{\rho}}{\rho} = -3\frac{\dot{a}}{a}(1+w) \quad . \tag{14}$$

Thus matter ($p \sim 0$) energy density evolves as a^{-3} whereas a cosmological constant stays constant, as expected. Why should they be presently of similar magnitude? This is known as the *cosmic coincidence problem*. In order to avoid any reference to us (and hence any anthropic interpretation, see below), we may rephrase the problem as follows. Why does the vacuum energy starts to dominate at a time t_Λ (redshift $z_\Lambda \sim 1$) which almost coincides with the epoch t_G (redshift $z_G \sim 3$ to 5) of galaxy formation?

3 Relaxation mechanisms

From the point of view of high energy physics, it is however difficult to imagine a rationale for a pure cosmological constant, especially if it is nonzero but small compared to the typical fundamental scales (electroweak, strong, grand unified or Planck scale). There should be dynamics associated with this form of energy.

For example, in the context of string models, any dimensionful parameter is expressed in terms of the fundamental string scale M_s and of vacuum expectation values of scalar fields. The physics of the cosmological constant would then the physics of the corresponding scalar fields.

Indeed, it is difficult to envisage string theory in the context of a true cosmological constant. The corresponding spacetime is known as de Sitter spacetime and has an event horizon. This is difficult to reconcile with the S-matrix approach of string theory in the context of conformal invariance. More precisely, in the S-matrix approach, states are asymptotically (i.e. at times $t \to \pm\infty$) free and interact only at finite times: the S-matrix element between an incoming set of free states and an outgoing set yields the probability associated with such a transition. In string theory, the states are strings and a diagram representing the tree level interaction of two closed strings has the form of a sphere with 4 tubes attached, corresponding to the 2 incoming and 2 outgoing string states. But conformal invariance imposes that the string world-sheet can be deformed at will: this is difficult to reconcile with the presence of a horizon and the requirement of asymptotically free states.

Steven Weinberg [2] has constrained the possible mechanisms for the relaxation of the cosmological constant by proving the following "no-go" theorem: *it is not possible to obtain a vanishing cosmological constant as a consequence of the equations of motion of a finite number of fields.*

Obviously, Weinberg's no-go theorem relies on a series of assumptions: Lorentz invariance, *finite* number of *constant* fields, possibility of globally redefining these fields...

3.1 Examples

An example of relaxation mechanism is provided by the Brown-Teitelboim mechanism [3] where the quantum creation of closed membranes leads to a reduction of the vacuum energy inside. This is easier to understand on a toy model with a single spatial dimension.

Let us thus consider a line and establish along it a constant electric field $E_0 > 0$: the corresponding (vacuum) energy is $E_0^2/2$. Quantum creation of a pair of $\pm q$-charged particles ($q > 0$) leads to the formation of a region (between the two charges) where the electric field is partially screened to the value $E_0 - q$ and thus the vacuum energy is decreased to the value $(E_0 - q)^2/2$. Quantum creation of pairs in the new region will subsequently decrease the value of the vacuum energy. The process ends in flat space when the electric field reaches the value $E \leq q/2$ because it then becomes insufficient to separate the pairs created.

In a truly three-dimensional universe, the quantum creation of pairs is replaced by the quantum creation of membranes and the one-dimensional electric field is replaced by a tensor field $A_{\mu\nu\rho}$. Tensor fields are typical of string theories. This mechanism has thus been studied in this context. In order to have the right amount of left-over vacuum energy, one needs very small membrane charges. It has been proposed that non-perturbative string effects are responsible of this smallness [4]. Alternatively, such small values appear more naturally in the presence of multiple tensor fields [5].

There are two potential problems with such a relaxation of the cosmological constant. First, since the region of small cosmological constant originates from regions with large vacuum energies, hence exponential expansion, it is virtually empty: matter has to be produced through some mechanism yet to be specified. The second problem has to do with the multiplicity of regions with different vacuum energies: why should we be in the region with the smallest value? Such questions are crying for an anthropic type of answer:

some regions of spacetime are preferred because they allow the existence of observers.

More generally, the anthropic principle approach can be sketched as follows. We consider regions of spacetime with different values of t_G (time of galaxy formation) and t_Λ, the time when the cosmological constant starts to dominate i.e. when the Universe enters a de Sitter phase of exponential expansion. Clearly galaxy formation must precede this phase otherwise no observer (similar to us) would be able to witness it. Thus $t_G \leq t_\Lambda$. On the other hand, regions with $t_\Lambda \gg t_G$ have not yet undergone any de Sitter phase of reacceleration and are thus "phase-space suppressed" compared with regions with $t_\Lambda \sim t_G$. Hence the regions favoured have $t_\Lambda \gtrsim t_G$ and thus $\rho_\Lambda \sim \rho_M$.

3.2 Brane universes and self-tuning

In a braneworld set up where matter is localized on a 3-brane (i.e. a surface with 3 spatial dimensions) plunged into a 5-dimensional spacetime, we have seen in (10) that the cosmological constant receives contributions both from the bulk vacuum energy Λ_B and from the brane vacuum energy or tension σ. Requiring a vanishing (or very small) cosmological constant requires a severe fine-tuning between the string tension and the bulk vacuum energy (known as the Randall-Sundrum [6] condition).

In the self-tuning scenario [7, 8], one introduces a bulk scalar field ϕ whose conformal coupling to the matter on the brane induces a relaxation mechanism which ensures Minkowski spacetime irrespective of the brane vacuum energy σ.

More precisely, one starts with the 5-dimensional action:

$$\begin{aligned}\mathcal{S} &= \mathcal{S}_{bk} + \mathcal{S}_{br} = M_5^3 \int d^5x \sqrt{|g_5|} \left[\frac{1}{2} R^{(5)} - \frac{3}{2} \partial^m \phi \, \partial_m \phi - 3\mathcal{V}(\phi) \right] \\ &\quad + \int_{\text{brane}} d^4x \sqrt{|g_4|} \, f^2(\phi) \, (-\sigma) \quad , \qquad (15)\end{aligned}$$

and one looks for static spatially flat solutions to the classical equations of motion, valid for any value of the brane tension. For example, in the case of vanishing potential $\mathcal{V}(\Phi) = 0$ and conformal coupling of the form $f^2(\phi) = Ce^{\mp\phi}$, one finds as solution a warped 5-dimensional metric

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \quad , \qquad (16)$$

$$A(y) = \frac{1}{2} \ln \left(1 - \frac{|y|}{y_c} \right) \quad , \quad \phi(y) = \phi_0 \pm \ln \left(1 - \frac{|y|}{y_c} \right) \quad . \qquad (17)$$

We note the presence of a naked singularity at $|y| = y_c > 0$.

The presence of this naked singularity has been associated with some hidden fine tuning associated with the presence of the singularity [9]. Indeed, one may cure the singularity by adding a second brane but the content of the second brane is then fine-tuned.

Alternatively, one may include the one-loop corrections to gravity in the bulk and see whether this smooths the singularity out [10]. The starting point is now the action:

$$\begin{aligned}S_{\text{bulk}} &= \frac{M_5^3}{2} \int d^4x \; dy \sqrt{-g} \left\{ R - \zeta (\nabla\phi)^2 + \alpha e^{-\zeta\phi} \left[\mathcal{L}_{GB} + c_2 (\nabla\phi)^4 \right] - 2\Lambda_B e^{\zeta\phi} \right\} \\ S_{\text{brane}} &= - \int d^4x \sqrt{|g_4|} \, \sigma e^{\chi(\phi)} \qquad (18)\end{aligned}$$

with the Gauss-Bonnet combination $\mathcal{L}_{GB} = R^2 - 4R_{ab}R^{ab} + R^{abcd}R_{abcd}$. There indeed exist solutions with no naked singularity at finite distance from the brane:

$$A(y) = A_0 + x \ln\left(1 + \frac{|y|}{y_c}\right) \quad , \quad \phi(y) = \phi_0 - \frac{2}{\zeta}\ln\left(1 + \frac{|y|}{y_c} \quad , \right) \tag{19}$$

where x is a number which is a given function of $\alpha\Lambda_B$. Gravity is localized on the brane for $x < -1/2$ as can be checked by computing the Planck mass of the effective 4-dimensional theory. However, one finds that an unwanted fine tuning among the parameters reappears. Hence the presence/absence of fine tuning seems to be correlated with the absence/presence of naked singularities in the bulk.

Another possibility is to try to hide the singularity behind a horizon, or in other words to replace the bulk scalar field by a black hole configuration in the bulk. Indeed, in a (bulk) black hole background, it is possible to find flat brane solutions without naked singularities, with enough parameters to avoid fine tuning of the theory [11]. For example, a charged (AdS-Reissner-Nordstrom) black hole has a mass and a charge: this gives enough parameters so that they can be tuned to take into account variations of the brane vacuum energy.

4 Dark energy

In this subsection, we will take a slightly different route. We assume that some unknown mechanism relaxes the vacuum energy to zero or to a very small value. We then introduce some new dynamical component which, in order to account for the present observations has negative pressure. and thus an equation of state[1]:

$$p = w\rho, \quad w < 0. \tag{20}$$

Experimental data may constrain such a dynamical component, referred to in the literature as dark energy, just as it did with the cosmological constant. For example, in a spatially flat Universe with only matter and an unknown component X with equation of state $p_X = w_X\rho_X$, one obtains from (12) with $\rho = \rho_M + \rho_X$, $p = w_X\rho_X$ the following form for the deceleration parameter at present time t_0

$$q_0 \equiv -\left.\frac{\ddot{a}a}{\dot{a}^2}\right|_{t_0} = \frac{\Omega_M}{2} + (1 + 3w_X)\frac{\Omega_X}{2}, \tag{21}$$

where $\Omega_X = \rho_X/\rho_c$. Supernovae results give a constraint on the parameter w_X.

A particularly interesting candidate in the context of fundamental theories is a scalar[2] field ϕ slowly evolving in its potential $V(\phi)$. Indeed, the corresponding energy density and pressure are, for a minimally coupled scalar field,

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad , \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad , \quad w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \quad . \tag{22}$$

[1]We recall that non-relativistic matter (dust) has an equation of state $p \sim 0$ whereas $p = \rho/3$ corresponds to radiation.

[2]A vector field or any field which is not a Lorentz scalar must have settled down to a vanishing value. Otherwise, Lorentz invariance would be spontaneously broken.

If the kinetic energy is subdominant ($\dot{\phi}^2/2 \ll V(\phi)$), we clearly obtain $-1 \leq w_\phi \leq 0$.

We will see below that the scalar field must be extremely light. We therefore have two possible situations:

- a scalar potential slowly decreasing to zero as ϕ goes to infinity [12, 13, 14]. This is often referred to as *quintessence* or runaway quintessence.
- a very light field (pseudo-Goldstone boson) which is presently relaxing to its vacuum state [15].

In both cases one is relaxing to a position where the vacuum energy is zero. This is associated with our assumption that some unknown mechanism wipes the cosmological constant out. We discuss the two cases in turn.

Runaway quintessence

A runaway potential is frequently present in models where supersymmetry is dynamically broken. We have seen that supersymmetric theories are characterized by a scalar potential with many flat directions, *i.e.* directions ϕ in field space for which the potential vanishes. The corresponding degeneracy is lifted through dynamical supersymmetry breaking. In some instances (dilaton or compactification radius), the field expectation value $< \phi >$ actually provides the value of the strong interaction coupling. Then at infinite ϕ value, the coupling effectively goes to zero together with the supersymmetry breaking effects and the flat direction is restored: the potential decreases monotonically to zero as ϕ goes to infinity.

Let us take the example of supersymmetry breaking by gaugino condensation in effective superstring theories. The value g_0 of the gauge coupling at the string scale M_S is provided by the vacuum expectation value of the dilaton field s (taken to be dimensionless by dividing by m_P) present among the massless string modes: $g_0^2 =< s >^{-1}$. If the gauge group has a one-loop beta function coefficient $b > 0$, then the running gauge coupling becomes strong at the scale

$$\Lambda \sim M_S e^{-8\pi^2/(bg_0^2)} = M_S e^{-8\pi^2 s/b} \ . \tag{23}$$

At this scale, the gaugino fields are expected to condense. Through dimensional analysis, the gaugino condensate $< \bar{\lambda}\lambda >$ is expected to be of order Λ^3. Terms quadratic in the gaugino fields thus yield in the effective theory below condensation scale a potential for the dilaton:

$$V \sim \left|< \bar{\lambda}\lambda >\right|^2 \propto e^{-48\pi^2 s/b}. \tag{24}$$

The s-dependence of the potential is of course more complicated and one usually looks for stable minima with vanishing cosmological constant. But the behavior (23) is characteristic of the large s region and provides a potential slopping down to zero at infinity as required in the quintessence approach. A similar behavior is observed for moduli fields whose *vev* describes the radius of the compact manifolds which appear from the compactification from 10 or 11 dimensions to 4 in superstring theories.

On general grounds, one considers the following action

$$\mathcal{S} = \int d^4x \sqrt{g} \left[-\frac{m_P^2}{2} R + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) \right], \tag{25}$$

which describes a real scalar field ϕ minimally coupled to gravity with self-interactions of which are described by the potential $V(\phi)$.

The energy density and pressure stored in the scalar field are given by (22). One assumes that the background (matter and radiation) energy density ρ_B and pressure p_B obey a standard equation of state

$$p_B = w_B \rho_B. \tag{26}$$

If one neglects the spatial curvature ($k \sim 0$), the equation of motion for ϕ simply reads

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi} \quad \text{or} \quad \dot{\rho}_\phi = -3H\dot{\phi}^2 \ , \tag{27}$$

with

$$H^2 = \frac{1}{3m_{P}^2}(\rho_B + \rho_\phi) \ . \tag{28}$$

We are looking for *scaling solutions i.e.* solutions where the ϕ energy density scales as a power of the cosmic scale factor: $\rho_\phi \propto a^{-3(1+w_\phi)}$ as one obtains from (27). If one can neglect the background energy ρ_B, then (28) yields a simple differential equation for $a(t)$ which is solved as:

$$a \propto t^{2/[3(1+w_\phi)]}. \tag{29}$$

One obtains for a potential $V(\phi) = V_0 e^{-\lambda\phi/m_P}$, where V_0 is a positive constant,

$$\phi = \phi_0 + \frac{2}{\lambda} m_{P} \ \ln(t/t_0) \quad \text{and} \quad w_\phi = \frac{\lambda^2}{3} - 1 \ . \tag{30}$$

It is clear from (30) that, for λ sufficiently small, the field ϕ can play the role of quintessence. We note that, even if we started with a small value ϕ_0, ϕ reaches a value of order m_{P}.

But the successes of the standard big-bang scenario indicate that clearly ρ_ϕ cannot have always dominated: it must have emerged from the background energy density ρ_B. If $\lambda^2 > 3(1 + w_B)$, which seems to be favored in the context of strings, the global attractor turns out to be a scaling solution [12, 16, 17] with the following properties:

$$\Omega_\phi \equiv \frac{\rho_\phi}{\rho_\phi + \rho_B} = \frac{3}{\lambda^2}(1 + w_B) \quad , \quad w_\phi = w_B \quad . \tag{31}$$

The second equation clearly indicates that this does not correspond to a dark energy solution (20).

Ways to obtain a quintessence component have been proposed however. Let us sketch some of them in turn.

One is the notion of *tracker field* [18]. This idea also rests on the existence of scaling solutions of the equations of motion which play the role of late time attractors, as illustrated above. An example is provided by a scalar field described by the action (25) with a potential

$$V(\phi) = \lambda \ \frac{\Lambda^{4+\alpha}}{\phi^\alpha} \tag{32}$$

with $\alpha > 0$. In the case where the background density dominates, one finds an attractor scaling solution [13, 19] $\phi \propto a^{3(1+w_B)/(2+\alpha)}$, $\rho_\phi \propto a^{-3\alpha(1+w_B)/(2+\alpha)}$. Thus ρ_ϕ decreases at a slower rate than the background density ($\rho_B \propto a^{-3(1+w_B)}$) and tracks it until it becomes of the same order at a given value a_Q. We thus have:

$$\frac{\phi}{m_P} \sim \left(\frac{a}{a_Q}\right)^{3(1+w_B)/(2+\alpha)} \quad , \quad \frac{\rho_\phi}{\rho_B} \sim \left(\frac{a}{a_Q}\right)^{6(1+w_B)/(2+\alpha)} \quad . \tag{33}$$

One finds

$$w_\phi = -1 + \frac{\alpha(1+w_B)}{2+\alpha}. \tag{34}$$

Shortly after ϕ has reached for $a = a_Q$ a value of order m_P, it satisfies the standard slow roll conditions ($m_P|V'/V| \ll 1$, $m_P^2|V''/V| \ll 1$) and therefore (34) provides a good approximation to the present value of w_ϕ. Thus, at the end of the matter-dominated era, this field may provide the quintessence component that we are looking for.

Two features are interesting in this respect. One is that this scaling solution is reached for rather general initial conditions, *i.e.* whether ρ_ϕ starts of the same order or much smaller than the background energy density [18]. Regarding the cosmic coincidence problem, it can be rephrased here as follows (since ϕ is of order m_P in this scenario): why is $V(m_P)$ of the order of the critical energy density ρ_c? It is thus the scale Λ which determines the time when the scalar field starts to emerge and the universe expansion reaccelerates. Indeed, using (33), the constraint reads:

$$\Lambda \sim \left(H_0^2 m_P^{2+\alpha}\right)^{1/(4+\alpha)} . \tag{35}$$

We may note that this gives for $\alpha = 2$, $\Lambda \sim 10$ MeV, not such an atypical scale for high energy physics.

A model [20] has been proposed which goes one step further: the dynamical component, a scalar field, is called k-essence and the model is based on the property observed in string models that scalar kinetic terms may have a non-trivial structure. Tracking occurs only in the radiation-dominated era; a new attractor solution where quintessence acts as a cosmological constant is activated by the onset of matter domination.

Models of dynamical supersymmetry breaking easily provide a model of the tracker field type just discussed [21]. Let us consider supersymmetric QCD with gauge group $SU(N_c)$ and $N_f < N_c$ flavors, *i.e.* N_f quarks Q_g (resp. antiquarks $\bar{Q}^g$), $g = 1 \cdots N_f$, in the fundamental $\mathbf{N_c}$ (resp. anti-fundamental $\bar{\mathbf{N}}_\mathbf{c}$) of $SU(N_c)$. At the scale of dynamical symmetry breaking Λ where the gauge coupling becomes strong, boundstates of the meson type form: ${M_f}^g = Q_f \bar{Q}^g$. The dynamics is described by a superpotential which can be computed non-perturbatively using standard methods:

$$W = (N_c - N_f)\frac{\Lambda^{(3N_c - N_f)/(N_c - N_f)}}{(\det M)^{1/(N_c - N_f)}} . \tag{36}$$

Such a superpotential has been used in the past but with the addition of a mass or interaction term (*i.e.* a positive power of M) in order to stabilize the condensate. One does not wish to do that here if M is to be interpreted as a runaway quintessence component. For illustration purpose, let us consider a condensate diagonal in flavor space: ${M_f}^g \equiv \phi^2 \delta_f^g$. Then the potential for ϕ has the form (32), with $\alpha = 2(N_c + N_f)/(N_c - N_f)$. Thus,

$$w_\phi = -1 + \frac{N_c + N_f}{2N_c}(1 + w_B), \tag{37}$$

which clearly indicates that the meson condensate is a potential candidate for a quintessence component.

One may note that, in the tracker model, when ϕ reaches values of order $m_{_P}$, it satisfies the slow roll conditions of an inflation model. The last possibility that I will discuss goes in this direction one step further. It is known under several names: deflation [22], kination [23], quintessential inflation [24]. It is based on the remark that, if a field ϕ is to provide a dynamical cosmological constant under the form of quintessence, it is a good candidate to account for an inflationary era where the evolution is dominated by the vacuum energy. In other words, are the quintessence component and the inflaton the same unique field?

In this kind of scenario, inflation (where the energy density of the Universe is dominated by the ϕ field potential energy) is followed by reheating where matter-radiation is created by gravitational coupling during an era where the evolution is driven by the ϕ field kinetic energy (which decreases as a^{-6}). Since matter-radiation energy density is decreasing more slowly, this turns into a radiation-dominated era until the ϕ energy density eventually emerges as in the quintessence scenarios described above.

Quintessential problems

However appealing, the quintessence idea is difficult to implement in the context of realistic models [25, 26]. The main problem lies in the fact that the quintessence field must be extremely weakly coupled to ordinary matter. This problem can take several forms :

• we have assumed until now that the quintessence potential monotonically decreases to zero at infinity. In realistic cases, this is difficult to achieve because the couplings of the field to ordinary matter generate higher order corrections that are increasing with larger field values, unless forbidden by a symmetry argument. For example, in the case of the potential (32), the generation of a correction term $\lambda_d\, m_{_P}^{4-d}\phi^d$ puts in jeopardy the slowroll constraints on the quintessence field, unless very stringent constraints are imposed on the coupling λ_d. But one typically expects from supersymmetry breaking $\lambda_d \sim M_{SB}^4/m_{_P}^4$ where M_{SB} is the supersymmetry breaking scale.

Similarly, because the *vev* of ϕ is of order $m_{_P}$, one must take into account the full supergravity corrections. One may then argue [27] that this could put in jeopardy the positive definiteness of the scalar potential, a key property of the quintessence potential. This may point towards models where $< W >= 0$ (but not its derivatives) or to no-scale type models.

• the quintessence field must be very light. If we return to our example of supersymmetric QCD in (32), $V''(m_P)$ provides an order of magnitude for the mass-squared of the quintessence component:

$$m_\phi \sim \Lambda \left(\frac{\Lambda}{m_P}\right)^{1+\alpha/2} \sim H_0 \sim 10^{-33} \text{ eV}. \tag{38}$$

using (35). This might argue for a pseudo-Goldstone boson nature of the scalar field that plays the rôle of quintessence. This field must in any case be very weakly coupled to matter; otherwise its exchange would generate observable long range forces. Eötvös-type experiments put very severe constraints on such couplings.

Again, for the case of supersymmetric QCD, higher order corrections to the Kähler potential of the type

$$\kappa(\phi_i, \phi_j^\dagger)\left[\beta_{ij}\left(\frac{Q^\dagger Q}{m_P^2}\right) + \bar{\beta}_{ij}\left(\frac{\bar{Q}\bar{Q}^\dagger}{m_P^2}\right)\right] \tag{39}$$

will generate couplings of order 1 to the standard matter fields ϕ_i, $\phi_j^\dagger$ since $< Q >$ is of order m_P [28].

• it is difficult to find a symmetry that would prevent any coupling of the form $\beta(\phi/m_P)^n F^{\mu\nu} F_{\mu\nu}$ to the gauge field kinetic term. Since the quintessence behavior is associated with time-dependent values of the field of order m_P, this would generate, in the absence of fine tuning, corrections of order one to the gauge coupling. But the time dependence of the fine structure constant for example is very strongly constrained: $|\dot{\alpha}/\alpha| < 5 \times 10^{-17}\text{yr}^{-1}$. This yields a limit [25]:

$$|\beta| \leq 10^{-6}\frac{m_P H_0}{< \dot{\phi} >}, \tag{40}$$

where $< \dot{\phi} >$ is the average over the last 2×10^9 years.

Pseudo-Goldstone boson

There exists a class of models [15] very close in spirit to the case of runaway quintessence: they correspond to a situation where a scalar field has not yet reached its stable ground-state and is still evolving in its potential.

More specifically, let us consider a potential of the form:

$$V(\phi) = M^4 v\left(\frac{\phi}{f}\right) , \tag{41}$$

where M is the overall scale, f is the vacuum expectation value $< \phi >$ and the function v is expected to have coefficients of order one. If we want the potential energy of the field (assumed to be close to its *vev* f) to give a substantial fraction of the energy density at present time, we must set

$$M^4 \sim \rho_c \sim H_0^2 m_P^2 . \tag{42}$$

However, requiring that the evolution of the field ϕ around its minimum has been over-damped by the expansion of the Universe until recently imposes

$$m_\phi^2 = \frac{1}{2}V''(f) \sim \frac{M^4}{f^2} \leq H_0^2. \tag{43}$$

Let us note that this is again one of the slowroll conditions familiar to the inflation scenarios.

From (42) and (43), we conclude that f is of order m_P (as the value of the field ϕ in runaway quintessence) and that $M \sim 10^{-3}$ eV (not surprisingly, this is the scale Λ typical of the cosmological constant, see (9)). As we have seen, the field ϕ must be very light: $m_\phi \sim h_0 \times 10^{-60} m_P \sim h_0 \times 10^{-33}$ eV. Such a small value is only natural in the context of an approximate symmetry: the field ϕ is then a pseudo-Goldstone boson. A typical example of such a field is provided by the string axion field. In this case, the potential simply reads:

$$V(\phi) = M^4 \left[1 + \cos(\phi/f)\right] . \tag{44}$$

Coupling dark energy with dark matter?

Even if quintessence has to be very weakly coupled with ordinary matter, the constraint may not be so strong with dark matter. Models of such coupled quintessence have been proposed [29]. They lead to violations of the equivalence principle and they predict an early acceleration (at $z > 1$).

All the preceding shows that there is extreme fine tuning in the couplings of the quintessence field to matter, unless they are forbidden by some symmetry. This is somewhat reminiscent of the fine tuning associated with the cosmological constant. In fact, *the quintessence solution does not claim to solve the cosmological constant (vacuum energy) problem* described above. If we take the example of a supersymmetric theory, the dynamical cosmological constant provided by the quintessence component clearly does not provide enough amount of supersymmetry breaking to account for the mass difference between scalars (sfermions) and fermions (quarks and leptons): at least 100 GeV. There must be other sources of supersymmetry breaking and one must fine tune the parameters of the theory in order not to generate a vacuum energy that would completely drown ρ_ϕ.

However, the quintessence solution shows that, once this fundamental problem is solved, one can find explicit fundamental models that effectively provide the small amount of cosmological constant that seems required by experimental data.

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